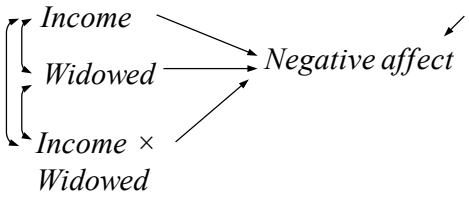
Moderation and Moderated Mediation Examples

Continuous Moderation Example (Mplus)

A subsequent handout ("Simple Slopes for Exploring a Significant Interaction in SEM") will illustrate simple slopes tests and plotting.




```
   title:  Moderator examples ;

     data:  file=C:\Jason\mplus\semclass\moderator.dat;
     format=free;


     define:
           center widow(grandmean);
           center income(grandmean);
           widxinc = widow*income;

     variable:  names = widow income social posaff negaff
              pos21 pos22 pos23 pos25
              panas1 panas2 panas3 panas4 panas5
              panas6 panas7 panas8 panas9 panas10 ;
              !  grouping is widow (0=notwidow,1=widow);

                missing = all (-99);

                usevariables = widow income negaff widxinc;

     analysis:  type=general; estimator=mlr;

     model:  negaff on widow income widxinc;


     output:  stdyx ;
```

```
MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 NEGAFF   ON
    WIDOW             -0.016      0.122     -0.132      0.895
    INCOME             0.007      0.017      0.439      0.661
    WIDXINC           -0.035      0.042     -0.829      0.407

 Intercepts
    NEGAFF             1.731      0.048     36.343      0.000

 Residual Variances
    NEGAFF             0.550      0.061      9.010      0.000


STANDARDIZED MODEL RESULTS
STDYX Standardization
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 NEGAFF   ON
    WIDOW             -0.009      0.070     -0.132      0.895
    INCOME             0.030      0.067      0.440      0.660
    WIDXINC           -0.056      0.068     -0.827      0.408

 Intercepts
    NEGAFF             2.330      0.096     24.238      0.000

 Residual Variances
    NEGAFF             0.996      0.008    117.715      0.000

R-SQUARE
    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    NEGAFF             0.004      0.008      0.497      0.619
```

Continuous Moderation Example (`lavaan`)

lavaan

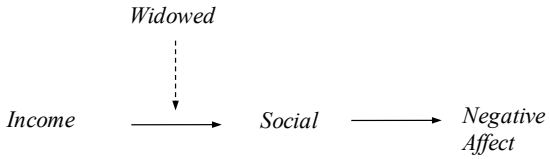
```
> #center predictors
> library(QuantPsyc)
> mydata$widow = meanCenter(mydata$widow)
> mydata$income = meanCenter(mydata$income)
> mydata$widxinc = mydata$widow*mydata$inc
>
> library(psych)
> #describe(mydata)
>
> library(lavaan)
> model = '
+ negaff ~ widow + income + widxinc
+
+     '
> #Below, MLM is used to request Satorra & Bentler (1988; 1994)
> #robust standard errors and scaled chi-square
> #Note: results similar to Mplus but adjusted chi-square and robust SEs differ
> fit = sem(model, data = mydata, estimator="mlr")
> summary(fit,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)

lavaan 0.6-18 ended normally after 1 iteration

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         4

                                                  Used       Total
  Number of observations                           256         275

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                                 0.000       0.000
  Degrees of freedom                                 0           0

Model Test Baseline Model:

  Test statistic                                 1.079       0.992
  Degrees of freedom                                 3           3
  P-value                                        0.782       0.803
  Scaling correction factor                                  1.088

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    1.000       1.000
  Tucker-Lewis Index (TLI)                       1.000       1.000

  Robust Comparative Fit Index (CFI)                            NA
  Robust Tucker-Lewis Index (TLI)                               NA

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)               -286.641    -286.641
  Loglikelihood unrestricted model (H1)       -286.641    -286.641

  Akaike (AIC)                                 581.281     581.281
  Bayesian (BIC)                               595.462     595.462
  Sample-size adjusted Bayesian (SABIC)        582.781     582.781

Root Mean Square Error of Approximation:

  RMSEA                                          0.000          NA
  90 Percent confidence interval - lower         0.000          NA
  90 Percent confidence interval - upper         0.000          NA
  P-value H_0: RMSEA <= 0.050                       NA          NA
  P-value H_0: RMSEA >= 0.080                       NA          NA

  Robust RMSEA                                                0.000
  90 Percent confidence interval - lower                      0.000
  90 Percent confidence interval - upper                      0.000
  P-value H_0: Robust RMSEA <= 0.050                             NA
  P-value H_0: Robust RMSEA >= 0.080                             NA

Standardized Root Mean Square Residual:

  SRMR                                           0.000       0.000

Parameter Estimates:

  Standard errors                             Sandwich
  Information bread                           Observed
  Observed information based on                Hessian
```

```
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  negaff ~
    widow            -0.017    0.122   -0.139    0.889   -0.017   -0.010
    income            0.007    0.017    0.445    0.656    0.007    0.030
    widxinc          -0.035    0.042   -0.829    0.407   -0.035   -0.057

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .negaff            0.550    0.061    9.010    0.000    0.550    0.996

R-Square:
                   Estimate
    negaff            0.004
```

