①
Example: 451
1-14

Assume that approximately 1 out of 1000 people are affected by a certain disease.

Suppose that, if a person has the disease, the test results will be positive 99% of the time. (1% false negative)

Suppose that, if a person doesn't have the disease, the test results will be negative 95% of the time (5% false positive)

②

	T	T'	
D	.00099	.00001	.001
D'	.04995	.94905	.999
	.05094	.94906	1

$\boxed{\text{Bayes' Rule}}$

Given that a person tests positive, find the probability that he/she has the disease.

$$P(D \mid T) = \frac{P(D \cap T)}{P(T)} = \frac{.00099}{.05094} = .019$$

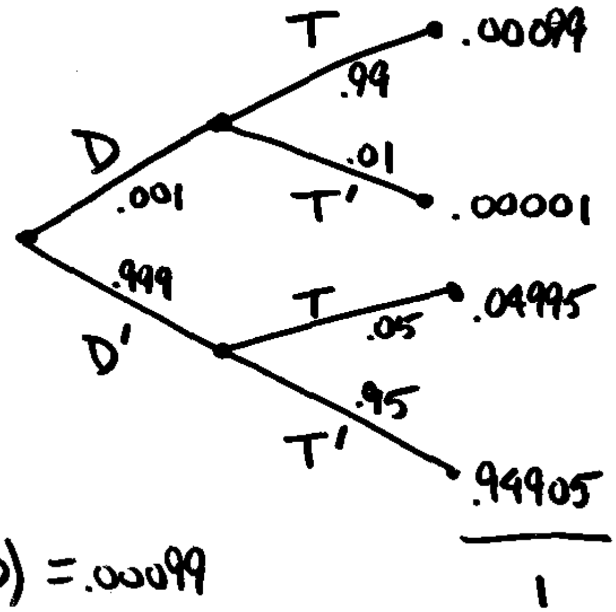
D: has the disease

T: test results are positive

$P(D) = .001$

$P(T \mid D) = .99$

$P(T' \mid D') = .95$



$P(D \cap T) = P(D)P(T \mid D) = .00099$

Random variables

Definition: A random variable is a variable whose value is determined by the outcome of an experiment.

Actually, a random variable is a function whose domain is S and whose range is a subset of $\mathbb{R}$

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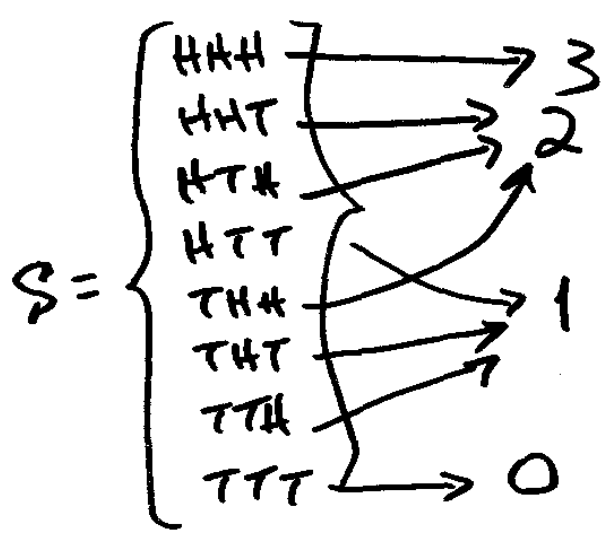
Example: Flip 3 coins. Let $X = \#\text{heads}$



If the range of X is finite or countably infinite, then X is called a discrete random variable.

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If the range of X is an interval, then X is called a continuous random variable.

Discrete case

Define a probability function

$$p(x) = P(X = x)$$

Properties: $0 \le p(x) \le 1$

$$\sum_x p(x) = 1$$

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3 coins example

x	$p(x)$
0	1/8
1	3/8
2	3/8
3	1/8
	1

$p(0) = P(X=0) = P(TTT)$

$p(1) = P(X=1)$

The <u>probability distribution</u> of a discrete random variable X consists of the set of all possible $(x, p(x))$

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<u>Example</u>: Roll 2 dice.

Let Y = minimum

y	$p(y)$
1	11/36
2	9/36
3	7/36
4	5/36
5	3/36
6	1/36
	1

$$p(y) = \frac{2(6-y)+1}{36}, \quad y = 1,2,3,4,5,6$$

HW #2 due 1/21

p.39 #14, 16

p.48 #30, 32, 38, 48, 50

p.56 #58, 60, 64