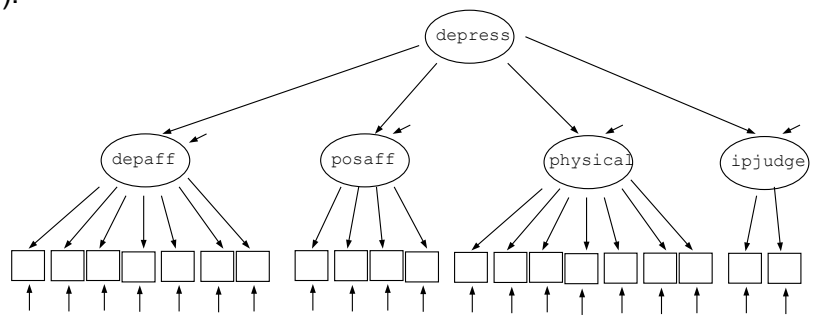
Example of Second-Order Factor Model

This example tests a second-order factor model of the full 20-item CESD. Listwise deletion was used for convenience, and missing data estimation is probably preferable (although there were very few missing values overall).



Mplus (some output deleted to save space)

```
 title:  Second Order Factor Model - Class Example;

  data:  file=c:\jason\mplus\semclass\cfa2nd.dat;  format=20f1.0;
      listwise=on;

  variable:  names = rcesde1 rcesdf1 rcesdg1 rcesdh1 rcesdl1 rcesdm1 rcesds1
                  rcesdc1 rcesdk1 rcesdn1 rcesdp1
                  rcesda1 rcesdb1 rcesdd1 rcesdi1 rcesdj1 rcesdo1 rcesdr1
                  rcesdq1 rcesdt1 ;

             missing=blank;

  analysis:  type=general; estimator=mlm;

  model: depaff by rcesde1-rcesds1;
         posaff by rcesdc1-rcesdp1;
         physical by rcesda1-rcesdr1;
         ipjudge by rcesdq1-rcesdt1;
         depress by depaff*1 posaff physical ipjudge;
         depress@1;

  output:  modindices (3.84) stdyx;


INPUT READING TERMINATED NORMALLY
Second Order Factor Model - Class Example;

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         294

   DEPAFF      POSAFF      PHYSICAL    IPJUDGE     DEPRESS


Estimator                                                      MLM
Information matrix                                        EXPECTED
Maximum number of iterations                                 1000
Convergence criterion                                   0.500D-04
Maximum number of steepest descent iterations                  20

Input data file(s)
  c:\jason\mplus\semclass\cfa2nd.dat

Input data format
  (20F1.0)

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters                       64

Loglikelihood

          H0 Value                       -6396.520
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          H1 Value                       -6225.553
Information Criteria

          Akaike (AIC)                   12921.040
          Bayesian (BIC)                 13156.790
          Sample-Size Adjusted BIC       12953.828
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                            280.889*
          Degrees of Freedom                  166
          P-Value                          0.0000
          Scaling Correction Factor        1.2173
            for MLM

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
    for chi-square difference testing in the regular way.  MLM, MLR and WLSM
    chi-square difference testing is described on the Mplus website.  MLMV, WLSMV,
    and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                         0.049
          90 Percent C.I.                  0.039  0.058
          Probability RMSEA <= .05         0.587

CFI/TLI

          CFI                              0.903
          TLI                              0.890

Chi-Square Test of Model Fit for the Baseline Model

          Value                         1380.240
          Degrees of Freedom                190
          P-Value                        0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                            0.049

WRMR (Weighted Root Mean Square Residual)

          Value                            1.042


MODEL RESULTS
                                                    Two-Tailed
                  Estimate       S.E.  Est./S.E.    P-Value

 DEPAFF   BY
    RCESDE1          1.000      0.000    999.000    999.000
    RCESDF1          0.327      0.074      4.447      0.000
