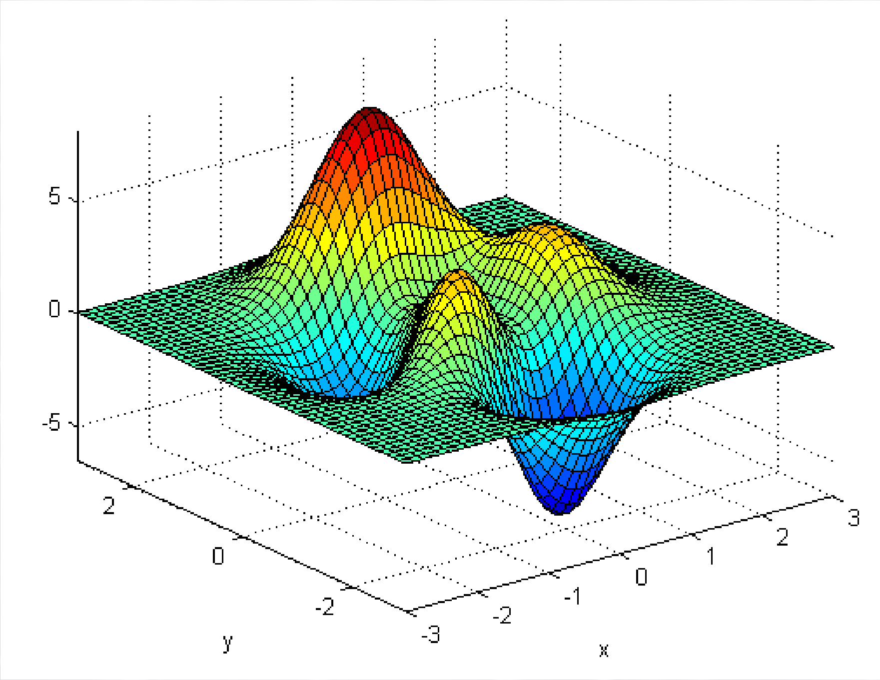
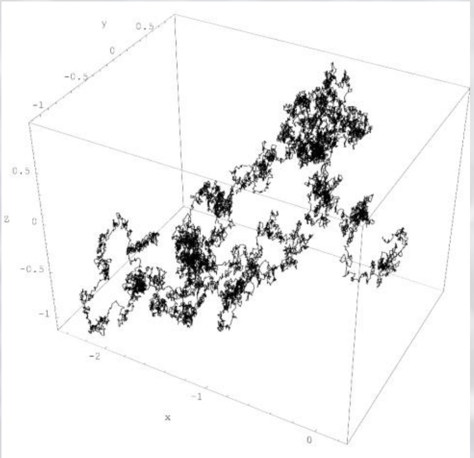
A.I.: Beyond Classical Search



Trivial Algorithms



- Random Sampling
  - Generate a state randomly
- Random Walk
  - Randomly pick a neighbor of the current state
- Both algorithms asymptotically complete.

Overview

- **Previously** we addressed a single category of problems: <u>**observable, deterministic, known environments**</u> where the solution is a sequence of actions.
- Now we consider what happens when these assumptions are **relaxed**.
- First we look at purely **local search** strategies, including methods inspired by statistical physics (**simulated annealing**) and evolutionary biology (**genetic algorithms**).
- Later, we examine what happens when we relax the assumptions of determinism and observability. The key idea is that the agent cannot predict exactly what percept it will receive, so it considers a **contingency** plan.
- Lastly, we discuss online search.

Local Search for Optimization

- Note that for many types of problems, the path to a goal is irrelevant (we simply want the solution – consider the 8-queens).
- If the path to the goal does not matter, we might consider a class of algorithms that don't concern themselves with paths at all.

- **Local search algorithms** operate using a single current node (rather than multiple paths) and generally move only to neighbors of that node.

- Typically, the paths followed by the search are not retained (the benefit being a potentially substantial memory savings).

Local Search for Optimization

- Local search
  - Keep track of single current state
  - Move only to neighboring states
  - Ignore paths

- Advantages:
  - Use very little memory
  - Can often find reasonable solutions in large or infinite (continuous) state spaces.

- “Pure optimization” problems
  - All states have an objective function
  - Goal is to find state with max (or min) objective value
  - Does not quite fit into path-cost/goal-state formulation
  - Local search can do quite well on these problems.

Local Search for Optimization

- In addition to finding goals, local search algorithms are useful for solving **pure optimization problems**, in which we aim to find the best state according to an **objective function**.

- Nature, for example, provides an objective function – *reproductive fitness*.

- To understand local search, we consider **state-space landscapes**, as shown next.

Optimization

- So what is optimization?

- Find the minimum or maximum of an objective function (usually: given a set of constraints):

$$
\begin{aligned}
&\arg\min_x f_0(x) \\
&\qquad \text{s.t.}\, f_i(x) \leq 0, i = \{1, \ldots, k\} \\
&\qquad\qquad h_j(x) = 0, j = \{1, \ldots l\}
\end{aligned}
$$

Why Do We Care?

