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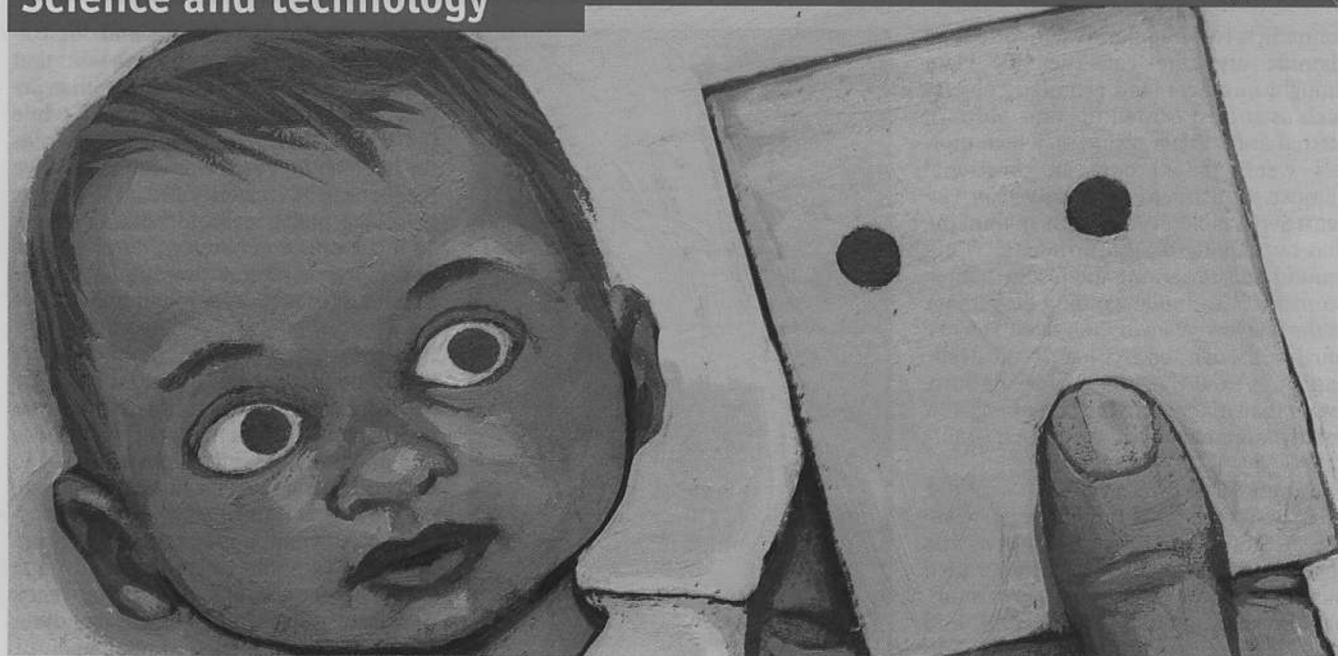

Numbers

Easy as 1, 2, 3

People come into the world ready to count its wonders

THE baby is just one day old and has not yet left hospital. She is quiet but alert. Twenty centimetres from her face researchers have placed a white card with two black spots on it. She stares at it intently. A researcher removes the card and replaces it by another, this time with the spots differently spaced. As the cards alternate, her gaze starts to wander—until a third, with three black spots, is presented. Her gaze returns: she looks at it for twice as long as she did at the previous card. Can she tell that the number two is different from three, just 24 hours after coming into the world?

Or do newborns simply prefer more to fewer? The same experiment, but with three spots preceding two, shows the same revival of interest when the number of spots changes. Perhaps it is just the newness? When slightly older babies were shown cards with pictures of household objects instead of dots (a comb, a key, an orange and so on), changing the number of items had an effect separate from changing the items themselves. Could it be the pattern that two things make, as opposed to three? No again. Babies paid more attention to rectangles moving randomly on a screen when their number changed from two to three, or vice versa. The effect even crosses between senses. Babies who were repeatedly shown two spots perked up more when they then heard three drumbeats than when they heard just two; likewise when the researchers started with drumbeats and moved to spots.