    RCESDG1          0.867      0.079     10.963      0.000
    RCESDH1          0.741      0.086      8.610      0.000
    RCESDL1          0.858      0.081     10.623      0.000
    RCESDM1          0.147      0.058      2.525      0.012
    RCESDS1          0.880      0.075     11.809      0.000

 POSAFF   BY
    RCESDC1          1.000      0.000    999.000    999.000
    RCESDK1          4.209      1.503      2.800      0.005
    RCESDN1          4.489      1.569      2.862      0.004
    RCESDP1          3.961      1.349      2.937      0.003

 PHYSICAL BY
    RCESDA1          1.000      0.000    999.000    999.000
    RCESDB1          1.398      0.181      7.742      0.000
    RCESDD1          0.997      0.155      6.413      0.000
    RCESDI1          0.802      0.125      6.406      0.000
    RCESDJ1          0.941      0.169      5.576      0.000
    RCESDO1          0.846      0.161      5.246      0.000
    RCESDR1          0.953      0.153      6.217      0.000

IPJUDGE  BY
```

RCESDQ1	1.000	0.000	999.000	999.000
RCESDT1	2.883	2.080	1.386	0.166
DEPRESS BY				
DEPAFF	0.738	0.052	14.231	0.000
POSAFF	0.118	0.043	2.760	0.006
PHYSICAL	0.417	0.060	6.941	0.000
IPJUDGE	0.024	0.018	1.327	0.184
Variances				
DEPRESS	1.000	0.000	999.000	999.000

STANDARDIZED MODEL RESULTS

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
DEPAFF BY				
RCESDE1	0.766	0.029	26.231	0.000
RCESDF1	0.323	0.065	4.960	0.000
RCESDG1	0.614	0.046	13.483	0.000
RCESDH1	0.610	0.049	12.406	0.000
RCESDL1	0.768	0.032	24.110	0.000
RCESDM1	0.252	0.083	3.042	0.002
RCESDS1	0.785	0.033	23.449	0.000
POSAFF BY				
RCESDC1	0.208	0.072	2.881	0.004
RCESDK1	0.738	0.050	14.634	0.000
RCESDN1	0.746	0.047	16.025	0.000
RCESDP1	0.586	0.053	11.048	0.000
PHYSICAL BY				
RCESDA1	0.554	0.056	9.904	0.000
RCESDB1	0.674	0.037	18.147	0.000
RCESDD1	0.484	0.054	8.914	0.000
RCESDI1	0.546	0.067	8.137	0.000
RCESDJ1	0.420	0.060	7.037	0.000
RCESDO1	0.449	0.058	7.812	0.000
RCESDR1	0.521	0.061	8.526	0.000
IPJUDGE BY				
RCESDQ1	0.214	0.113	1.897	0.058
RCESDT1	0.749	0.266	2.818	0.005
DEPRESS BY				
DEPAFF	0.957	0.039	24.409	0.000
POSAFF	0.782	0.051	15.489	0.000
PHYSICAL	0.806	0.049	16.614	0.000
IPJUDGE	0.332	0.135	2.459	0.014
Variances				
DEPRESS	1.000	0.000	999.000	999.000

R-SQUARE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
RCESDE1	0.587	0.045	13.116	0.000
RCESDF1	0.104	0.042	2.480	0.013
RCESDG1	0.377	0.056	6.741	0.000
RCESDH1	0.372	0.060	6.203	0.000
RCESDL1	0.589	0.049	12.055	0.000
RCESDM1	0.064	0.042	1.521	0.128
RCESDS1	0.617	0.053	11.725	0.000
RCESDC1	0.043	0.030	1.440	0.150
RCESDK1	0.544	0.074	7.317	0.000
RCESDN1	0.556	0.069	8.013	0.000
RCESDP1	0.344	0.062	5.524	0.000
RCESDA1	0.307	0.062	4.952	0.000
RCESDB1	0.454	0.050	9.074	0.000
RCESDD1	0.235	0.053	4.457	0.000
RCESDI1	0.298	0.073	4.068	0.000
RCESDJ1	0.177	0.050	3.518	0.000

```
    RCESDO1            0.202      0.052      3.906      0.000
    RCESDR1            0.272      0.064      4.263      0.000
    RCESDQ1            0.046      0.048      0.948      0.343
    RCESDT1            0.561      0.398      1.409      0.159

     Latent                                          Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    DEPAFF             0.916      0.075     12.205      0.000
    POSAFF             0.612      0.079      7.744      0.000
    PHYSICAL           0.650      0.078      8.307      0.000
    IPJUDGE            0.110      0.090      1.229      0.219

Minimum M.I. value for printing the modification index     3.840