Linear Classification

$$\arg\min_{w} \sum_{i=1}^{n} ||w||^2 + C \sum_{i=1}^{n} \xi_i$$

$$\text{s.t. } 1 - y_i x_i^T w \leq \xi_i$$

$$\xi_i \geq 0$$

Maximum Likelihood

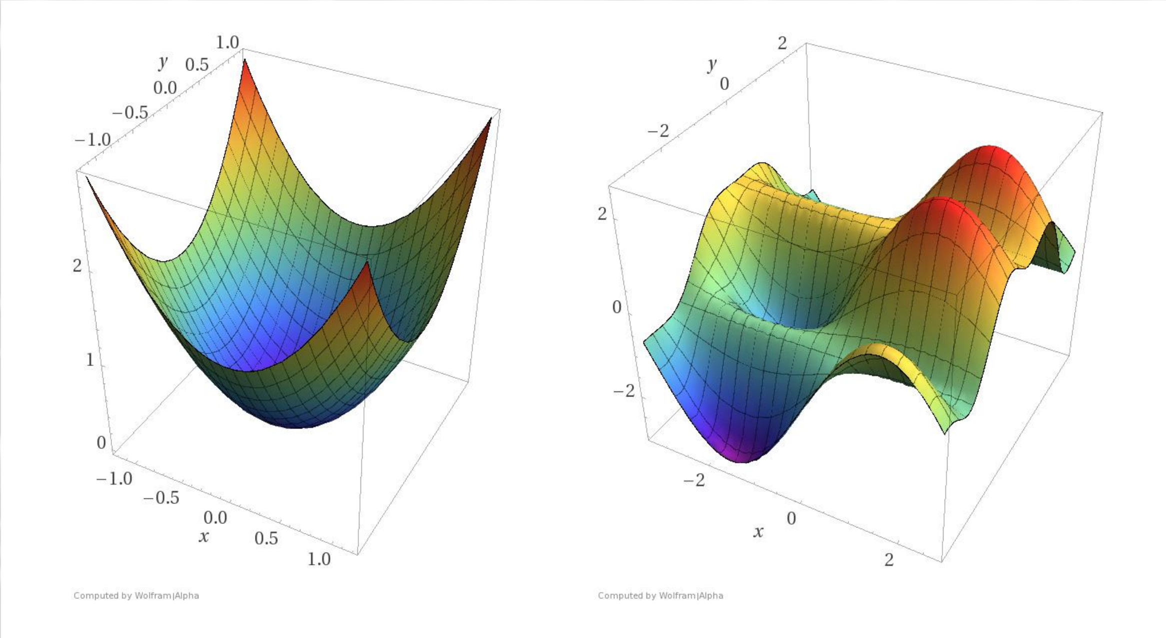
$$\arg\max_{\theta} \sum_{i=1}^{n} \log \, p_\theta(x_i)$$

K-Means

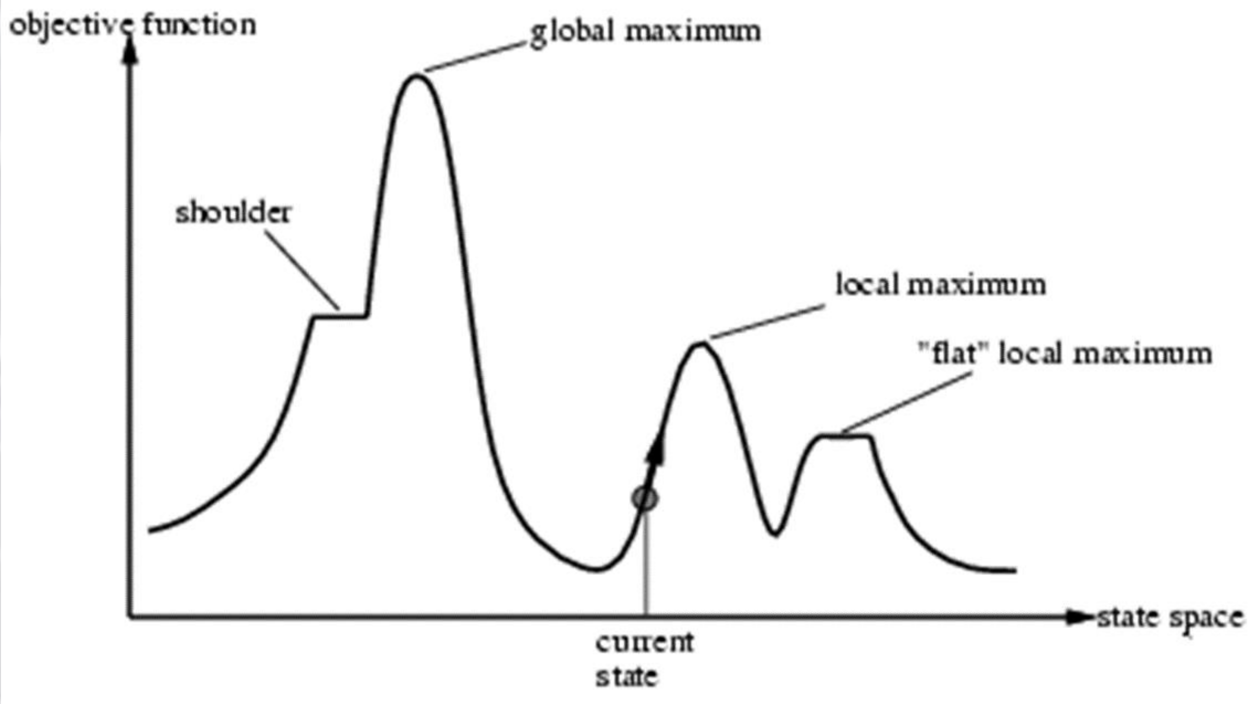
$$\arg \min_{\mu_1, \mu_2, \ldots, \mu_k} J(\mu) = \sum_{j=1}^{k} \sum_{i \in C_j} ||x_i - \mu_j||^2$$

Convexity

- We prefer convex problems.



