① 451 2-23

p.157

5.32 Hypergeometric$(N=10, n=4, k=3)$

X = # def in sample

a) $P(X=0) = \dfrac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}}$

b) $P(X\le 2) = p(0) + p(1) + p(2)$

$$= \frac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} + \frac{\binom{3}{1}\binom{7}{3}}{\binom{10}{4}} + \frac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}}$$

OR $1 - P(X=3)$

②

p.151

5.16

4 engines

$\text{Bino}(n=4, p=.6)$

$P(X=2) + P(X=3) + P(X=4)$

$\binom{4}{2}(.6)^2(.4)^2$

$+ \binom{4}{3}(.6)^3(.4)$

$+ \binom{4}{4}(.6)^4$

2 engines

$\text{Bino}(n=2, p=.6)$

$P(X=1) + P(X=2)$

$\binom{2}{1}(.6)^1(.4)^1 + \binom{2}{2}(.6)^2$

p.157

5.30 Hyper $(N = 15, n = 3, k = 6)$ (3)

$X =$ # narcotics in sample

$$P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - \frac{\binom{6}{0}\binom{9}{3}}{\binom{15}{3}}$$

$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n$ Let $y = \left(1 + \frac{x}{n}\right)^n$

Then $\ln y = n \ln\left(1 + \frac{x}{n}\right)$

$$= \frac{\ln\left(1 + \frac{x}{n}\right)}{1/n}$$

$$\lim_{n\to\infty} \ln y = \lim_{n\to\infty} \frac{\ln\left(1 + \frac{x}{n}\right)}{1/n} \quad \left(\frac{0}{0}\right)$$ (4)

$$= \lim_{n\to\infty} \frac{\frac{1}{1+\frac{x}{n}} \cdot \left(\frac{-x}{n^2}\right)}{-\frac{1}{n^2}}$$ by L'Hopital

$$= \lim_{n\to\infty} \frac{x}{1 + \frac{x}{n}} = x$$

So $\ln y \longrightarrow x$ as $n \to \infty$

Therefore $y \to e^x$ as $n \to \infty$

(5)

Consider the binomial distribution

$$p(x) = \binom{n}{x} p^x q^{n-x}$$

Take the limit as $n \to \infty$ and hold np constant.

Set $np = \mu$, so $p = \frac{\mu}{n}$

$$p(x) = \binom{n}{x} \left(\frac{\mu}{n}\right)^x \left(1 - \frac{\mu}{n}\right)^{n-x}$$

$$\lim_{n\to\infty} p(x) = \lim_{n\to\infty} \binom{n}{x} \left(\frac{\mu}{n}\right)^x \left(1 - \frac{\mu}{n}\right)^{n-x}$$

(6)

$$= \lim_{n\to\infty} \frac{n!}{x!(n-x)!} \frac{\mu^x}{n^x} \frac{\left(1-\frac{\mu}{n}\right)^n}{\left(1-\frac{\mu}{n}\right)^x}$$

$$= \frac{\mu^x}{x!} \lim_{n\to\infty} \frac{\left(1-\frac{\mu}{n}\right)^n}{\left(1-\frac{\mu}{n}\right)^x} \frac{n(n-1)\cdot\cdots\cdot(n-x+1)}{n\cdot n \quad \cdots\cdots \quad n}$$

$$= \frac{\mu^x}{x!} \lim_{n\to\infty} \frac{\left(1-\frac{\mu}{n}\right)^n}{\left(1-\frac{\mu}{n}\right)^x} \cdot 1\cdot\left(1-\frac{1}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)$$

$$= \frac{\mu^x}{x!} e^{-\mu}$$

$$\boxed{p(x) = \frac{\mu^x e^{-\mu}}{x!}}$$ is called the Poisson distribution

Poisson experiment

(7)

- (1) observe a process over a fixed amount of time (or space)
- (2) X counts the number of occurrences of a particular type
- (3) Observations in non-overlapping intervals are independent
- (4) The probability of an occurrence within an interval is proportional to the length of the interval.

(8)

- (5) In a very small interval, the probability of more than 1 occurrence is approximately zero.

Check the validity of $p(x) = \frac{\mu^x e^{-\mu}}{x!}$ as a probability distribution.

$$1 \stackrel{?}{=} \sum_{x=0}^{\infty} \frac{\mu^x e^{-\mu}}{x!}$$

$$= e^{-\mu}\left[1 + \mu + \frac{\mu^2}{2!} + \frac{\mu^3}{3!} + \dots\right]$$

$$= e^{-\mu}[e^{\mu}] = 1$$

Let $\mu = \lambda t$, so $p(x) = \dfrac{(\lambda t)^x e^{-\lambda t}}{x!}$ (9)

$x = 0, 1, 2, \ldots$

Example: Watch the bank entrance for 20 minutes and count customers. We know that they average 30 customers per hour.

Poisson$\left(\mu = \lambda t = 30 \cdot \frac{1}{3} = 10\right)$

(10)

Find the probability of seeing exactly 5 customers in the 20 minute period.

$$P(X=5) = \frac{10^5 e^{-10}}{5!} = .0378$$

Example: A fire station gets 2.83 calls per day. Find the probability that it gets 4 or fewer calls in a given day.

Poisson$\left(\mu = \lambda t = (2.83)(1) = 2.83\right)$

(11)

$$P(X \le 4) = p(0) + p(1) + p(2) + p(3) + p(4)$$
$$= \frac{2.83^0 e^{-2.83}}{0!} + \dots + \frac{2.83^4 e^{-2.83}}{4!}$$
$$= .8429$$

For the Poisson distribution, $E[X] = \mu = \lambda t$

Variance of binomial was npq

$$\lim_{n\to\infty} npq = \lim_{n\to\infty} \cancel{n}\left(\frac{\mu}{\cancel{n}}\right)\left(1 - \frac{\mu}{n}\right) = \mu$$

(12)

Catalog of discrete distributions

Name	parameters	μ	σ^2
Uniform			
Bernoulli			
Binomial	n, p	np	npq
Hypergeometric			
Multinomial			
Multivar. Hyper.			
Geometric			
Negative Bino (Pascal)			
Poisson	$\mu = \lambda t$	μ	μ

HW#7 p.165 #52, 58, 76

p.186 # 4, 10, 12, 22