Math 256

Mini Project # 2

Due:

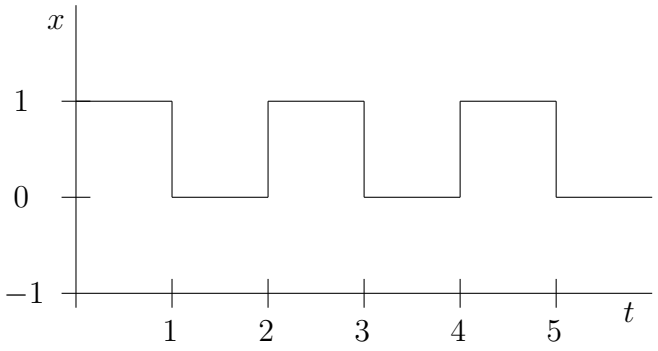
Objective : Observe the behavior of RLC circuits and forced RLC circuits.

Recall we have learned from the book that the 2nd order linear differential equation for an RLC circuit is given by,

$$L\,C\,\frac{d^2x}{dt^2} + R\,C\,\frac{dx}{dt} + x = V_S(t),$$

where x is the voltage across the capacitor. We will explore the solutions to this D.E. by hand and by the computer. If we force the circuit by applying a nonzero $V_S(t)$. We are going the observe the response of the circuit to the forcing function.

One forcing function we are going the consider is the *square wave* function. This function consists of periodic pulses of constant values. Consider the following representation.



This representation shows why the name square wave is used. We will make use of the square wave function below.

1. Solve the following differential equation (by hand). Show work.

$$L\,C\,\frac{d^2x}{dt^2} + R\,C\,\frac{dx}{dt} + x = 0 \tag{1}$$

where $R = 20$, $C = 0.01$, $L = 10$, $x(0) = 10$, $x'(0) = 0$.

2. Use technology to draw the solution curve in the phase plane. You will need to print this out for the project. (Winplot has the **end pt** button if you need to change the view window.)

3. Use technology to draw the component graph of $x(t)$ over the interval $0 \leq t \leq 5$. Winplot does this under the **Misc** tab using **Deq miscellany**.

4. Solve the following forced D.E. by hand, using complexification. Show work.

$$L\,C\,\frac{d^{\,2}x}{dt^2} + R\,C\,\frac{dx}{dt} + x = 2\,\sin(t) \tag{2}$$

where $L = 10$, $R = 20$, $C = 0.01$, $x(0) = 10$, $x\,'(0) = 0$.

5. Use technology to draw the solution curve in the phase plane.

6. Use technology to draw the component graph of $x(t)$ over the interval $0 \leq t \leq 25$.

7. Use technology to draw the component graph of $x\,'(t)$ over the interval $0 \leq t \leq 25$.

8. Suppose we use a square wave function to force the D.E. The square wave may be built using the **hvs** function. Here is one way to define the the square wave function.

   f(x) = sum((-1)^n∗hvs(x − n) , n , 0 , 50)

   Consider the following forced D.E.

$$L\,C\,\frac{d^{\,2}x}{dt^2} + R\,C\,\frac{dx}{dt} + x = 2 * f(t/3) \tag{3}$$

   where $R = 20$, $C = 0.01$, $L = 10$, $x(0) = 10$, $x\,'(0) = 0$.

9. Use technology to draw the solution curve in the phase plane. (For Winplot don't forget the **end pt** button.)

10. Use technology to draw the component graph of $x(t)$ over the interval $0 \leq t \leq 25$.

You **must** give Captions to your graphs.

Include in your write-up:

The solution to eq. 1, done by hand.
The print out of the solution curve for eq. 1.
The print out of the component graph graph for eq. 1.
The solution to eq. 2, done by hand.
The print out of the solution curve for eq. 2.
The print out of the two component graphs for eq. 2.
The print out of the solution curve for eq. 3.
The print out of the component graph for eq. 3.