KEY

Differential Equations

Solve the following differential equations.

1. $\frac{dy}{dt} = t \cdot y^2$

$y = 0$ is the equilibrium solution

$$\int y^{-2}\,dy = \int t\,dt$$

$$-y^{-1} = \frac{1}{2}t^2 + C$$

$$(-1)\quad -\frac{1}{y} = \frac{1}{2}t^2 + C \quad (-1)$$

$$\frac{1}{y} = -\frac{1}{2}t^2 + C$$

$$y = \frac{1}{-\frac{1}{2}t^2 + C}$$

2. $\frac{dy}{dt} = t^2 e^{-y}$

No equilibrium solutions

$$\int e^y \, dy = \int t^2 \, dt$$

$$e^y = \frac{1}{3} t^3 + C$$

$$\ln(e^y) = \ln\left(\frac{1}{3}t^3 + C\right)$$

$$y = \ln\left(\frac{1}{3}t^3 + C\right) \quad \checkmark$$

3. $\frac{dy}{dt} = \frac{1+y}{t^2}$

$y = -1$ is the equilibrium solution.

$$\int \frac{1}{1+y}\,dy = \int t^{-2}\,dt$$

$u = 1+y$
$du = dy$

$$\ln|1+y| = -1t^{-1} + C$$

$$e^{\ln|1+y|} = e^{-\frac{1}{t}+C}$$

$$|1+y| = e^{C}e^{-\frac{1}{t}}$$

$$1+y = \pm e^{C}e^{-\frac{1}{t}}$$

$$y = -1 + Ce^{-\frac{1}{t}}$$

key

4. $\frac{dy}{dx} = \frac{\sec(y)}{x}$

Note

No equilibrium solutions

$$\int \frac{1}{\sec(y)}\,dy = \int \frac{1}{x}\,dx$$

$$\int \frac{1}{(1/\cos(y))}\,dy = \int \frac{1}{x}\,dx$$

$$\int \cos(y)\,dy = \int \frac{1}{x}\,dx$$

$$\sin(y) = \ln|x| + C$$

$$\sin^{-1}(\sin(y)) = \sin^{-1}(\ln|x| + C)$$

$$y = \sin^{-1}(\ln|x| + C) \quad \checkmark$$