Local Search for Optimization



State-Space Landscape

- If elevation corresponds with cost, then the aim is to find the lowest valley – **a global minimum**; if elevation corresponds to an objective function, then the aim is to find the highest peak – **a global maximum**.
- A **complete** local search algorithm always finds a goal (if one exists); an **optimal** algorithm always finds a global minimum/maximum.

Hill-Climbing (Greedy Local Search)

function HILL-CLIMBING(*problem*) **return** a state that is a local maximum
input: *problem*, a problem
local variables: *current*, **a node.**
neighbor, **a node.**

current ← MAKE-NODE(INITIAL-STATE[*problem*])
loop do
neighbor ← a highest valued successor of *current*
if VALUE [*neighbor*] ≤ VALUE[*current*] **then return** STATE[*current*]
current ← *neighbor*

Minimum version will reverse inequalities and look for lowest valued successor

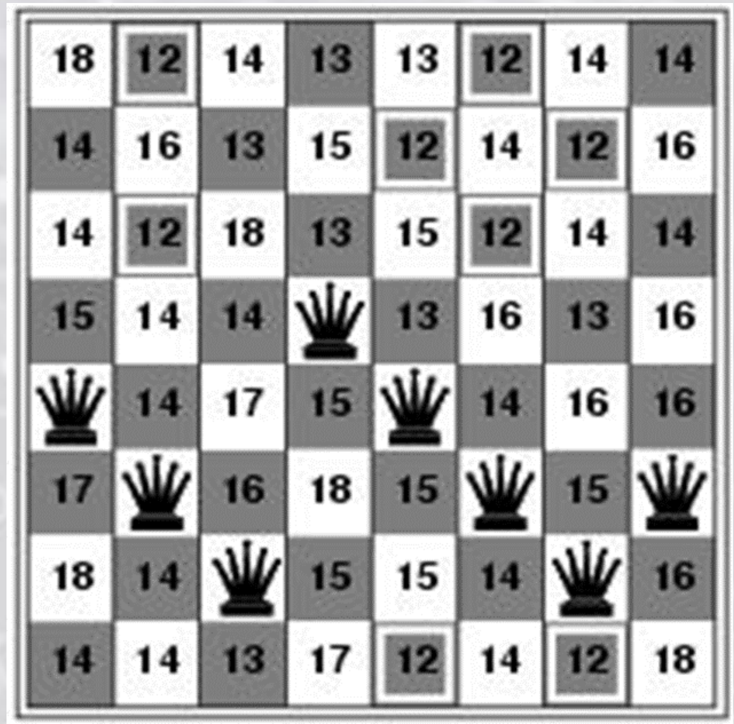
Hill-Climbing

- "A loop that continuously moves towards increasing value"
  - terminates when a peak is reached
  - Aka greedy local search
- Value can be either
  - Objective function value
  - Heuristic function value (minimized)

- Hill climbing does not look ahead of the immediate neighbors
- Can randomly choose among the set of best successors
  - if multiple have the best value

- "Climbing Mount Everest in a thick fog with amnesia"

Hill-Climbing (8-queens)

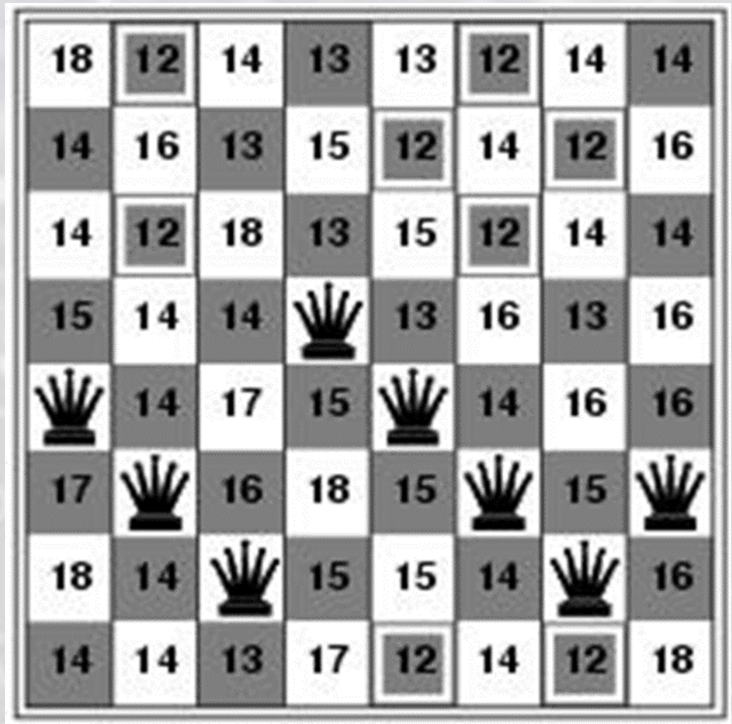


- Need to convert to an optimization problem!

