Comprehensive Physics Notes

Emma O'Neil

Contents

Chapter 1

PH631

1.1 Classical Mechanics

Question 1: Charged particle in a constant magnetic field with harmonic oscillator potential

We consider a particle of charge q in a constant magnetic field $B_0 e_z$ and corresponding vector potential $-\frac{1}{2}\mathbf{r} \times \mathbf{B}$, with a harmonic oscillator potential $V(\rho) = \frac{1}{2}m\omega_0^2\rho^2$. Additionally, recall that the velocity dependent potential is given by $q\mathbf{\Phi} - q\mathbf{v} \cdot \mathbf{A}$. We will give a full solution in Newtonian, Lagrangian, and Hamiltonian frameworks.

Solution: We make use of the following constants throughout this problem:

- $\omega_B = -\frac{qB_0}{m}$
- $\Omega = \sqrt{\omega_0^2 + \frac{\omega_B^2}{4}}$
- $\Omega_\pm = \frac{\omega_B}{2} \pm \Omega$

Newtonian: First, the acceleration in cylindrical coordinates is given by:

$$\begin{aligned}
\frac{d^2}{dt^2}\mathbf{r} &= \frac{d^2}{dt^2}\left[\rho e_\rho + z e_z\right] \\
&= \frac{d}{dt}[\dot{\rho} e_\rho + \rho\dot{\phi} e_\phi + \dot{z} e_z] \\
&= \ddot{\rho} e_\rho + \dot{\rho}\dot{\phi} e_\phi + \dot{\rho}\dot{\phi} e_\phi + \rho(\ddot{\phi} e_\phi - \dot{\phi}^2 e_\rho) + \ddot{z} e_z \\
&= (\ddot{\rho} - \rho\dot{\phi}^2) e_\rho + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) e_\phi + \ddot{z} e_z
\end{aligned}$$

We begin from the Lorentz force equation combined with the harmonic oscillator force:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - m\omega_0^2 \rho e_\rho$$

and Newton's law:

$$\mathbf{F} = m\ddot{\mathbf{r}} = m((\ddot{\rho} - \rho\dot{\phi}^2) e_\rho + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) e_\phi + \ddot{z} e_z),$$

where $\mathbf{r}$ is the position vector.

The electric field in this example is 0, so we compute the force due to the magnetic field:

$$q\mathbf{v} \times \mathbf{B} = q\begin{vmatrix} e_\rho & e_\phi & e_z \\ \dot{\rho} & \rho\dot{\phi} & \dot{z} \\ 0 & 0 & B_0 \end{vmatrix} = q(\rho\dot{\phi} B_0 e_\rho - \dot{\rho} B_0 e_\phi)$$

Matching components, we have:

$$\begin{aligned} m\ddot{\rho} - m\rho\dot{\phi}^2 &= q\rho\dot{\phi}B_0 - m\omega_0^2\rho \\ \ddot{\rho} - \rho\dot{\phi}^2 &= \frac{qB_0}{m}\rho\dot{\phi} - \omega_0^2\rho \end{aligned}$$

$$\ddot{\rho} - \rho\dot{\phi}^2 + \omega_B\rho\dot{\phi} + \omega_0^2\rho = 0 \tag{1.1}$$

And for the radial component:

$$\begin{aligned} m\rho\ddot{\phi} + 2m\dot{\rho}\dot{\phi} &= -q\dot{\rho}B_0 \\ \rho\ddot{\phi} + 2\dot{\rho}\dot{\phi} &= \omega_B\dot{\rho} \end{aligned}$$

$$\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi} - \omega_B\dot{\rho} = 0 \tag{1.2}$$

Lagrangian: The Lagrangian $\mathcal{L}(q, \dot{q})$ of this system is given by:

$$\begin{aligned} \mathcal{L} &= T - V \\ &= \frac{1}{2}m(\dot{\rho}^2 + \rho^2\dot{\phi}^2) - q\Phi + q\mathbf{v}\cdot\mathbf{A} - \frac{1}{2}m\omega_0^2\rho^2 \end{aligned}$$

