Math 261: Practice Problems for the Final

1. (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation that reflects each vector $x = (x_1, x_2, x_3)$ through the plane $x_3 = 0$ onto $T(x) = (x_1, x_2, -x_3)$. Show that T is a linear transformation.

   (b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(x_1, x_2) = (x_1 + x_2, 4x_1 + 5x_2)$. Find $x \in \mathbb{R}^2$ such that $T(x) = (3, 8)$.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation given by

$$T(x_1, x_2, x_3) = (x_2 + x_3, 2x_2 + 4x_3, x_3 - x_2, 4x_3 - 2x_2).$$

   What is a basis of the kernel of T? How about the image of T?

3. Suppose that λ_1 and λ_2 are two distinct eigenvalues of a square matrix A. Let v_1 be an eigenvector corresponding to λ_1, and v_2 an eigenvector corresponding to λ_2. Is the set $\{v_1, v_2\}$ linearly independent? Justify your answer.

4. Let $A = \begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}$. Compute A^{100}.