"One great blooming, buzzing confusion" was how William James, a 19th-century psychologist, described the way he thought the world looked to a newborn baby. But these experiments, and many others like them over the past few decades, have convinced researchers that, on the contrary, babies are born with many ways of making sense of what they see and hear. The trick is to use their love of novelty to work out what is happening inside their brains: when shown the same things repeatedly, babies' eyes wander; when the scene changes, their gaze returns. That makes visible what to them constitutes a change in the world around them worthy of notice.

Dot and carry one

One of those ways of understanding the world is by number. People are born with an innate sense of how many items there are in small collections. Experiments in which older children and adults are shown randomly arranged dots and asked to say quickly how many there are show this sense is retained throughout life. Up to three or four items, and the number is immediately visible without counting. Within a limited range, humans are born arithmeticians, too. When babies a few months old were shown dolls placed and removed from behind a screen they had correct expectations of the number of dolls they would see when the curtain was drawn aside, and were surprised when trickery meant those expectations were violated. In fact, they were more surprised to see the wrong number of dolls than the right number, but different-looking ones.

Some animals also seem able to perceive and understand small numbers. From the 1930s Otto Köhler, a German zoologist, trained ravens to open boxes with the same number of dots on the lid as a card held by a researcher. One raven learnt to distinguish two, three, four, five and six dots. Rats can learn to ignore a certain number of doors in a maze before choosing which one to enter. Chimpanzees have been taught to match the numerals 1 to 6 to the number of objects in a display and to find oranges hidden in two different places and point to the numeral that indicates their total number.

Even more strikingly, some wild animals appear to understand and use numerical facts without training. Karen McComb of the University of Sussex, in England, played a variety of recordings of lions roaring at night in the Serengeti National Park—different numbers of lions; their roars in sequence and overlapping; and so on. She wanted to test the theory that, since fights between lions are very costly, when lions heard large numbers of intruders' roars they would withdraw unless they were in superior numbers. The best explanation of what she observed was that lions estimated the number of intruders from the number of different-sounding roars, compared that number to the number in their own group and then decided whether to attack or slink away.

That humans (and perhaps other animals) come ready-supplied with numbers ▸▸

contradicts two popular rival theories: the Platonic and the constructivist. Plato thought numbers (and geometric objects such as circles) existed in some abstract, eternal and perfect realm, of which mortals were granted only an occasional glimpse. Constructivists follow Jean Piaget, a Swiss child psychologist, in thinking that by moving things in the real world around and observing the results people "construct" an understanding of number in the first few years of their lives. The distinction, though abstract, has practical relevance too. Could "maths-phobes" be born, rather than made? Can they be cured? And could mathematics be taught better to all?

Numbers on the brain

Brian Butterworth, a cognitive neuroscientist at University College London, has spent much of his career teasing out which bits of humans' understanding of numbers are innate—and which learnt, and how. He thinks people are born with brain circuits that are dedicated to recognising and understanding the number of items in small collections. On this foundation an entire "number sense" is built, as children realise that bigger and bigger numbers can be reached by adding "one more" and learn by experience how these bigger numbers behave.

His most recent work has confirmed that to develop a better understanding of numbers than that of a newborn baby, it is not necessary to be able to count with words. He collaborated with some Australian researchers to test aboriginal children in the country's Northern Territory who were monolingual speakers of one of two languages, Warlpiri and Anindilyakwa, in which the only number words are one, two, few and many. (Words for numbers have generally arisen when and where people grow crops or keep herds; hunter-gatherer bands, who have no herds or other stores of wealth, need not keep track of surpluses, or balances of trade.)

Since the children were too old for the baby-staring trick, but unable to answer the question: "How many?", researchers laid out counters, then put them away and asked the children to "do as I did". To check that they were using the number of the counters, rather than mimicking their pattern, the researchers banged sticks together and asked them to "make the counters like the noises". The children performed about as well as English-speaking aboriginal children living in Melbourne.

Historically, one common method of counting has been to use body parts to keep track of a running total. The base-ten system used in modern arithmetic originates with the fingers, and linguistic traces of that fact remain in the similarity of "five", "finger" and "fist", and the dual meaning of "digit". Some think that the original inhabitants of Europe were 20-counters who used fingers and toes—the use of "score" for both 20 and keeping count may be a remnant. And there remain tribal peoples who have elaborate methods using eyes, nostrils, elbows and so on.

