Differential Equations

Maple has a built in **solve** feature. Try the following.

$$>$$
 solve($\ln(x)^{\wedge}2 = 16$, x) ;

This returns $x = e^{\pm 4}$.

For differential equations Maple has a built in command to solve differential equations. This is the **dsolve** command. You must type y(t).

$$> \mathrm{de} := \mathrm{diff}(\mathrm{y}(\mathrm{t}),\mathrm{t}) = 2^{*}\mathrm{t}^{*}\mathrm{y}(\mathrm{t})/(\mathrm{t}^{\wedge}2{+}4) \; ;$$

> dsolve(de , y(t)) ;

Take note that *Maple* represents the constant of integration as $_C1$. To verify that the computer got a correct solution we can assign the solution to a function of t. Do the following.

$$> y := unapply(rhs(\%) , t) ;$$

 $> y(t) ;$
 $> diff(y(t) , t) ;$
 $> simplify(rhs(de)) ;$

Don't forget to **restart:** before you work on a new problem to erase the definition for y(t).

Also, $Maple\ {\rm can}\ {\rm solve}\ {\rm initial-value}\ {\rm problems}.$ We can find a particular solution.

> restart:

- $> de := diff(y(t),t) = (\cos(t) + y(t)) / \cos(t);$
- > dsolve({de , y(0) = 0} , y(t)) ;
- > y := unapply(rhs(%) , t);
- > plot(y(t) , t=-6..12 , y=-10..10) ;

Use *Maple* to solve the following differential equations.

$$\frac{dy}{dt} = y^2 \qquad \qquad \frac{dy}{dt} = -5 y + 2 t \\ \frac{dy}{dt} = -.25 y^2 + 1.5 y \qquad \qquad \frac{dy}{dt} = -.016 y$$

Now, solve the following initial value problems.

$$\begin{aligned} \frac{dy}{dt} &= .08 \, y \ , \ y(0) = 150 \\ \\ \frac{dy}{dt} &= \frac{2 \, y + 1}{t} \ , \ y(1) = 0 \\ \\ \frac{dy}{dt} &= \frac{t}{y - t^2 \, y} \ , \ y(3) = 4 \\ \\ \frac{dy}{dt} &= (y^2 + 1) \, t \ , \ y(0) = 1 \\ \\ \frac{dy}{dt} &= \frac{2}{t} \, y + 2 \, t^2 \ , \ y(-2) = 4 \\ \\ \\ \frac{dy}{dt} &= \frac{2 \, t}{1 + t^2} \, y + \frac{2}{1 + t^2} \ , \ y(0) = 2 \end{aligned}$$

Graphical Stuff

> restart:

> with(DEtools):

$$>\mathrm{de}:=\mathrm{diff}(\mathrm{y}(\mathrm{t}),\mathrm{t})=-\mathrm{y}(\mathrm{t})^{\wedge}2+5^{*}\mathrm{y}(\mathrm{t})\;;$$

$$> DEplot(de, y(t), t=-3..3, y=-2..7, color=black);$$

$$> ext{ init } := \{ \ [0,1], [1,1], [-1,-1], [-1,6] \} \ ;$$

$> DEplot(de \ , \ y(t) \ , \ t=-3..3 \ , \ init \ , \ y=-2..7 \ , \ color=black \ , \ linecolor=red) \ ;$

Do the same for the following. You may need to change initial conditions and the t-interval.

$$\frac{dy}{dt} = -\frac{1}{t}y + 3t \qquad , \quad t = 1..5$$
$$\frac{dy}{dt} = \frac{3t^2 + 4t + 2}{2(y-1)} \qquad , \quad t = -3..3$$

Done in T_EX .