

# Math 261: Practice Midterm 1

**Instructions:** Exam time is 75 minutes. You are allowed one sheet of hand-written notes (both sides). Calculators, the textbook, and additional notes are not allowed. Justify all your answers carefully.

---

1. (a) Write down the augmented matrix for the following system of linear equations and find the reduced row echelon form of the matrix. Indicate which row operation you are performing at each step.

$$\begin{aligned}x_1 + 5x_2 - x_3 + 3x_4 &= 2 \\ -2x_1 - 10x_2 + 3x_3 - 8x_4 &= -1 \\ 3x_1 + 15x_2 - 3x_3 + 9x_4 &= 6.\end{aligned}$$

- (b) Using the reduced row echelon matrix in your answer from Part (a):
- (i) Identify the pivot columns of the augmented matrix.
  - (ii) Identify the basic and free variables in the linear system.
  - (iii) Does the system have no solutions, exactly one solution, or infinitely many solutions? Why?

2. Consider the vectors:

$$v_1 = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} -4 \\ 4 \\ -8 \end{pmatrix}.$$

Clearly, one can write the zero vector  $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  as a linear combination of  $v_1, v_2$  and  $v_3$  by selecting all the scalars to be zero. Is there another linear combination of  $v_1, v_2$  and  $v_3$  that gives the zero vector, yet not all the scalars are zero?

3. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 7 \\ -1 & 1 & 5 \end{bmatrix}$$

(b) Use Part (a) to solve the following linear system:

$$\begin{aligned}x_1 + x_2 + 2x_3 &= 2 \\2x_1 + 3x_2 + 7x_3 &= 3 \\-x_1 + x_2 + 5x_3 &= 5.\end{aligned}$$

4. Let  $A = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$ .

(a) Do these matrices commute? In other words, is it true that  $AB = BA$ ?

(b) Find  $B^t A^t$ .

5. Let  $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 5 \\ 4 & 1 & 1 \end{bmatrix}$ .

(a) Find  $\det A$ .

(b) Is  $A$  invertible?