① 451 1-19

Another random variable example:

Observe 4 independent components of a system. Each part has a 20% chance of failing. Let X = number of parts that fail.

Find the probability distribution for X.

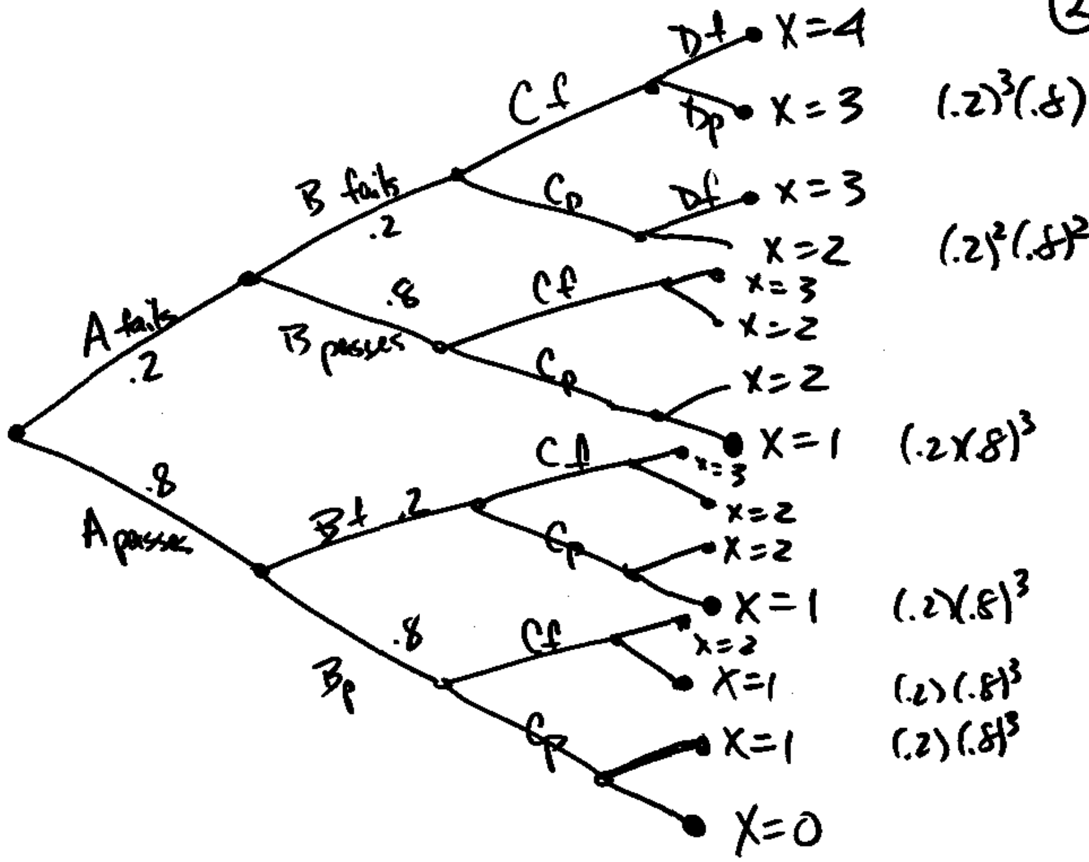
x	$p(x)$	
0	$(.8)^4$	$P(X=0)$
1	$4(.2)(.8)^3$	$P(X=1)$
2	$6(.2)^2(.8)^2$	
3	$4(.2)^3(.8)$	
4	$(.2)^4$	$P(X=4)$

Note:

$p(x) = \binom{4}{x}(.2)^x(.8)^{4-x}$

$x = 0,1,2,3,4$

②



③

Pascal's Triangle

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1 \\
\binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4}
\end{array}
$$

④

Example: Fire 10 independent shots at a target. Each shot has a 70% chance of hitting the target. $X = \#$ hits.

$$p(x) = \binom{10}{x} (.7)^x (.3)^{10-x}$$

$$x = 0, 1, 2, \ldots, 10$$

Some characteristics of probability distributions

$$\mu = E[X] = \sum_x x\,p(x)$$ This is the expected value of X.

5

Definition of expectation, in general:

Let X be a discrete random variable,

and let $g(X)$ be any function of X.