Arithmetically, bases 12, 24 and 60 have their appeal (they have more factors than ten's measly two). All three are still employed when telling the time, and 4,000 years ago the Babylonians used base 60 to do some pretty advanced mathematics. But fingers are particularly obvious and useful for keeping count. In another recent piece of work, Dr Butterworth and Robert Reeve of the University of Melbourne watched (English-speaking) five- and six-year-olds counting and doing simple sums. Most used their fingers, but around a quarter did not. Slightly more than half of the non-finger-counters were good arithmeticians, who presumably had outgrown needing to use their fingers. The others, who were decidedly weak, did not seem to have realised that their fingers could help.

More than 80 years ago Josef Gerstmann, an Austrian neurologist, described a set of problems that seem to arise simultaneously in people who have suffered damage to the left parietal lobe of the brain: finding writing difficult or impossible, being unable to understand arithmetic or tell right from left, and having difficulty in identifying one's fingers. There is still no agreement on whether these symptoms constitute a syndrome, but the bits of the brain used for storing facts about numbers and for representing the fingers are close to each other. Mental representations of numbers and of fingers may therefore be functionally connected.

In 2005 Dr Butterworth and his colleagues asked people to perform tasks that required dexterity, and others that involved matching pairs of numbers, while the area of their parietal lobes known as the left angular gyrus was stimulated by a magnetic field. Dexterity and recall of facts involving numbers were both impaired. So the connection between numbers and fingers may be more profound than the handiness of fingers for keeping count.

Easy for some

If numbers had been invented by some prehistoric genius, then learning how to use them would be a matter of intelligence and practice. But what comes naturally to most is lacking in a few. Just as some people are born colour-blind, or lose colour vision after a brain injury, others are "number-blind": unable to comprehend what everyone else sees effortlessly. That deficit may leave other abilities—including other mathematical abilities—unimpaired.

Dr Butterworth tells the story of Charles, a young man with lifelong mathematical difficulties. He could add two one-digit numbers only if he used his fingers. Sums involving two-digit numbers or multiplication or subtraction were beyond him. When shopping, he understood neither prices nor change. Tests showed he was not merely maths-phobic. Not only was he far slower than the average, but the pattern of his results was strange.

In one test Charles was shown a pair of digits and asked to name the larger number. The bigger the gap, the faster most people can do this: they say "nine" faster when shown 9 and 2 than when shown 9 and 7. But with Charles, the reverse was the case—and the researchers could see why. Rather than telling the answer directly, he was counting on from one number (on his fingers) until he got to the other, which meant he must have started at the smaller, or he got to ten, in which case he must have started at the bigger. Most strikingly, he lacked the fundamental numerical ability possessed by most newborns: being able to tell the number of objects in a small group simply by looking. When asked how many dots were on a sheet of paper, he counted on his fingers—even when there were only two.

Charles's deficit, though severe, seemed to affect his numerical abilities alone. Numerical deficits in people of otherwise normal abilities can be even more striking in cases of brain damage. Lisa Cipolotti, a neuropsychologist, studied a Signora Gaddi, who used to run a hotel and keep its accounts. After a stroke she could find the number of things in a small group only by counting—when asked how many arms a crucifix had, she got Dr Cipolotti to hold out her arms so she could count them. Signora Gaddi's problems seemed to affect only numbers. She could still read, speak ▸▸

and reason, remember historical and geographical facts, and order objects by their physical size.

In fact, Signora Gaddi's difficulties went even deeper than Charles's. The stroke which damaged her innate understanding of small numbers also robbed her of the entire numerical edifice built on that foundation. For her, numbers stopped at four. When asked to count up from one, she got to four and no further. If there were more than four dots on a page she could not count them. She could not say how old she was or how many days were in a week, or even tell the time.