Hill-Climbing (8-queens)

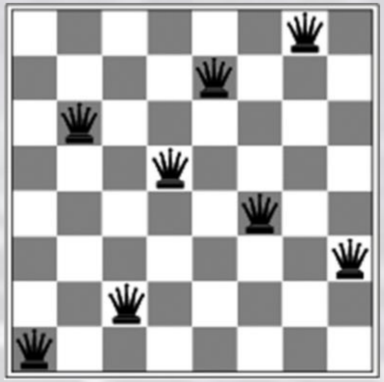
- State
  - All 8 queens on the board in some configuration
- Successor function
  - move a single queen to another square in the same column.
- Example of a heuristic function *h(n)*:
  - the number of pairs of queens that are attacking each other
  - (so we want to minimize this)

Hill-Climbing (8-queens)



- h = number of pairs of queens that are attacking each other
- $h = 17$ for the above state

Hill-Climbing (8-queens)

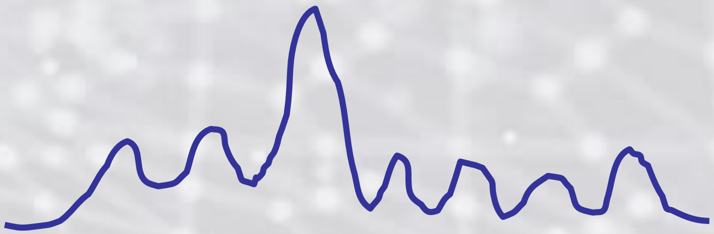


- Randomly generated 8-queens starting states…
- 14% the time it solves the problem
- 86% of the time it get stuck at a local minimum

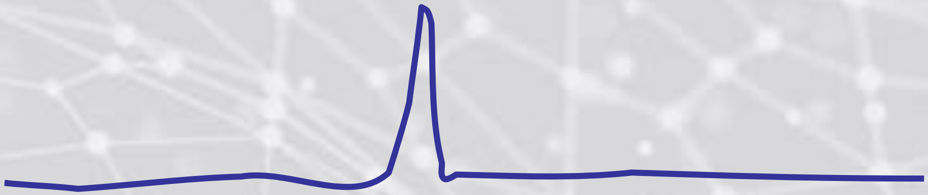
- However…
  - Takes only 4 steps on average when it succeeds
  - And 3 on average when it gets stuck
  - (for a state space with 8^8 =~17 million states)

Hill Climbing Drawbacks

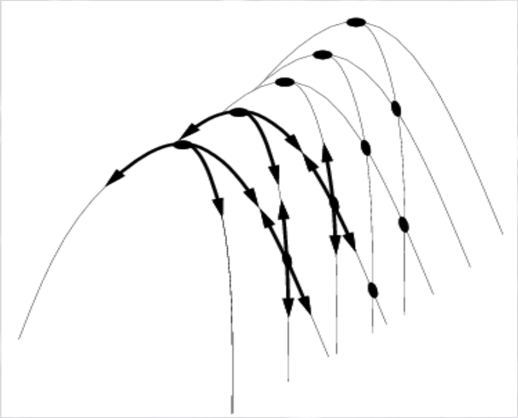
- Local maxima



- Plateaus



- Diagonal ridges



Escaping Shoulders: Sideways Move

- If no downhill (uphill) moves, allow sideways moves in hope that algorithm can escape
  - Need to place a limit on the possible number of sideways moves to avoid infinite loops
- For 8-queens
  - Now allow sideways moves with a limit of 100
  - Raises percentage of problem instances solved from 14 to 94%
  - However….
    - 21 steps for every successful solution
    - 64 for each failure

Tabu Search

- Prevent returning quickly to the same state
- Keep fixed length queue ("tabu list")
- Add most recent state to queue; drop oldest
- Never make the step that is currently tabu'ed

- Properties:
  - As the size of the tabu list grows, hill-climbing will asymptotically become "non-redundant" (won't look at the same state twice)
  - In practice, a reasonable sized tabu list (say 100 or so) improves the performance of hill climbing in many problems

Escaping Shoulders/local Optima Enforced Hill Climbing

- Perform breadth first search from a local optima
  - to find the next state with better h function
- Typically,
  - prolonged periods of exhaustive search
  - bridged by relatively quick periods of hill-climbing
- Middle ground b/w local and systematic search

Hill-climbing: stochastic variations

- Stochastic hill-climbing
  - Random selection among the uphill moves.
  - The selection probability can vary with the steepness of the uphill move.

- To avoid getting stuck in local minima
  - Random-walk hill-climbing
  - Random-restart hill-climbing
  - Hill-climbing with both

Hill Climbing: stochastic variations

→When the state-space landscape has local minima, any search that moves only in the greedy direction cannot be complete

→Random walk, on the other hand, is asymptotically complete

Idea: Put random walk into greedy hill-climbing

Hill-climbing with random restarts

- If at first you don't succeed, try, try again!
- Different variations
  - For each restart: run until termination vs. run for a fixed time
  - Run a fixed number of restarts or run indefinitely
- Analysis
  - Say each search has probability p of success
    - E.g., for 8-queens, $p = 0.14$ with no sideways moves
  - Expected number of restarts?
  - Expected number of steps taken?
- If you want to pick one local search algorithm, learn this one!!