Then $$E[g(X)] = \sum_x g(x)\, \underbrace{p(x)}_{\text{probability distribution of } X.}$$

So $$E[X] = \sum_x x\, p(x)$$

$$E[X^2] = \sum_x x^2 p(x)$$

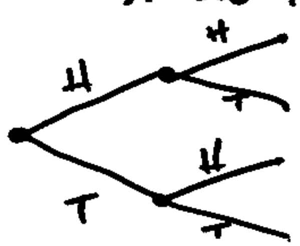
$$E[\sqrt{X}] = \sum_x \sqrt{x}\, p(x)$$

6

Definition: The variance of a discrete random variable X is

$$\sigma^2 = V[X] = Var[X]$$
$$= E[(X-\mu)^2] = \sum_x (x-\mu)^2 p(x)$$

Example: Flip 2 coins. Let $X = \#$heads.



(7)

x	$p(x)$
0	.25
1	.50
2	.25

$$E[X] = \sum_x x\,p(x)$$
$$= 0(.25) + 1(.50) + 2(.25)$$
$$= 1 = \mu$$

$$\sigma^2 = E[(X-\mu)^2] = \sum_x (x-\mu)^2 p(x)$$
$$= (0-1)^2(.25) + (1-1)^2(.50) + (2-1)^2(.25)$$
$$= .5 \qquad \sigma = \sqrt{.5} = .707$$

The standard deviation of X is $\sigma = \sqrt{\sigma^2}$

(8)

Alternate formula for σ^2:

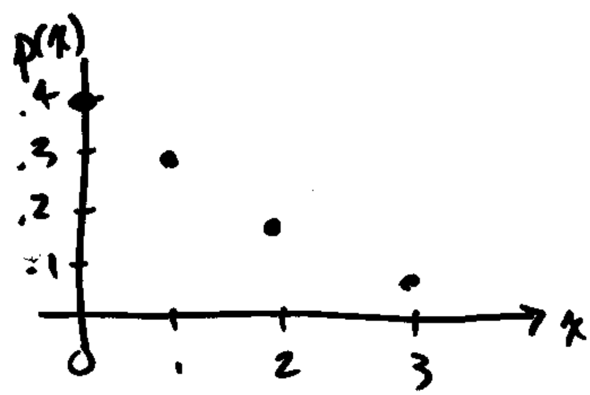
$$\sigma^2 = \sum_x (x-\mu)^2 p(x)$$
$$= \sum_x (x^2 - 2x\mu + \mu^2)\,p(x)$$
$$= \sum_x x^2 p(x) - 2\sum_x x\mu\, p(x) + \sum_x \mu^2 p(x)$$
$$= E[X^2] - 2\mu \underbrace{\sum_x x\,p(x)}_{E[X]} + \mu^2 \underbrace{\sum_x p(x)}_{1}$$
$$= E[X^2] - \mu^2 = E[X^2] - (E[X])^2$$

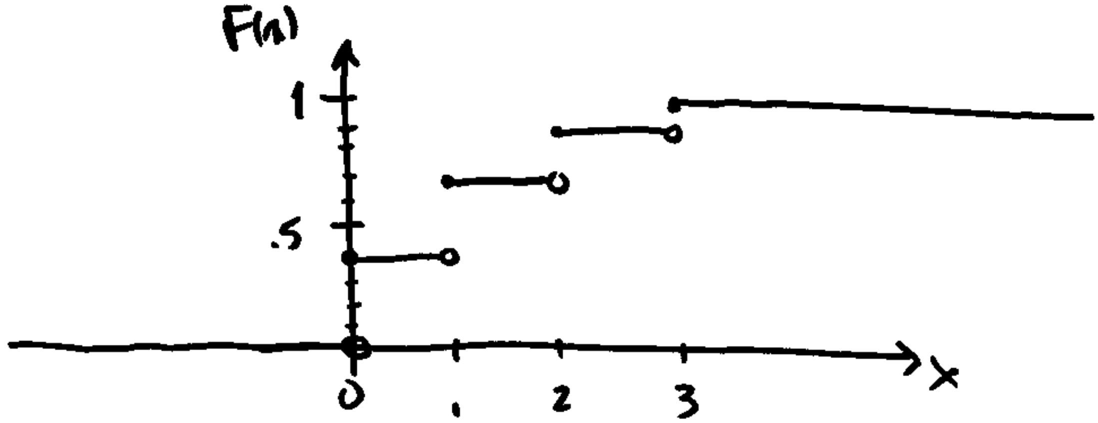
<u>Definition</u>: The cumulative distribution function for a discrete random variable X is

$$F(x) = \sum_{t \le x} p(t) = P(X \le x)$$

<u>Example</u>:

x	$p(x)$	$F(x)$
0	.4	.4
1	.3	.7
2	.2	.9
3	.1	1





Note: $F(x)$ is nondecreasing and continuous from the right.