Chapter 5 Some Discrete Distributions

① 451 2-16

1. Discrete Uniform

X takes on a finite set of values

$x_1, x_2, x_3, \ldots, x_k$

with probability $1/k$ each

Special case: X takes on values $1, 2, 3, \ldots, k$

Find μ and σ^2

②

$$\mu = E[X] = \sum_{\text{all } x} x\,p(x) = \sum_{x=1}^{k} x \cdot \frac{1}{k}$$

$$= \frac{1}{k}\sum_{x=1}^{k} x = \frac{1}{k}\,\frac{k(k+1)}{2} = \frac{k+1}{2}$$

$$E[X^2] = \sum_{\text{all } x} x^2 p(x) = \sum_{x=1}^{k} x^2 \cdot \frac{1}{k}$$

$$= \frac{1}{k}\sum_{x=1}^{k} x^2 = \frac{1}{k}\,\frac{k(k+1)(2k+1)}{6} = \frac{(k+1)(2k+1)}{6}$$

$$\sigma^2 = Var[X] = E[X^2] - \mu^2 = \frac{(k+1)(2k+1)}{6} - \left(\frac{k+1}{2}\right)^2$$

$$= \frac{(k+1)(2k+1)}{6} - \frac{(k+1)^2}{4}$$

$$= \frac{4k+2 - 3k - 3}{12}(k+1)$$

$$= \frac{k-1}{12}(k+1) = \frac{k^2-1}{12}$$

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2. <u>Bernoulli Distribution</u>

$$X = \begin{cases} 1 & \text{w/ prob. } p \\ 0 & \text{w/ prob. } 1-p = q \end{cases}$$

Find μ and σ^2.

$$\mu = E[X] = \sum_{\text{all } x} x\,p(x) = (0 \cdot q) + (1 \cdot p)$$
$$= p$$

$$E[X^2] = \sum_{\text{all } x} x^2 p(x) = (0^2 \cdot q) + (1^2 \cdot p) = p$$

$$\sigma^2 = E[X^2] - \mu^2 = p - p^2 = p(1-p)$$
$$= pq$$

3. <u>Binomial Distribution</u>

- Run a sequence of independent trials $n = \#$ trials
- Each trial results in either 0 or 1 with probabilities q and p

⑤

– X counts the number of 1s.

$$\left.\begin{aligned} X &= 0, 1, 2, \ldots, n \\ p(x) &= \binom{n}{x} p^x q^{n-x} \end{aligned}\right\} \text{Binomial Distribution}$$

Find μ and σ^2

Try using the definition:

$$\mu = E[X] = \sum_{\text{all } x} x\, p(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}$$

This is too difficult.

⑥

But $X = X_1 + X_2 + \ldots + X_n$,

where each X_i is Bernoulli

and they are independent

$$E[X] = E[X_1] + \ldots + E[X_n]$$
$$= np$$

$$V[X] = V[X_1] + \ldots + V[X_n]$$
$$= npq$$

Example: Aim a projectile at a target + take 20 shots.

Suppose you have a 70% probability of success on each shot.

Let $X = \#$ hits.

Binomial $(\underbrace{n=20,\ p=.7}_{\text{parameters}})$

Find $E[X] = np = 20(.7) = 14$

Find $V[X] = npq = 20(.7)(.3) = 4.2$

The standard deviation $= \sigma = \sqrt{4.2} = 2.05$ 8

Find the probability of getting exactly 15 hits.

$$p(15) = \binom{20}{15}(.7)^{15}(.3)^{5} = .1789$$

Find the probability of getting at least 15 hits

$$P(X \geq 15) = p(15) + p(16) + \ldots + p(20)$$

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4. Hypergeometric Distribution

- Population consisting of N items
- K of those items have a particular characteristic
- Take a sample of n items, without replacement (WOR)
- X counts the number of items in the sample, with the particular characteristic.

$$X = 0, 1, 2, \ldots, \min(n, k)$$

$$p(x) = \frac{\binom{K}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

Hypergeom. Distr.

(10)

Example: 200 potential jurors

120 females, 80 males

Select 12 WoR

Find the prob. that exactly 6 are female

Hypergeom $(N = 200, k = 120, n = 12)$

$$p(6) = \frac{\binom{120}{6}\binom{80}{6}}{\binom{200}{12}} = .18$$

$$\mu = E[X] = n\frac{k}{N}, \quad \sigma^2 = n\frac{k}{N}\left(1 - \frac{k}{N}\right)\left(\frac{N-n}{N-1}\right)$$

(11)

HW #6 due 2/23

p.150 # 2, 12, 16

p.157 # 30, 32, 48