                                  M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
BY Statements
DEPAFF   BY RCESDI1               4.534     0.284      0.219        0.288
POSAFF   BY RCESDG1               5.708    -1.979     -0.300       -0.275
POSAFF   BY RCESDL1               5.341     1.372      0.208        0.241
POSAFF   BY RCESDS1               4.971     1.316      0.199        0.230
IPJUDGE  BY RCESDA1               4.421     2.131      0.156        0.167
DEPRESS  BY RCESDG1               8.360    -1.832     -1.832       -1.682
DEPRESS  BY RCESDL1               4.695     1.014      1.014        1.177
DEPRESS  BY RCESDS1               4.136     0.952      0.952        1.102
DEPRESS  BY RCESDI1               4.141     0.246      0.246        0.323

WITH Statements
RCESDG1  WITH RCESDE1            16.759     0.177      0.177        0.319
RCESDH1  WITH RCESDF1             6.102    -0.092     -0.092       -0.168
RCESDH1  WITH RCESDG1             7.830     0.127      0.127        0.199
RCESDL1  WITH RCESDE1             3.919    -0.063     -0.063       -0.177
RCESDL1  WITH RCESDG1             8.979    -0.111     -0.111       -0.234
RCESDM1  WITH RCESDE1             7.161    -0.054     -0.054       -0.194
RCESDM1  WITH RCESDG1            10.471    -0.082     -0.082       -0.219
RCESDM1  WITH RCESDL1             9.943     0.055      0.055        0.228
RCESDS1  WITH RCESDH1            14.044    -0.118     -0.118       -0.298
RCESDC1  WITH RCESDE1             5.376    -0.076     -0.076       -0.165
RCESDN1  WITH RCESDF1             3.958    -0.067     -0.067       -0.149
RCESDN1  WITH RCESDH1            12.384    -0.123     -0.123       -0.274
RCESDN1  WITH RCESDS1             3.861     0.054      0.054        0.167
RCESDN1  WITH RCESDK1             4.155     0.106      0.106        0.300
RCESDA1  WITH RCESDS1             4.947    -0.070     -0.070       -0.169
RCESDB1  WITH RCESDA1             4.902     0.112      0.112        0.181
RCESDI1  WITH RCESDH1             5.198     0.076      0.076        0.160
RCESDI1  WITH RCESDL1             3.992     0.053      0.053        0.149
RCESDI1  WITH RCESDP1             5.584    -0.089     -0.089       -0.169
RCESDI1  WITH RCESDB1             4.826    -0.090     -0.090       -0.178
RCESDO1  WITH RCESDH1             8.998    -0.133     -0.133       -0.206
RCESDO1  WITH RCESDS1             5.220     0.079      0.079        0.169
RCESDO1  WITH RCESDA1             6.761    -0.126     -0.126       -0.186
RCESDO1  WITH RCESDB1            12.028     0.185      0.185        0.268
RCESDR1  WITH RCESDI1             4.415     0.079      0.079        0.154
RCESDR1  WITH RCESDJ1             6.138     0.148      0.148        0.174
RCESDT1  WITH RCESDM1             4.253     0.016      0.016        0.194
```

lavaan

```
> d = read.fortran('c:/jason/mplus/semclass/cfa2nd.dat',
+                  c("F1.0","F1.0","F1.0","F1.0","F1.0","F1.0","F1.0",
+                    "F1.0","F1.0","F1.0","F1.0",
+                    "F1.0","F1.0","F1.0","F1.0","F1.0","F1.0","F1.0",
+                    "F1.0","F1.0"))
> names(d) = c("rcesde1", "rcesdf1", "rcesdg1", "rcesdh1", "rcesdl1", "rcesdm1", "rcesds1",
+                  "rcesdc1", "rcesdk1", "rcesdn1", "rcesdp1",
+                  "rcesda1", "rcesdb1", "rcesdd1", "rcesdi1", "rcesdj1", "rcesdo1", "rcesdr1",
+                  "rcesdq1", "rcesdt1")
>
> model1 = '
+         depaff =~ rcesde1 + rcesdf1 + rcesdg1 + rcesdh1 + rcesdl1 + rcesdm1 + rcesds1
+         posaff =~ rcesdc1 + rcesdk1 + rcesdn1 + rcesdp1