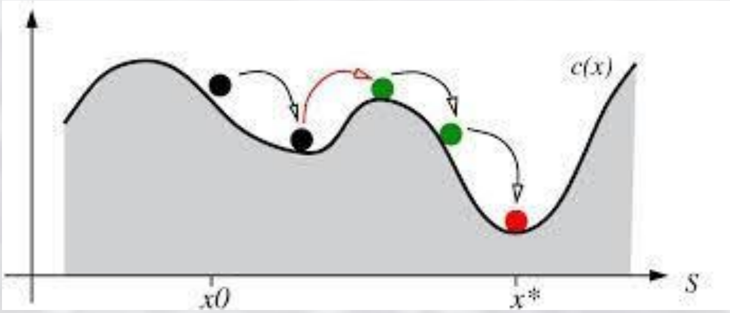
Hill-climbing with random walk

- At each step do one of the two
  - Greedy: With prob p move to the neighbor with largest value
  - Random: With prob 1-p move to a random neighbor

Hill-climbing with both

- At each step do one of the three
  - Greedy: move to the neighbor with largest value
  - Random Walk: move to a random neighbor
  - Random Restart: Resample a new current state

Simulated Annealing



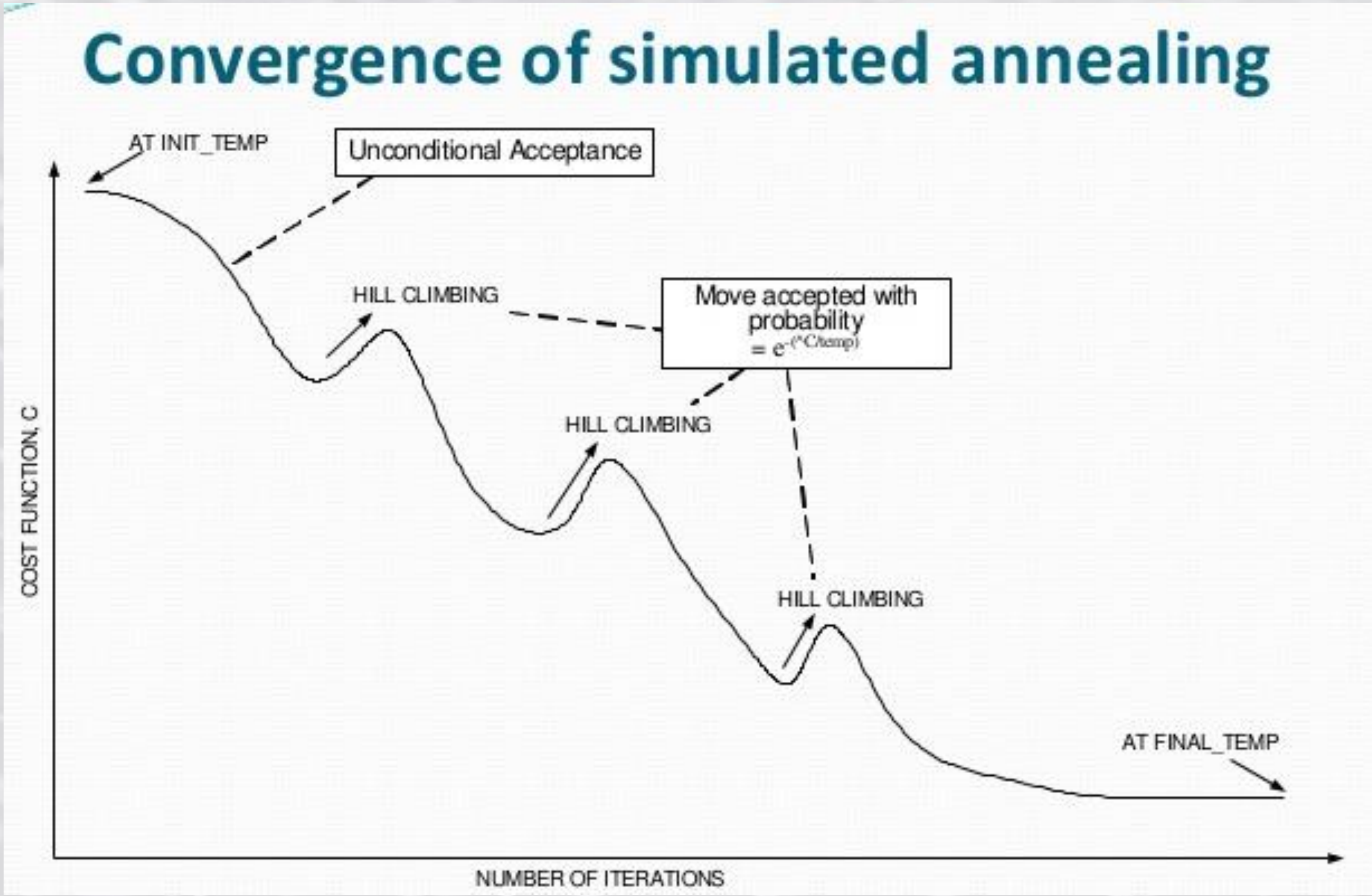
- Simulated Annealing = physics inspired twist on random walk
- Basic ideas:
  - like hill-climbing identify the quality of the local improvements
  - instead of picking the best move, pick one randomly
  - say the change in objective function is d
  - if d is positive, then move to that state
  - otherwise:
    - move to this state with probability proportional to d
    - thus: worse moves (very large negative d) are executed less often
  - however, there is always a chance of escaping from local maxima
  - over time, make it less likely to accept locally bad moves
  - (Can also make the size of the move random as well, i.e., allow "large" steps in state space)

