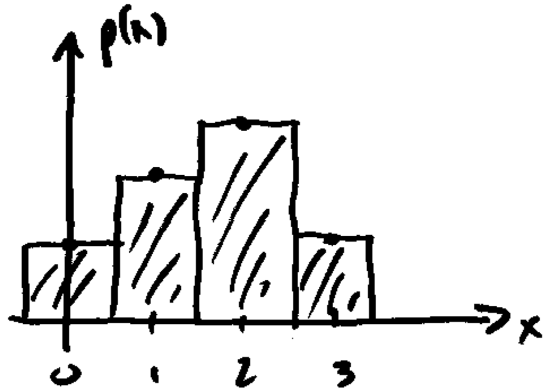
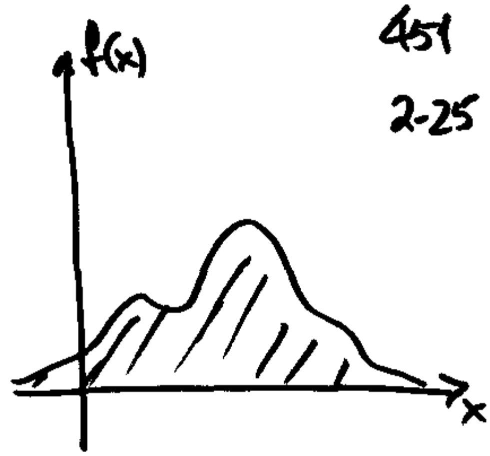
Continuous Distributions

① 451 2-25



Discrete

total area = 1

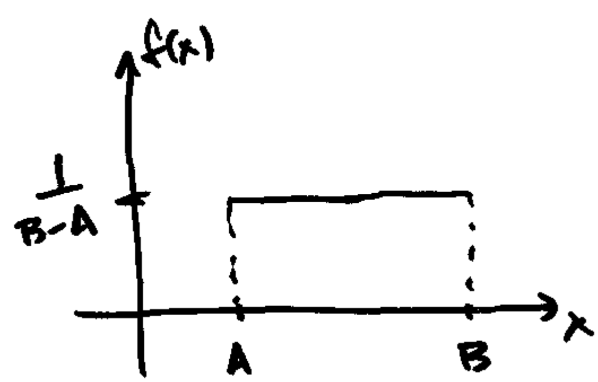


Continuous

total area = 1

②

Continuous Uniform distribution



$$f(x) = \begin{cases} \dfrac{1}{B-A} & A < x < B \\ 0 & \text{elsewhere} \end{cases}$$

Find $\mu = E[X] = \displaystyle\int_{\text{all } x} x\, f(x)\, dx = \int_A^B x \frac{1}{B-A}\, dx$

$$\frac{1}{B-A} \left. \frac{x^2}{2} \right|_A^B = \frac{1}{B-A}\left(\frac{B^2}{2} - \frac{A^2}{2}\right) = \frac{1}{2}\frac{1}{\cancel{B-A}}\cancel{(B-A)}(B+A) = \frac{B+A}{2}$$

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$$E[X^2] = \int_A^B x^2 \frac{1}{B-A}\,dx = \frac{1}{B-A}\,\frac{1}{3}x^3\Big|_A^B$$

$$= \frac{1}{B-A}\,\frac{1}{3}\left(B^3 - A^3\right) = \frac{1}{\cancel{B-A}}\,\frac{1}{3}\left(\cancel{B-A}\right)\left(B^2 + AB + A^2\right)$$

$$= \frac{1}{3}\left(B^2 + AB + A^2\right)$$

$$\sigma^2 = E[X^2] - \mu^2 = \frac{1}{3}(B^2 + AB + A^2) - \left(\frac{B+A}{2}\right)^2$$

$$= \frac{4B^2 + 4AB + 4A^2 - \left(3B^2 + 6AB + 3A^2\right)}{12}$$

$$= \frac{B^2 - 2AB + A^2}{12} = \frac{(B-A)^2}{12}$$

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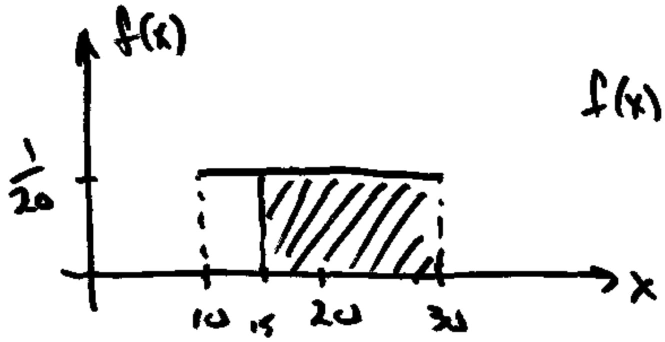
Example: My travel time to work is uniformly distributed between 10 and 30 minutes.