+         physical =~ rcesda1 + rcesdb1 + rcesdd1 + rcesdi1 + rcesdj1 + rcesdo1 + rcesdr1
+         ipjudge =~ rcesdq1 + rcesdt1
+         depress =~ NA*depaff + posaff + physical + ipjudge
+         depress~~1*depress
+         '
>         fit = sem(model1, data = d, missing="listwise", mimic = "Mplus", estimator="mlm")
> summary(fit,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)
```

```
lavaan 0.6-18 ended normally after 91 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        64

  Number of observations                           294

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                               341.934     280.889
  Degrees of freedom                               166         166
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.217
    Satorra-Bentler correction (Mplus variant)

Model Test Baseline Model:

  Test statistic                              1695.685    1380.240
  Degrees of freedom                               190         190
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.229

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.883       0.903
  Tucker-Lewis Index (TLI)                       0.866       0.890

  Robust Comparative Fit Index (CFI)                         0.904
  Robust Tucker-Lewis Index (TLI)                            0.891

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -6396.520   -6396.520
  Loglikelihood unrestricted model (H1)      -6225.553   -6225.553

  Akaike (AIC)                               12921.040   12921.040
  Bayesian (BIC)                             13156.790   13156.790
  Sample-size adjusted Bayesian (SABIC)      12953.828   12953.828

Root Mean Square Error of Approximation:

  RMSEA                                          0.060       0.049
  90 Percent confidence interval - lower         0.051       0.040
  90 Percent confidence interval - upper         0.069       0.057
  P-value H_0: RMSEA <= 0.050                    0.035       0.598
  P-value H_0: RMSEA >= 0.080                    0.000       0.000

  Robust RMSEA                                               0.054
  90 Percent confidence interval - lower                     0.043
  90 Percent confidence interval - upper                     0.064
  P-value H_0: Robust RMSEA <= 0.050                         0.286
  P-value H_0: Robust RMSEA >= 0.080                         0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.049       0.049

Parameter Estimates:

  Standard errors                             Robust.sem
  Information                                   Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  depaff =~
    rcesde1           1.000                               0.771    0.766
    rcesdf1           0.327    0.074    4.447    0.000    0.252    0.323
    rcesdg1           0.867    0.079   10.963    0.000    0.669    0.614
    rcesdh1           0.741    0.086    8.610    0.000    0.571    0.610
    rcesdl1           0.858    0.081   10.623    0.000    0.661    0.768
    rcesdm1           0.147    0.058    2.525    0.012    0.113    0.252
    rcesds1           0.880    0.075   11.809    0.000    0.679    0.785
  posaff =~
    rcesdc1           1.000                               0.151    0.208
    rcesdk1           4.209    1.503    2.800    0.005    0.637    0.738
    rcesdn1           4.489    1.569    2.862    0.004    0.679    0.746
    rcesdp1           3.961    1.349    2.937    0.003    0.599    0.586
  physical =~
    rcesda1           1.000                               0.518    0.554