And the vector potential $\mathbf{A}$ is given by:

$$\mathbf{A} = -\frac{1}{2}\mathbf{r}\times\mathbf{B} = -\frac{1}{2}\begin{vmatrix} e_\rho & e_\phi & e_z \\ \rho & 0 & z \\ 0 & 0 & B_0 \end{vmatrix} = \frac{1}{2}\rho B_0$$

Then $q\mathbf{v}\cdot\mathbf{A} = \frac{1}{2}q\rho^2\dot{\phi}B_0$, giving us a final Lagrangian of:

$$\mathcal{L} = \frac{1}{2}m(\dot{\rho}^2 + \rho^2\dot{\phi}^2) + \frac{1}{2}q\rho^2\dot{\phi}B_0 - \frac{1}{2}m\omega_0^2\rho^2$$

Lagrange's equations give us the equations of motion:

$$\begin{aligned} \frac{d}{dt}\left[\frac{\partial\mathcal{L}}{\partial\dot{\rho}}\right] - \frac{\partial\mathcal{L}}{\partial\rho} &= 0 \\ m\ddot{\rho} - m\rho\dot{\phi}^2 - q\rho\phi\dot{B}_0 + m\omega_0^2\rho &= 0 \\ \ddot{\rho} - \rho\dot{\phi}^2 + \omega_B\rho\dot{\phi} + \omega_0^2\rho &= 0 \end{aligned} \tag{1.3}$$

$$\begin{aligned} \frac{d}{dt}\left[\frac{\partial\mathcal{L}}{\partial\dot{\phi}}\right] - \frac{\partial\mathcal{L}}{\partial\phi} &= 0 \\ \frac{d}{dt}[m\rho^2\dot{\phi} + \frac{1}{2}q\rho^2 B_0] &= 0 \\ 2m\rho\dot{\rho}\dot{\phi} + m\rho^2\ddot{\phi} + q\rho\dot{\rho}B_0 &= 0 \end{aligned}$$

$$\rho^2\ddot{\phi} + 2\rho\dot{\rho}\dot{\phi} - \omega_B\rho\dot{\rho} = 0 \tag{1.4}$$

Hamiltonian: We begin from the Lagrangian $\mathcal{L} = T - V = \frac{1}{2}m(\dot{\rho}^2 + \rho^2\dot{\phi}^2) - \frac{1}{2}m\omega_B\rho^2\dot{\phi} - \frac{1}{2}m\omega_0^2\rho^2$. To perform a Legendre transform, we first need the conjugate momenta:

$$\begin{aligned} p_\rho &= \frac{\partial L}{\partial\dot{\rho}} = m\dot{\rho} \\ p_\phi &= \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = m\rho^2\dot{\phi} + \frac{1}{2}q\rho^2 B_0 = \boxed{m\rho^2\dot{\phi} - \frac{1}{2}m\omega_B\rho^2} \end{aligned}$$

Then, from $p_\phi = m\rho^2\dot{\phi} - \frac{1}{2}m\omega_B\rho^2$, we obtain:

$$\begin{gathered}
p_\phi + \frac{1}{2}m\omega_B\rho^2 = m\rho^2\dot{\phi} \\
(p_\phi + \frac{1}{2}m\omega_B\rho^2)\frac{1}{m\rho^2} = \dot{\phi} \\
\frac{1}{2m\rho^2}(p_\phi^2 + p_\phi m\omega_B\rho^2 + \frac{1}{4}m^2\omega_B^2\rho^4) = \frac{1}{2}m\rho^2\dot{\phi}^2 \\
\frac{p_\phi^2}{2m\rho^2} + \frac{p_\phi\omega_B}{2} + \frac{m\omega_B^2\rho^2}{8} = \frac{1}{2}m\rho^2\dot{\phi}^2
\end{gathered}$$

