Chapter 2 Probability

① 451 1-7

Experiment: A process that leads to one of several possible outcomes

Sample space (S): the set of all possible outcomes of an experiment

Event: a subset of S

Example 1: Flip a coin $S = \{H, T\}$

②

Example 2: Roll 1 6-sided die

$$S = \{1, 2, 3, 4, 5, 6\}$$

Example 3: Flip 2 coins in sequence

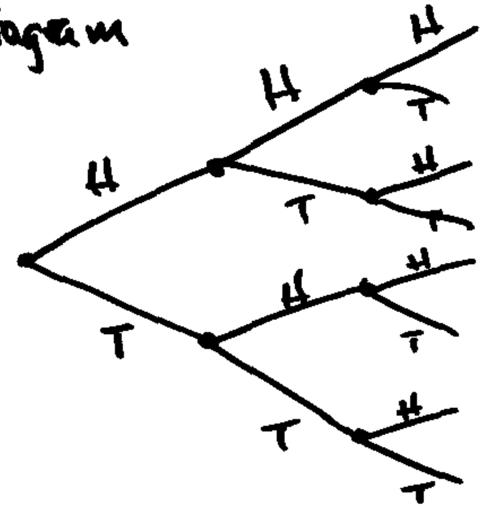
$$S = \{HH, HT, TH, TT\}$$

Example 4: Flip 3 coins in sequence

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

③

Tree diagram



Use example 4.

Let A be the event of getting "heads" exactly once. $A = \{HTT, THT, TTH\}$

④

For finite sample spaces, the probability of an event A is $P(A) = \dfrac{n(A)}{n(S)}$

In our example, $P(A) = \dfrac{3}{8}$

<u>Example 5</u>: Roll 2 6-sided dice in sequence

Find the probability that the sum is 9

⑤

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & \cdots & & & \vdots \\ \vdots & & & & & \vdots \\ (6,1) & (6,2) & \cdots & & & (6,6) \end{array} \right\}$$

$n(S) = 36$

$A = \{ (3,6), (4,5), (5,4), (6,3) \}$

$n(A) = 4 \qquad P(A) = \frac{4}{36} = \frac{1}{9}$

⑥

Counting Rules

① Multiplication principle

If a process can be broken down into a sequence of operations, then the total number of outcomes is the product of the number of outcomes at each stage.

Example: Randomly select a license plate of the form AAA111. $n(S) = 26^3 10^3$

② Factorial rule

The number of ways of ordering or arranging n items is

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 1$$

③ Permutation rule

Start with n items. Select and rank r of them. The number of possible outcomes is $P^n_r = n \cdot (n-1) \cdots (n-r+1) = {}_nP_r$

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Note: $$P^n_r = \frac{n(n-1)\ldots(n-r+1)(n-r)\cdots 1}{(n-r)\cdots 1} = \frac{n!}{(n-r)!}$$

Example: 20 items, select and rank 3

$$n(S) = P^{20}_3 = 20 \cdot 19 \cdot 18 = \frac{20!}{17!}$$

4 Combination rule

Start with n items. Select r of them, without ordering. The number of outcomes is

$$C_r^n = {}_nC_r = \binom{n}{r} = \frac{P_r^n}{r!} = \frac{n!}{r!(n-r)!}$$

Example: 5 items A, B, C, D, E Select 3, no order

$$S = \begin{Bmatrix} ABC & ACD & BCD & CDE \\ ABD & ACE & BCE & \\ ABE & ADE & BDE & \end{Bmatrix} \quad n(S) = 10$$

Example: Receive a batch of 50 items. You choose 4 for inspection.

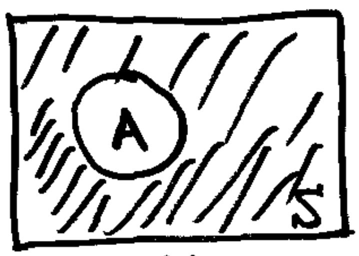
$$n(S) = C_4^{50} = \binom{50}{4} = \frac{50!}{4!\,46!}$$

Example: Deal 5 cards cards from a deck of 52. Find the probability of getting a royal flush.

$$n(S) = \binom{52}{5} = 2{,}598{,}960$$

$n(A) = 4 \qquad P(A) = \dfrac{4}{2598960}$ (11)

The <u>complement</u> of an event A is the set A' consisting of all items in S that are <u>not</u> in A.



A'

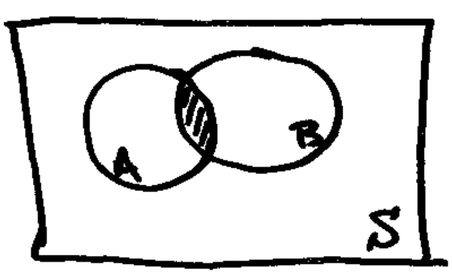
$$\frac{n(A)}{n(S)} + \frac{n(A')}{n(S)} = \frac{n(S)}{n(S)}$$

$$P(A) + P(A') = 1$$

(12)

$$\boxed{P(A') = 1 - P(A)}$$ Complement Rule

The <u>intersection</u> of events A and B is $A \cap B$, which is the set of items appearing in both A <u>and</u> B.



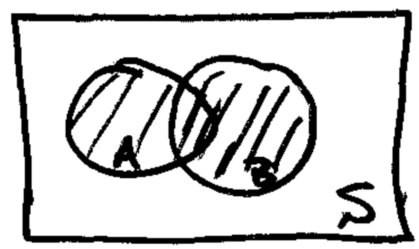
$A \cap B$

(13)

A and B are disjoint or mutually exclusive if they do not intersect, i.e.

$$A \cap B = \emptyset = \{\ \}$$

The union of events A and B is the set of items contained in either A or B (or both)



$A \cup B$

(14)

HW #1 due 1/14/10

p. 13 #2
p. 17 #8
p. 28 #18