    rcesdb1           1.398    0.181    7.742    0.000    0.724    0.674
    rcesdd1           0.997    0.155    6.413    0.000    0.516    0.484
    rcesdi1           0.802    0.125    6.406    0.000    0.415    0.546
    rcesdj1           0.941    0.169    5.576    0.000    0.487    0.420
```

```
    rcesdo1           0.846    0.161    5.246    0.000    0.438    0.449
    rcesdr1           0.953    0.153    6.217    0.000    0.493    0.521
  ipjudge =~
    rcesdq1           1.000                               0.073    0.214
    rcesdt1           2.883    2.081    1.386    0.166    0.212    0.749
  depress =~
    depaff            0.738    0.052   14.231    0.000    0.957    0.957
    posaff            0.118    0.043    2.760    0.006    0.782    0.782
    physical          0.417    0.060    6.941    0.000    0.806    0.806
    ipjudge           0.024    0.018    1.327    0.184    0.332    0.332


Variances:
                  Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    depress           1.000                               1.000    1.000
   .rcesde1           0.418    0.043    9.677    0.000    0.418    0.413
   .rcesdf1           0.548    0.053   10.422    0.000    0.548    0.896
   .rcesdg1           0.739    0.059   12.478    0.000    0.739    0.623
   .rcesdh1           0.550    0.047   11.721    0.000    0.550    0.628
   .rcesdl1           0.305    0.038    8.108    0.000    0.305    0.411
   .rcesdm1           0.189    0.040    4.690    0.000    0.189    0.936
   .rcesds1           0.286    0.032    8.960    0.000    0.286    0.383
   .rcesdc1           0.508    0.076    6.732    0.000    0.508    0.957
   .rcesdk1           0.340    0.061    5.545    0.000    0.340    0.456
   .rcesdn1           0.368    0.057    6.408    0.000    0.368    0.444
   .rcesdp1           0.687    0.078    8.812    0.000    0.687    0.656
   .rcesda1           0.605    0.055   10.945    0.000    0.605    0.693
   .rcesdb1           0.631    0.056   11.217    0.000    0.631    0.546
   .rcesdd1           0.869    0.065   13.415    0.000    0.869    0.765
   .rcesdi1           0.407    0.052    7.851    0.000    0.407    0.702
   .rcesdj1           1.106    0.081   13.719    0.000    1.106    0.823
   .rcesdo1           0.758    0.048   15.773    0.000    0.758    0.798
   .rcesdr1           0.653    0.077    8.508    0.000    0.653    0.728
   .rcesdq1           0.112    0.043    2.633    0.008    0.112    0.954
   .rcesdt1           0.035    0.033    1.057    0.291    0.035    0.439
   .depaff            0.050    0.045    1.112    0.266    0.084    0.084
   .posaff            0.009    0.006    1.402    0.161    0.388    0.388
   .physical          0.094    0.030    3.117    0.002    0.350    0.350
   .ipjudge           0.005    0.005    1.048    0.295    0.890    0.890

R-Square:
                  Estimate
    rcesde1           0.587
    rcesdf1           0.104
    rcesdg1           0.377
    rcesdh1           0.372
    rcesdl1           0.589
    rcesdm1           0.064
    rcesds1           0.617
    rcesdc1           0.043
    rcesdk1           0.544
    rcesdn1           0.556
    rcesdp1           0.344
    rcesda1           0.307
    rcesdb1           0.454
    rcesdd1           0.235
    rcesdi1           0.298
    rcesdj1           0.177
    rcesdo1           0.202
    rcesdr1           0.272
    rcesdq1           0.046
    rcesdt1           0.561
    depaff            0.916
    posaff            0.612
    physical          0.650
    ipjudge           0.110
```