Math 261: Practice Midterm 2

1. Let W be the plane in $\mathbb{R}^3$ spanned by $u_1 = \begin{pmatrix} 4 \\ -1 \\ -8 \end{pmatrix}$ and $u_2 = \begin{pmatrix} -7 \\ 4 \\ -4 \end{pmatrix}$.

   (a) Find the orthogonal projection $\widehat{v}$ of the vector $v = \begin{pmatrix} 8 \\ 7 \\ -7 \end{pmatrix}$ onto W.

   (b) Find the distance from v to W, i.e., the length of the vector $v - \widehat{v}$.

2. (a) Consider the vectors: $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}$. Do these vectors span $\mathbb{R}^3$?

   (b) Find a basis for the set of vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$ that belong to the plane $2x + 3y + 5z = 0$.

3. Let $v_1 = \begin{pmatrix} 1 \\ 0 \\ t \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, where t is some real number.

   (a) Find all the values of t for which v_1, v_2, v_3 are linearly dependent, and find a linear dependence relation in each case.

   (b) Let $A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ t & 1 & 0 \end{pmatrix}$, where t is a real number. For which values of t does the equation $Ax = 0$ have a non-trivial solution?

4. (a) Show that $W = \left\{ \begin{pmatrix} x \\ x - y \\ y \end{pmatrix} : x, y \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^3$.

   (b) Let $A = \begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{pmatrix}$. Find a basis of $Col(A), Row(A)$, and $Nul(A)$.