"I hated maths at school"

From Barbie dolls programmed to say "math class is tough" to ministers of state who will parse and analyse a sentence but refuse to answer "what's half of three-quarters?", maths-phobia is everywhere. One reason is that mathematics builds on itself, so that one missed step can lead to a lifetime of failure. Nor does it help that sums have unambiguously right and wrong answers, making it all too clear to schoolmates just what a child does and doesn't know. But amidst the stragglers are those whose problem runs deeper than fear and loathing: the "dyscalculic", as researchers have taken to calling those whose number sense is impaired. Numerical tests given to a representative sample of children in Havana suggest their proportion in the general population is 3-6%.

Sceptics may feel this is a learning disability too far—another chance for middle-class parents to classify little Johnny as different, rather than thick. And perhaps dyscalculia will collect a penumbra of dubious cases around it, as dyslexia has. But perhaps not. Dyslexia manifests itself as a difficulty with a highly unnatural activity: reading. The best single predictor of dyscalculia, by contrast, is abnormal slowness in counting a few dots on a page, a task that most find trivially easy.

The researchers at University College have created a dyscalculia screener, which they think should be used to test all children early in life. With luck, diagnosis will progress to treatment: they are working on a remedial programme too. But even if dyscalculics never fully develop the sense of numbers they were born without, their mathematical careers need not be over before they have started. There are entire fields of mathematics where numerical manipulation is peripheral: logic and geometry, for example. Dr Butterworth recalls an eminent geometrician ("I won't say his name; it would embarrass him") who approached him after he had given a talk on his research. "He said: 'You know, I have always been dreadful at arithmetic.' So I asked: 'What's seven eights?' He just mumbled: 'Oh, that's trivial, there's an algorithm for that,' and walked away." ■

More numbers

When 1, 2, 3... is not enough

Arguments over what counts as a number

EVERY now and then mathematics has been convulsed by a row, not over where numbers come from—but over what should be allowed to count as one. Two millennia ago, inspired by such discoveries as the relationship between musical pitch and the lengths of vibrating strings (double the length of the string and the note falls by an octave), the followers of Pythagoras decided that all of Nature must be expressible as ratios of whole numbers. Their discovery that one very simple geometric ratio—that of the length of a square's diagonal to the length of its side—could not was, according to legend, so shocking that it was kept a secret on pain of death.

Such "irrational" numbers were bad enough, but what to make of negative ones? Although they had been widely employed since at least the mid-1500s, in particular to represent debts, many mathematicians refused to use them, claiming that quantities less than zero were an absurdity. A number "submits to be taken away from another number greater than itself but to attempt to take it away from a number less than itself is ridiculous," wrote William Frend, a Cambridge mathematician, in 1796.

And what, too, to make of the square roots of negative numbers? Since both negative and positive numbers, when multiplied by themselves, give positive answers, such numbers were labelled "imaginary", and regarded by many as meaningless. "The symbol $\sqrt{-1}$ is beyond the power of arithmetical computation," wrote Robert Woodhouse, another Cambridge mathematician, in 1801. It took the brilliant idea, of Carl Friedrich Gauss and others, of regarding imaginary numbers as perpendicular to "real" ones before the latest variety of number could be accepted into the swelling menagerie.

By the end of the 19th century irrational, negative and imaginary numbers were widely accepted—not least because they were so very useful; turning one's back on such delights meant voluntarily abstaining from doing some very interesting mathematics. But still the numerical controversies raged. Georg Cantor, a German mathematician, had developed a "transfinite arithmetic" to calculate with the infinitely many infinities he had discovered, each infinitely larger than the previous one. Leopold Kronecker, a prominent German mathematician (and one of Cantor's teachers) described his student as a "scientific charlatan", a "renegade" and a "corrupter of youth"; his work was a "disease" from which mathematics would surely be cured some day, thought French mathematician Henri Poincaré.

Between 1910 and 1913 Bertrand Russell and Alfred North Whitehead published their three-volume "Principia Mathematica", in which, among other things, they sought to solve certain paradoxes that arose from Cantor's work. Their main aim, though, was to provide a firm foundation for all of mathematics—a hopeless quest, it turned out, when Kurt Gödel published his "incompleteness theorem" in 1931.

Russell and Whitehead suggested no new numbers or arithmetical rules, but they did try to show how the simplest numbers—integers—could be built using the principles of logic. But the methods they proposed for even the simplest sums were desperately cumbersome. And for the proof that 1+1=2, readers had to wait until volume II, page 83.