Uniform$(A = 10, B = 30)$



$$f(x) = \begin{cases} \frac{1}{20} & 10 < x < 30 \\ 0 & \text{elsewhere} \end{cases}$$

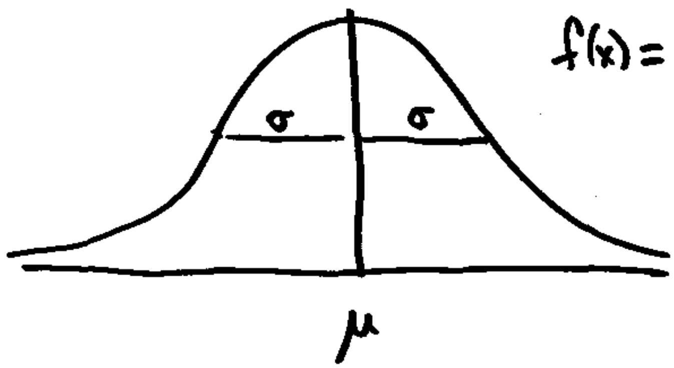
Find the probability that my travel time is more than 15 minutes.

$$\text{Area} = (30-15)\,\frac{1}{20} = .75$$

OR $\quad P(X>15) = \int_{15}^{30} \frac{1}{20}\,dx = \frac{1}{20}x \Big|_{15}^{30}$

$= \frac{1}{20}(30-15) = .75$

The normal distribution



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

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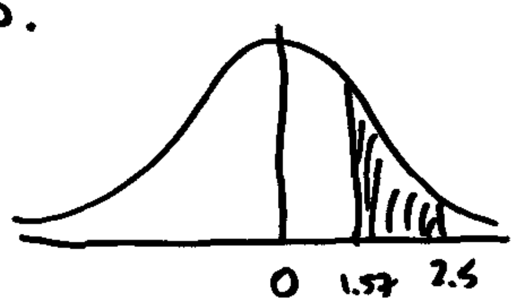
The standard normal distribution
has $\mu = 0$ and $\sigma = 1$

$$f(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}x^2}$$

<u>Example</u>: In the standard normal distribution, find the probability of seeing a value between 1.57 and 2.5.

Normal$(\mu=0, \sigma=1)$



⑦

$$P(1.57 < X < 2.5) = \int_{1.57}^{2.5} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\,dx$$

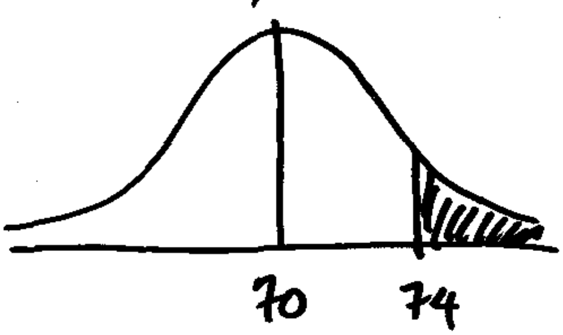
$$= F(2.5) - F(1.57) = .9938 - .9418$$
$$= .052$$

Using Table A.3

<u>Example</u>: We have a set of measurements that are normally distributed with $\mu = 70$ mm and $\sigma = 2$ mm. What percentage of the items are larger than 74 mm?

⑧

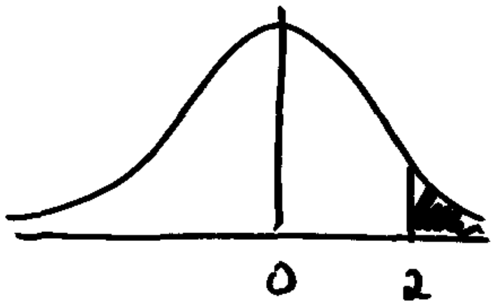
Normal$(\mu = 70, \sigma = 2)$ $\qquad$ $P(X > 74)$



$$P(X > 74) = \int_{74}^{\infty} \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-70}{2}\right)^2} dx$$

Z-score transformation: $\quad z = \dfrac{x - \mu}{\sigma}$

After transformation: $N(\mu = 0, \sigma = 1)$ (9)



$P(Z > 2)$

$= F(\infty) - F(2)$

$= 1 - F(2)$

$= 1 - .9772$

$= .0228 = 2.28\%$

(10)

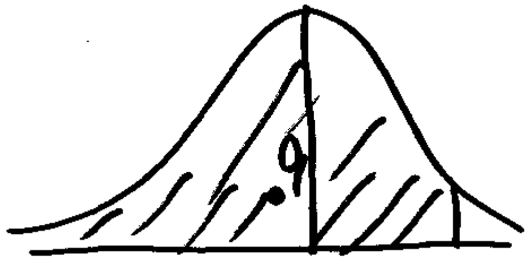
<u>Example</u>: Assume that speeds on I5 are normally distributed with $\mu = 68$ and $\sigma = 4$. Find the 90^{th} percentile of the speeds.

$N(\mu = 68, \sigma = 4)$

$P(X < x) = .9$

Find x.

(11)



$0 \qquad z = 1.28$ using Table A.3

$$Z = \frac{X-\mu}{\sigma} \qquad 1.28 = \frac{X - 68}{4}$$

$$x = 1.28(4) + 68 = 73.12$$

(12)

Use the normal distribution to approximate the binomial

$Binu(n = 20, p = .6)$

$P(10 \le X \le 12)$

$= p(10) + p(11) + p(12)$

$= .4565$

Normal : $P(9.5 < X < 12.5)$ $\qquad \mu = np = 12$

$\qquad\qquad ||\qquad\qquad\qquad\qquad\qquad \sigma = \sqrt{npq}$

$P\left(\frac{9.5-12}{\sqrt{4.8}} < Z < \frac{12.5-12}{\sqrt{4.8}}\right) \qquad = \sqrt{20(.6)(.4)}$

$= \sqrt{4.8}$

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$= P(-1.14 < z < 0.23)$

$= F(0.23) - F(-1.14)$

$= .5910 - .1271 = .4639$