Physical Interpretation of Simulated Annealing

- A Physical Analogy:
    - imagine letting a ball roll downhill on the function surface
        - this is like hill-climbing (for minimization)
    - now imagine shaking the surface, while the ball rolls, gradually reducing the amount of shaking
        - this is like simulated annealing

- Annealing = physical process of cooling a liquid or metal until particles achieve a certain frozen crystal state
    - simulated annealing:
        - free variables are like particles
        - seek "low energy" (high quality) configuration
        - slowly reducing temp. T with particles moving around randomly

Simulated Annealing



Simulated annealing

function SIMULATED-ANNEALING(*problem, schedule*) **return** a solution state

input: *problem*, a problem

schedule, a mapping from time to temperature

local variables: *current*, a node.

next, a node.

T, a "temperature" controlling the prob. of downward steps

current $\leftarrow$ MAKE-NODE(INITIAL-STATE[*problem*])

for t $\leftarrow$ 1 to ∞ do

T $\leftarrow$ *schedule*[*t*]

if *T = 0* **then return** *current*

next $\leftarrow$ a randomly selected successor of *current*

$\Delta E \leftarrow$ VALUE[*next*] - VALUE[*current*]

if $\Delta E > 0$ **then** *current* $\leftarrow$ *next*

else *current* $\leftarrow$ *next* only with probability $e^{\Delta E / T}$

Temperature T

- High T: probability of "locally bad" move is higher.
- Low T: probability of "locally bad" move is lower.
- Typically, T is decreased as the algorithm runs longer, i.e., there is a "temperature schedule".

- In statistical mechanics, the **Boltzmann distribution** is a probability distribution that gives the probability of a certain state as a function of that state's energy and temperature of the system to which the distribution is applied. It is given as:

$$p_i = \frac{e^{-\varepsilon_i / kT}}{\sum_{j=1}^{M} e^{-\varepsilon_j / kT}}$$

Simulated Annealing in Practice

- method proposed in 1983 by IBM researchers for solving VLSI layout problems (Kirkpatrick et al, *Science*, 220:671-680, 1983).
  - theoretically will always find the global optimum

- Other applications: Traveling salesman, Graph partitioning, Graph coloring, Scheduling, Facility Layout, Image Processing, …

- useful for some problems, but can be very slow
  - slowness comes about because T must be decreased very gradually to retain optimality

Local beam search

- Idea: Keeping only one node in memory is an extreme reaction to memory problems.

- Keep track of *k* states instead of one
  - Initially: *k* randomly selected states
  - Next: determine all successors of *k* states
  - If any of successors is goal $\rightarrow$ finished
  - Else select *k* best from successors and repeat

Local Beam Search (contd)

- Not the same as *k random-start searches run in parallel!*
- Searches that find good states recruit other searches to join them

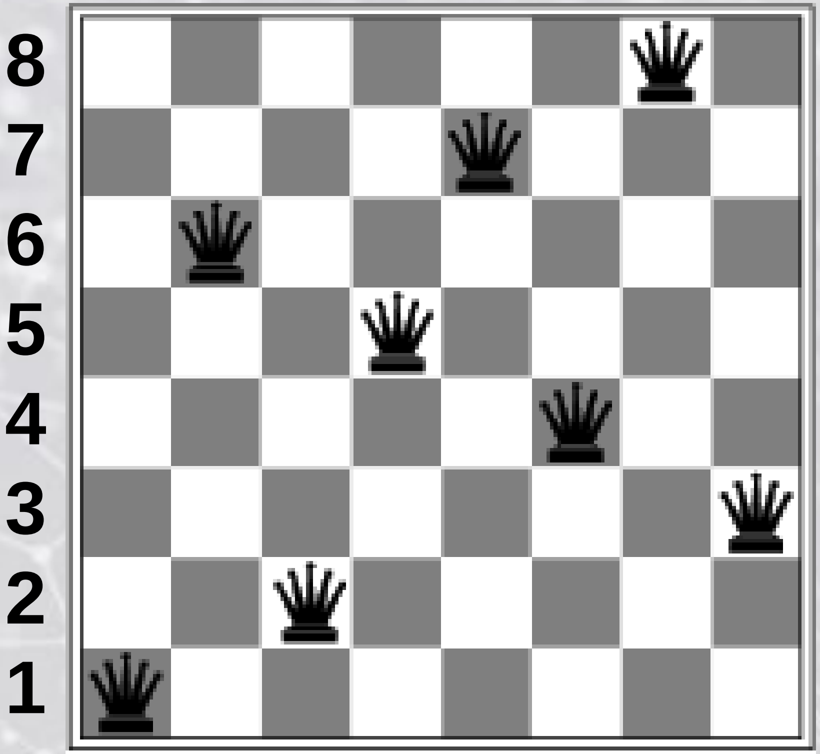
- Problem: quite often, all *k states end up on same local hill*
- Idea: Stochastic beam search
  - Choose *k successors randomly, biased towards good ones*

- Observe the close analogy to natural selection!

