Stat 451

10-11-16

Some properties of μ and σ^2:

(1)

Recall $\mu = E[X] = \sum_{\text{all } x} x\, p(x)$ and

$$\sigma^2 = E[(X-\mu)^2] = E[X^2] - \mu^2$$

$$E[aX+b] = \sum_{\text{all } x} (ax+b)\, p(x)$$

$$= \sum_{\text{all } x} ax\, p(x) + \sum_{\text{all } x} b\, p(x)$$

$$= aE[X] + b$$

(2)

$$\mathrm{Var}[aX+b] = E[(aX+b)^2] - (E[aX+b])^2$$

$$= E[a^2X^2 + 2abX + b^2] - (aE[X] + b)^2$$

$$= a^2E[X^2] + 2abE[X] + b^2$$

$$- [a^2(E[X])^2 + 2abE[X] + b^2]$$

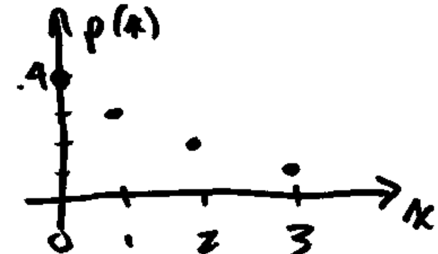
$$= a^2[E[X^2] - (E[X])^2]$$

$$= a^2\,\mathrm{Var}[X]$$

3

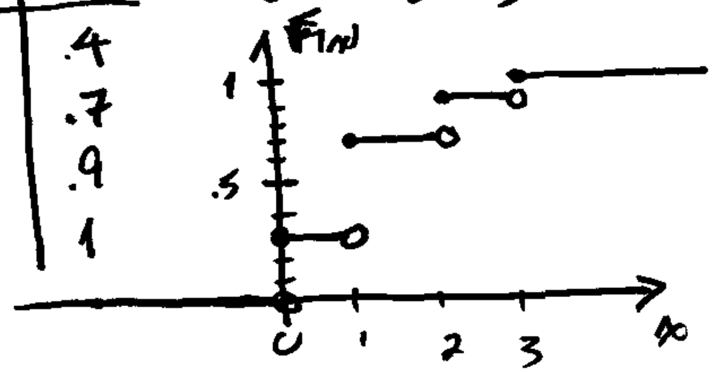
Defn: The <u>cumulative</u> <u>distribution</u> <u>function</u> (cdf) for a random variable X is

$$F(x) = P(X \le x)$$



Example:

x	$p(x)$	$F(x)$
0	.4	.4
1	.3	.7
2	.2	.9
3	.1	1



4

Note: The cdf is always non-decreasing + right-continuous

The <u>Bernoulli</u> <u>distribution</u> (p)

$$X = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p = q \end{cases}$$

x	$p(x)$
0	q
1	p
	1

$$\mu = E[X] = \sum_{\text{all } x} x\, p(x) = 0 \cdot q + 1 \cdot p = p$$

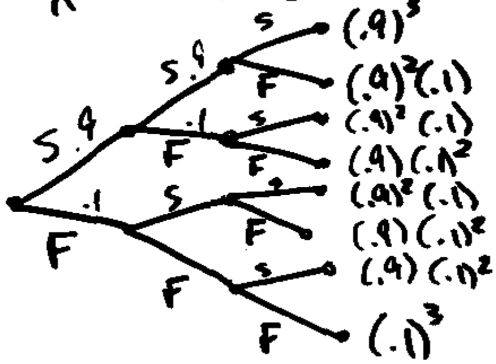
$$\sigma^2 = E[X^2] - \mu^2 = 0^2 \cdot q + 1^2 \cdot p - p^2 = p - p^2$$
$$= p(1-p) = pq$$

(5)

The Binomial distribution (n, p)

Consider a system with 3 independent components. Suppose that each component has a .1 chance of failure (or .9 chance of success)

Let X = # successes



x	$p(x)$
0	$(.1)^3$
1	$3(.9)(.1)^2$
2	$3(.9)^2(.1)$
3	$(.9)^3$
	1

(6)

In general, a binomial experiment has these properties:

1. Run a sequence of n independent trials
2. The success probability on each trial is p
3. X is counting the # of successes

Then X will take on values $0, 1, 2, \ldots, n$

and $p(x) = \binom{n}{x} p^x q^{n-x}$

(7)

2 more properties of μ & σ^2, to be derived later

$$E[X+Y] = E[X] + E[Y]$$

$$V[X+Y] = V[X] + V[Y] + 2\,\mathrm{Cov}(X,Y)$$

and if X & Y are independent, then $\mathrm{Cov}(X,Y) = 0$

If $X \sim \mathrm{Bino}(n,p)$ then $X = X_1 + X_2 + \dots + X_n$
where each $X_i \sim \mathrm{Bern}(p)$, all independent

So $E[X] = np$ and $\mathrm{Var}[X] = npq$

(8)

The Hypergeometric distribution (N, N_1, n)

- Population consists of N items
- N_1 of those items are of a particular type, and the remaining N_2 items are of a different type $(N_1 + N_2 = N)$
- Take a random sample of n of items from the population
- X = # of typed items in the sample.

X takes on values

$$0, 1, 2, \ldots, \min(n, N_1)$$

$$p(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}$$

Example: 100 monitors, 3 defective

take a sample of 10 $\quad X = \#$ defects in sample.

Find $P(X \geq 1) = 1 - P(X=0) = 1 - \dfrac{\binom{3}{0}\binom{97}{10}}{\binom{100}{10}}$

$$1 - \frac{\binom{97}{10}}{\binom{100}{10}} = .273$$