This will be useful to express the Hamiltonian in terms of positions and conjugate momenta. We now perform a Legendre transform to find H:

$$\begin{aligned}
H &= \sum \dot{q}p - \mathcal{L} \\
&= m\dot{\rho}^2 + m\rho^2\dot{\phi}^2 - \frac{1}{2}m\omega_B\rho^2\dot{\phi} - \frac{1}{2}m\dot{\rho}^2 - \frac{1}{2}m\rho^2\dot{\phi}^2 + \frac{1}{2}m\omega_B\rho^2\dot{\phi} + \frac{1}{2}m\omega_0^2\rho^2 \\
&= \frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}m\rho^2\dot{\phi}^2 + \frac{1}{2}m\omega_0^2\rho^2
\end{aligned}$$

$$H = \frac{p_\rho^2}{2m} + \frac{p_\phi^2}{2m\rho^2} + \frac{p_\phi\omega_B}{2} + \frac{m\omega_B^2\rho^2}{8} + \frac{m\omega_0^2\rho^2}{2} \tag{1.5}$$

Now we obtain the four first-order differential equations from Hamilton's equations:

$$\dot{\rho} = \frac{\partial H}{\partial p_\rho} = \frac{p_\rho}{m} \tag{1.6}$$

$$\dot{p}_\rho = -\frac{\partial H}{\partial \rho} = \frac{p_\phi^2}{2m\rho^3} - \frac{m\omega_B^2\rho}{4} - m\omega_0^2\rho \tag{1.7}$$

$$\dot{\phi} = \frac{\partial H}{\partial p_\phi} = \frac{p_\phi}{m\rho^2} + \frac{\omega_B}{2} \tag{1.8}$$

$$\dot{p}_\phi = 0 \text{ (From Lagrangian case previously)} \tag{1.9}$$

1.2 Special Relativity

1.2.1 Time Dilation And Length Contraction

As a consequence of the Lorentz transformation, we have time dilation and length contraction effects. This suggests the notion of a *proper time* of an object, which is the time coordinate in an inertial frame where the object is stationary.

Recall the Lorentz transformation for a boost in the x-direction:

$$\begin{gathered}
ct \overset{L}{\mapsto} \gamma(ct - \frac{v}{c}x) = ct' \\
x \overset{L}{\mapsto} \gamma(x - \frac{v}{c}ct) = x'
\end{gathered}$$

The length measured in a frame S is the distance between the endpoints as measured at equal times in S. We'll derive the length contraction effect observed when measuring the length of a moving rod.

Our frame is S, so we are measuring the length of the rod at equal times $ct_1 = ct_2$. The transformation to the primed frame gives us the proper length of the rod.

$$\begin{aligned}\Delta x' = L[\Delta x] &= \gamma(x_2 - \frac{v}{c}ct) - \gamma(x_1 - \frac{v}{c}ct)\\ &= \gamma(x_2 - x_1) - \gamma(\frac{v}{c}ct - \frac{v}{c}ct)\\ \Delta x' &= \gamma \Delta x\end{aligned}$$

$$\Delta x = \frac{1}{\gamma}\Delta x' \leqslant \Delta x' \tag{1.10}$$

So to us, the rod appears shorter than its proper length $\Delta x'$.

To derive time dilation, we want to assume a clock is stationary in its own rest frame S', so $x_1' = x_2'$, where we are in another frame S moving at velocity v relative to S'. Thus, $ct_2' - ct_1'$ is the proper time $c\tau$.

$$\begin{aligned}\Delta ct = L^{-1}[ct_2' - ct_1'] &= \gamma(ct_2' + \frac{v}{c}x') - \gamma(ct_1' + \frac{v}{c}x')\\ &= \gamma(ct_2' - ct_1')\end{aligned}$$

$$\Delta ct = \gamma c\tau \tag{1.11}$$

The following example illustrates that it is impossible to have a single synchronized time for clocks on a rotating disk.

Question 2: Rotating Clocks

Consider a rapidly rotating disk of radius R with angular velocity ω. Find:
(a) The infinitesimal length units ds_r and ds_θ in a frame where a small piece of the disk is at rest.
(b) The ratio between radius and circumference as measured in a frame rotating with the disk.
(c) The time dilation effect at different radii on the disk.
(d) The discrepancy between times at $\theta = 0$ and $\theta = 2\pi$ if we suppose the clocks can be synchronized around the entire disk.