Genetic algorithms

- Twist on Local Search: successor is generated by combining two parent states
- A state is represented as a string over a finite alphabet (e.g. binary)
  - 8-queens
    - State = position of 8 queens each in a column
- Start with *k* randomly generated states (population)
- Evaluation function (fitness function):
  - Higher values for better states.
  - Opposite to heuristic function, e.g., # non-attacking pairs in 8-queens
- Produce the next generation of states by "simulated evolution"
  - Random selection
  - Crossover
  - Random mutation

Genetic algorithms & 8-queens



String representation
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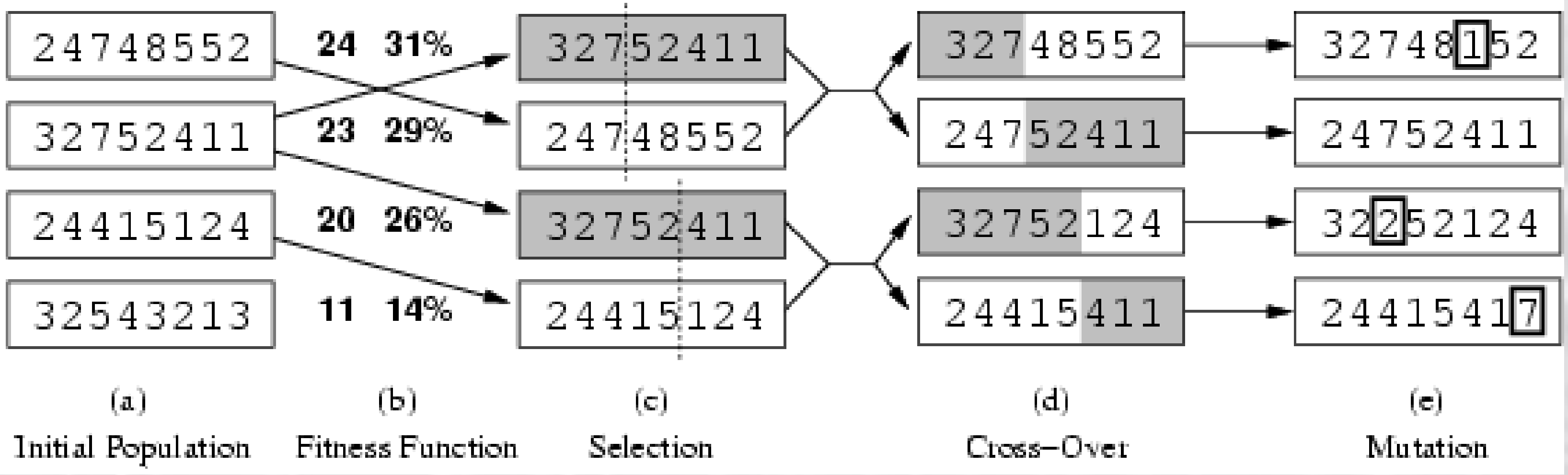
Can we evolve 8-queens through genetic algorithms?

Genetic algorithms



4 states for 8-queens problem

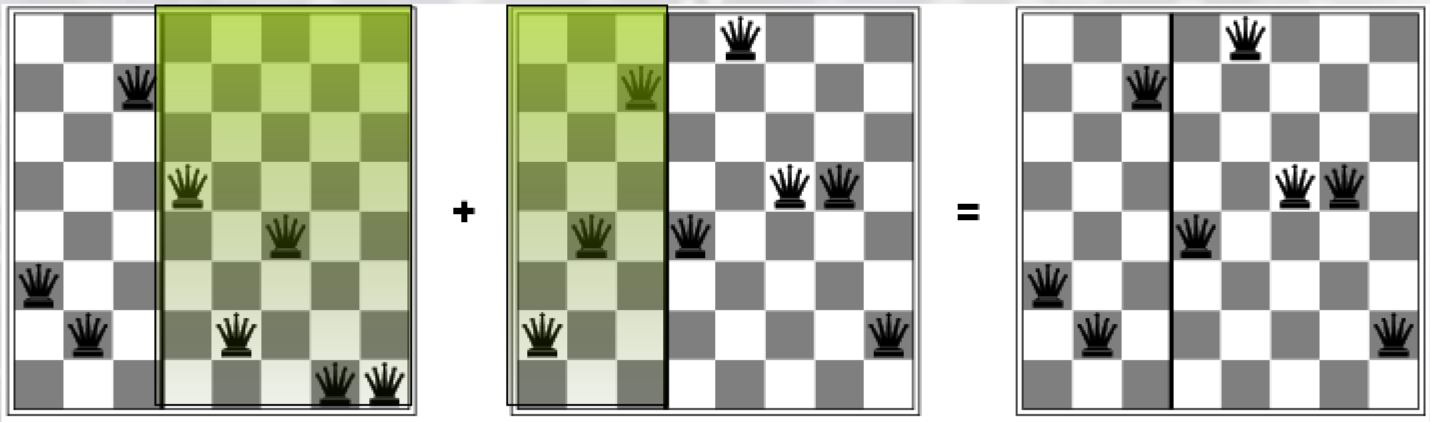
2 pairs of 2 states randomly selected based on fitness. Random crossover points selected

New states after crossover

Random mutation applied

- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

Genetic algorithms



Has the effect of “jumping” to a completely different new part of the search space (quite non-local)

