Conditional probability example:

①

Suppose that 10 out of 1000 people are affected by a certain disease.

Suppose that, if a person has the disease, the test results will be positive 97% of the time.
(3% false negatives)

Also suppose that, if the person doesn't have the disease, the test results will be negative 95% of the time.
(5% false positive)

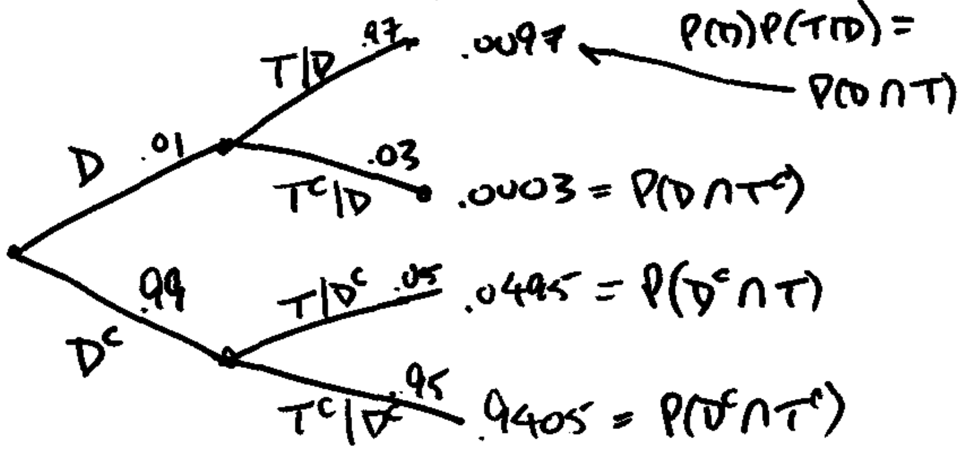
②

D: disease $\quad P(D) = \frac{10}{1000} = .01$

T: test results are positive

$$P(T \mid D) = .97$$

$$P(T^c \mid D^c) = .95$$



(3)

true positives

false negatives

Bayes' Rule

	T	T^c	
D	.0097	.0003	.01
D^c	.0495	.9405	.99
	.0592	.9408	1

false positives

true negatives

Question: If a person tests positive, what is the probability that they have the disease?

$$P(D \mid T) = \frac{P(D \cap T)}{P(T)} = \frac{.0097}{.0592} = .164$$

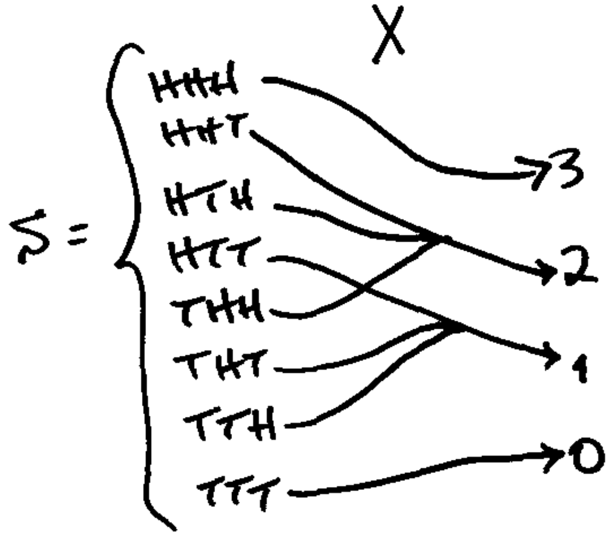
(4)

Random variables

Defn: A random variable is a variable whose value is determined by the outcome of an experiment.

Actually, a random variable is a function whose domain is S and whose range is a subset of $\mathbb{R}$

Example: Flip 3 coins. Let $X = \#$ heads 5



If the range of X is finite or countably infinite, then X is called a discrete random variable.

If the range of X is an interval, then X is called a continuous random variable.

Discrete Random variables 6

Define a probability (mass) function

$$p(x) = P(X = x)$$

Properties: $0 \le p(x) \le 1$

$$\sum_{\text{all } x} p(x) = 1$$

3 coin example:

x	$p(x)$
0	1/8
1	3/8
2	3/8
3	1/8 /1

$p(0) = P(X=0) = P(TTT)$
$p(1) = P(X=1) = P(HTT, THT, TTH)$

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Example: Roll 2 dice

Let Y = smaller of the two

y	$p(y)$
1	$11/36$
2	$9/36$
3	$7/36$
4	$5/36$
5	$3/36$
6	$1/36$
	1

$$S = \left\{ \begin{matrix} 11 & 12 & \cdots & 16 \\ 21 & 22 & \cdots & 26 \\ \vdots & & & \\ 61 & 62 & \cdots & \underline{66} \end{matrix} \right\}$$

44, 45, 54, 46, 64

55, 56, 65

Note:
$$p(y) = \frac{2(6-y)+1}{36} \quad y = 1,2,3,\ldots,6$$

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Defn: For a discrete random variable X,

the expected value of X is

$$\mu = E(X) = \sum_{\text{all } x} x\,p(x) \quad (1^{st} \text{ moment})$$

"mu"

3-coin example:
$$\begin{aligned} \mu &= \sum x p(x) \\ &= 0 \cdot \tfrac{1}{8} + 1 \cdot \tfrac{3}{8} + 2 \cdot \tfrac{3}{8} + 3 \cdot \tfrac{1}{8} \\ &= \frac{0+3+6+3}{8} = \frac{12}{8} = 1.5 \end{aligned}$$

(9)

Defn: $E(X^2)$ is called the 2nd moment

$$= \sum_{\text{all } x} x^2 p(x)$$

3 coin example:

$$E(X^2) = 0^2 \cdot \frac{1}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{3}{8} + 3^2 \cdot \frac{1}{8}$$

$$= \frac{0+3+12+9}{8} = \frac{24}{8} = 3$$

(10)

Defn: $E[(X-\mu)^2]$ is the variance of X

$$\sigma^2 = Var(X) = \sum_{\text{all } x} (x-\mu)^2 p(x)$$

"sigma squared"

Note: $\sigma^2 = \sum (x-\mu)^2 p(x) = \sum (x^2 - 2\mu x + \mu^2) p(x)$

$$= \sum x^2 p(x) - 2\mu \underbrace{\sum x p(x)}_{\mu} + \mu^2 \underbrace{\sum p(x)}_{1}$$

$$= E(X^2) - \mu^2$$

3 coin example:

$$\sigma^2 = E(X^2) - \mu^2 = 3 - (1.5)^2$$
$$= .75$$

Defn: The standard deviation of X is $\sigma = \sqrt{\sigma^2}$

HW #2 due 10/13

p.72 # 7, 8, 9, 13

p.78 # 4, 6