Solution: (a) The length units in the unprimed frame (lab frame) are $rd\theta$ and dr. So, the angular and radial components in the primed frame (proper length) are $ds_\theta = \gamma_r r d\theta$ and $ds_r = dr$, where dr remains unchanged between frames as it is perpendicular to the direction of motion.

Here, $v = \omega r$, so $\gamma_r = 1/\sqrt{1 - \frac{\omega^2 r^2}{c^2}}$.

(b) The radius in the rest frame S' of the disk remains unchanged, and we find the circumference by integrating the length unit ds_θ from 0 to 2π:

$$\begin{aligned}\int_0^{2\pi} ds_\theta &= \int_0^{2\pi} \gamma_r r d\theta\\ &= \gamma_r r\theta \Big|_0^{2\pi}\\ &= 2\pi r\gamma_r\end{aligned}$$

So, the ratio is $2\pi r\gamma_r / r = 2\pi\gamma_r$.

(c) Assume clocks are attached to the disk at distances r and angle $\theta = 0$ at lab time $t = 0$, each synchronized at times $t_r = 0$. Then, at later time $t = t_0$, equation (1.11) tells us that $\gamma t_{0r} = t_0$, so the clock at radius r on the disk will display the local time $t_{0r} = \frac{1}{\gamma_r}t_0$.

(d) Suppose we can place clocks around the edge of the rotating disk with distance ds_θ between them, and synchronize them in sequence so that neighboring clocks display the same time for simultaneous events in the rest frames of the clocks. Calculating from the lab frame, there should be an infinitesimal discrepancy between two adjacent clocks, following the inverse Lorentz transformation from the rest frame of the clocks to the lab frame.

$$\mathsf{c}dt = \gamma_R(\underbrace{c d\tau}_{0} + \frac{v ds_\theta}{c}) = \frac{\gamma_R v ds_\theta}{c} = \frac{\gamma(\omega r)(\gamma_R r d\theta)}{c}$$

Set the constant $K = \gamma_R^2 \omega r^2/c$. Now, we integrate with respect to θ from 0 to 2π to find the total time discrepancy around the entire disk:

$$\Delta ct = \int_0^{2\pi} K d\theta = K\theta \Big|_0^{2\pi} = 2\pi K = \frac{2\pi\gamma_R^2 \omega r^2}{c}$$

Converting this to the proper time (rotating frame), we have $\Delta c\tau = \frac{2\pi\gamma_R\omega r^2}{c}$, which is nonzero.

1.2.2 Tensors

When we index by i, the sum runs from 1 to 3. When we index by μ, it instead runs from 0 to 3.

We have a set of orthogonal basis vectors $\{e_\mu\}$ in spacetime. Then we write

$$\underline{\mathbf{x}} = x^\mu \underline{\mathbf{e}}_\mu \tag{1.12}$$

In Einstein summation, we can write the following products:

$$\mathbf{a} \cdot \mathbf{b} = a_i b_i$$

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \epsilon_{ijk} a_i b_j c_k$$

Definition 1.2.1: Metric Tensor

The metric tensor is given by

$$g = (g_{\mu\nu}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Recall the Lorentz invariant line element $d\underline{x}^2 \equiv ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$. It can instead be written as:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1.13}$$

Considering this spacetime interval as a generalized scalar product (Minkowski inner product), we can see that the basis vectors are orthogonal, from $\langle \underline{\mathbf{e}}_\mu, \underline{\mathbf{e}}_\nu \rangle = g_{\mu\nu}$. That is, 0 when μ and ν differ, 1 when they are the same and nonzero, and -1 when they are both 0.

Definition 1.2.2: Upper and Lower Indices

The precise definition of subscript coordinates is

$$x_\mu = g_{\mu\nu} x^\nu, x_\nu = g_{\mu\nu} x^\mu$$

This is essentially matrix multiplication. It gives us the relations:

$$x_0 = -x^0, x_1 = x^1, x_2 = x^2, x_3 = x^3$$

This will be useful to simplify relativistic equations later. The upper indices are called *contravariant* components, and lower indices *covariant* components. From example 1.2.3, it immediately follows that

$$ds^2 = dx_\mu dx^\mu \tag{1.14}$$

The metric tensor is its own inverse, as it squares to the identity. Thus we can also raise indices, via $g_{\mu\nu} = g^{\mu\nu}$, and $x^\mu = g^{\mu\nu} x_\nu$, $x^\nu = g^{\mu\nu} x_\mu$.