Comments on Genetic Algorithms

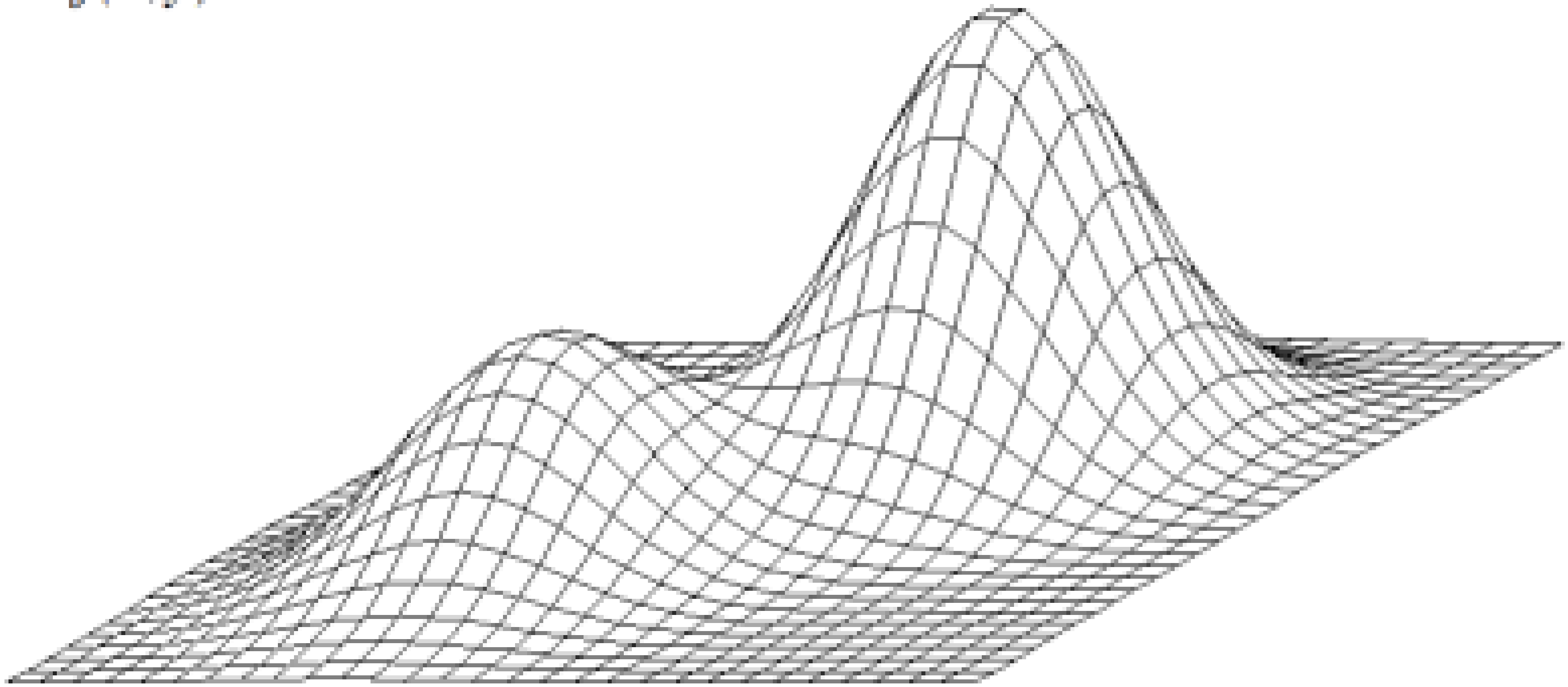
- Genetic algorithm is a variant of "stochastic beam search"
- Positive points
  - Random exploration can find solutions that local search can't
    - (via crossover primarily)
  - Appealing connection to human evolution
    - "neural" networks, and "genetic" algorithms are **metaphors**!
- Negative points
  - Large number of "tunable" parameters
    - Difficult to replicate performance from one problem to another
  - Lack of good empirical studies comparing to simpler methods
  - Useful on some (small?) set of problems but no convincing evidence that GAs are better than hill-climbing w/random restarts in general

Optimization of Continuous Functions

- Discretization
  - use hill-climbing
- Gradient descent
  - make a move in the direction of the gradient
    - gradients: closed form or empirical

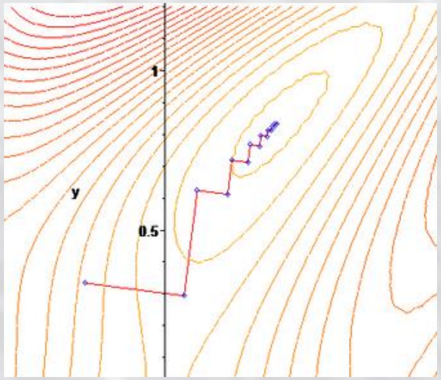
Objective Function in Continuous Search Space

$$f(x,y)=e^{-(x^2+y^2)} + 2e^{-((x-1.7)^2+(y-1.7)^2)}$$



Gradient Descent

Assume we have a continuous function: $f(x_1,x_2,\ldots,x_N)$
and we want minimize over continuous variables X1,X2,..,Xn



1. Compute the *gradients* for all i: $\partial f(x_1,x_2,\ldots,x_N) / \partial x_i$

2. Take a small step downhill in the direction of the gradient:

$$x_i \leftarrow x_i - \lambda \partial f(x_1,x_2,\ldots,x_N) / \partial x_i$$

3. Repeat.

- How to select λ
  - Line search: successively double
    - until f starts to increase again