Note:-

We will be using the metric signature $(-,+,+,+)$ which describes the eigenvalues of g. In particle physics, $(+,-,-,-)$ is more common.

Definition 1.2.3: Lorentz Transformation

A Lorentz transformation from frames S to S' with relative velocity v in component form can be written $x'^{\mu} = L^{\mu}{}_{\nu}x^{\nu}$. Note that the first index of L is the row, and the second index is the column. Then $x' = Lx$, where L is given by

$$L = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In which $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$

We derive a condition on the Lorentz transformation via the invariance of the spacetime interval. It must be the case that

$$g_{\mu\nu}dx'^{\mu}dx'^{\nu} = g_{\mu\nu}L^{\mu}{}_{\rho}L^{\nu}{}_{\sigma}dx^{\rho}dx^{\sigma} = g_{\rho\sigma}dx^{\rho}dx^{\sigma} \tag{1.15}$$

The outer two expressions are equal by invariance of the spacetime interval, and the middle is the left expression written in coordinates from the unprimed frame. Thus, $g_{\mu\nu}L^{\mu}{}_{\rho}L^{\nu}{}_{\sigma} = g_{\rho\sigma}$ as our choice of dx was arbitrary.

Then in matrix form, where the transpose is due to the repeated μ in the leftmost index which corresponds to rows,

$$L^T g L = g. \tag{1.16}$$

Definition 1.2.4: Four-vector

A general four-vector is a spacetime vector $\underline{\mathbf{A}}$ which has four components A^{μ} and transforms as $A^{\mu} \mapsto L^{\mu}{}_{\nu}A^{\nu} = A'^{\nu}$

We can decompose a four-vector in any set of basis vectors:

$$\underline{\mathbf{A}} = A^{\mu}\underline{\mathbf{e}}_{\mu} = A'^{\mu}\underline{\mathbf{e}}'_{\mu}$$

Thus a Lorentz transformation of a four-vector is a change of basis.

Definition 1.2.5: Minkowski inner product

The Minkowski inner product or Lorentz invariant scalar product, denoted $\langle \underline{A}, \underline{B} \rangle$ or $\underline{A} \cdot \underline{B}$, is defined as

$$\underline{A} \cdot \underline{B} = g_{\mu\nu}A^{\mu}B^{\nu} = A^{\mu}B_{\nu}$$

The square norm is not always positive.

Timelike vectors satisfy $\underline{A}^2 > 0$, lightlike $\underline{A}^2 = 0$, and spcaelike $\underline{A}^2 < 0$

Note that orthogonality in the sense of $\underline{A} \cdot \underline{B} = 0$ does not necessarily appear orthogonal on a Minkowski diagram. For example, a lightlike vector is perpendicular to itself. Two orthogonal four-vectors have equal angles with the light cone in opposite directions, which is why a lightlike vector is perpendicular to itself (0 both ways)

Example 1.2.1 (Lorentz transformation on lower indices)

Recall that $x_{\mu} = g_{\mu\nu}x^{\nu}$, $x_{\nu} = x^{\mu}g_{\mu\nu}$, $x^{\mu} = g^{\mu\nu}x_{\nu}$, and $x^{\nu} = x_{\mu}g^{\mu\nu}$. Consider a lower-indexed four-vector

$A_\mu = g_{\mu\nu}A^\nu$. Then we have:

$$\begin{aligned} A'_\mu &= g_{\mu\nu}A'^\nu \\ &= g_{\mu\nu}L^\nu{}_\rho A^\rho \\ &= g_{\mu\nu}L^\nu{}_\rho g^{\rho\sigma}A_\sigma \\ &= L_{\mu\rho}g^{\rho\sigma}A_\sigma \\ &= L_\mu{}^\sigma A_\sigma \end{aligned}$$

When the entries of L are $L^\mu{}_\nu$, the components $L_\mu{}^\nu$ represent the matrix $\tilde{L} = gLg^{-1} = (L^T)^{-1}$, since $L^T gLg^{-1} = gg^{-1} = \mathbb{I}$.

Thus, four-vectors transform as $\underline{\mathbf{A}} \mapsto (L^T)^{-1}\underline{\mathbf{A}}$

[Insert tensor notes here... will condense that section later]

Question 3: Spacelike and timelike vectors are orthogonal.

Consider two four-vectors $\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ such that $\langle \underline{\mathbf{A}}, \underline{\mathbf{B}} \rangle = 0$, where the brackets denote the Minkowski inner product.. Assume that $\underline{\mathbf{A}}$ is timeline, that is, $\underline{A}^2 = A^\mu A_\mu < 0$.

Solution: We decompose into space and time components:

$$\begin{aligned} \underline{\mathbf{A}}^2 &= -(A^0)^2 + \mathbf{A}\cdot\mathbf{A} < 0 && (1.17) \\ (A^0)^2 &> |\mathbf{A}|^2 && (1.18) \\ \langle \underline{\mathbf{A}}, \underline{\mathbf{B}} \rangle &= -A^0B^0 + \mathbf{A}\cdot\mathbf{B} = 0 && (1.19) \\ B^0 &= \frac{\mathbf{A}\cdot\mathbf{B}}{A^0} && (1.20) \end{aligned}$$

This gives the following chain of equalities and inequalities:

$$\begin{aligned} \underline{\mathbf{B}} &= -(B^0)^2 + |\mathbf{B}|^2 \\ &= -\frac{(\mathbf{A}\cdot\mathbf{B})^2}{(A^0)^2} + |\mathbf{B}|^2, \text{ by equation 1.16} \\ &= \frac{-(\mathbf{A}\cdot\mathbf{B})^2 + (A^0)^2|\mathbf{B}|^2}{(A^0)^2} \\ &\geqslant \frac{-|\mathbf{A}|^2|\mathbf{B}|^2 + (A^0)^2|\mathbf{B}|^2}{(A^0)^2}, \text{ by the Cauchy-Schwarz inequality.} \\ &> \frac{-|\mathbf{A}|^2|\mathbf{B}|^2 + |\mathbf{A}|^2|\mathbf{B}|^2}{(A^0)^2} = 0, \text{ by equation 1.14} \end{aligned}$$

Therefore $\underline{\mathbf{B}}$ is a spacelike vector. ☐

Question 4: Raising and lowering indices practice

From $g_{\mu\nu}L^\mu{}_\rho L^\nu{}_\sigma = g_{\rho\sigma}$, show that $L_\mu{}^\rho L^\mu{}_\sigma = \delta^\rho_\sigma$.

Solution:

$$\begin{aligned} g_{\mu\nu}L^\mu{}_\rho L^\nu{}_\sigma &= g_{\rho\sigma} \\ L_{\nu\rho}L^\nu{}_\sigma &= g_{\rho\sigma} \\ g^{\mu\rho}L_{\nu\rho}L^\nu{}_\sigma &= g^{\mu\rho}g_{\rho\sigma} \\ L_\nu{}^\mu L^\nu{}_\sigma &= \delta^\mu_\sigma \\ L_\mu{}^\rho L^\mu{}_\sigma &= \delta^\rho_\sigma, \text{ by relabeling } \mu \to \rho \text{ and } \nu \to \mu \end{aligned}$$

□

Example 1.2.2 (Tensor field derivative identities)

The following identities are useful for computing derivatives of tensor fields:

$$\frac{\partial x^{\mu}}{\partial x^{\nu}} = \delta^{\mu}_{\nu} \tag{1.21}$$

$$g_{\mu\nu}\delta^{\nu}_{\rho} = g_{\mu\rho} \tag{1.22}$$

Chapter 2

PH632

Chapter 3

Quantum Stochastics

Chapter 4

Quantum Mechanics