I recommend following a significant interaction with simple slopes analyses and plots. See handout "Simple Slopes for Continuous Measured and Latent Variable Interactions" from this class for examples.[1]

Moderated Mediation with Z as Moderator of the $X - M$ Relationship

I have only illustrated these moderated mediation examples in Mplus. See https://cehs-research.github.io/EDUC-7610/Slides/15_EDUC7610_mediation_example (Tyson Barrett) for lavaan examples.




```
title:  Moderator examples;

  data:  file=C:\Jason\mplus\semclass\moderator.dat;
format=free;


  define:
        center widow(grandmean);
        center income(grandmean);
        widxinc = widow*income;

  variable:  names = widow income social posaff negaff
           pos21 pos22 pos23 pos25
           panas1 panas2 panas3 panas4 panas5
           panas6 panas7 panas8 panas9 panas10 ;
           !  grouping is widow (0=notwidow,1=widow);

             missing = all (-99);

             usevariables = widow income social negaff widxinc;

  analysis:  type=general; !estimator=mlr;
      !bootstrap estimation of indirect effects is also possible
        !but a sepearte run withou estimator=mlr is needed for
the indirect test;
       bootstrap = 1000;

  !follows the first example with moderation X predicting M;
  model:  negaff on widow income widxinc social;
          social on widow income widxinc;

       model indirect:
       negaff ind income;
       negaff ind widow;
       negaff ind widxinc;

  output:  stdyx; cinterval(bootstrap);
```

```
MODEL RESULTS
```

[1] For additional illustrations on testing and plotting simple slopes in Mplus, see Muthén, Muthén, & Asparouhov (2016) and material by Chris Stride and colleagues (Stride, C.B., Gardner, Catley, & Thomas, (2015), https://www.figureitout.org.uk/models_and_index.pdf (p. 15).

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
NEGAFF ON				
WIDOW	-0.040	0.120	-0.337	0.736
INCOME	0.024	0.017	1.411	0.158
WIDXINC	-0.034	0.042	-0.822	0.411
SOCIAL	-0.187	0.054	-3.462	0.001
SOCIAL ON				
WIDOW	-0.198	0.143	-1.384	0.166
INCOME	0.083	0.019	4.286	0.000
WIDXINC	-0.024	0.049	-0.489	0.625
Intercepts				
SOCIAL	2.825	0.062	45.693	0.000
NEGAFF	2.263	0.170	13.314	0.000
Residual Variances				
SOCIAL	0.809	0.077	10.545	0.000
NEGAFF	0.522	0.053	9.770	0.000

STANDARDIZED MODEL RESULTS

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
NEGAFF ON				
WIDOW	-0.023	0.069	-0.338	0.735
INCOME	0.098	0.069	1.432	0.152
WIDXINC	-0.056	0.068	-0.833	0.405
SOCIAL	-0.238	0.065	-3.637	0.000
SOCIAL ON				
WIDOW	-0.090	0.066	-1.370	0.171
INCOME	0.266	0.059	4.465	0.000
WIDXINC	-0.031	0.063	-0.493	0.622
Intercepts				
SOCIAL	2.999	0.181	16.538	0.000
NEGAFF	3.045	0.212	14.386	0.000
Residual Variances				
SOCIAL	0.912	0.033	27.758	0.000
NEGAFF	0.945	0.030	31.735	0.000

R-SQUARE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
SOCIAL	0.088	0.033	2.688	0.007
NEGAFF	0.055	0.030	1.857	0.063

Results from bootstrap analysis of indirect effects (unstandandardized only included here)

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

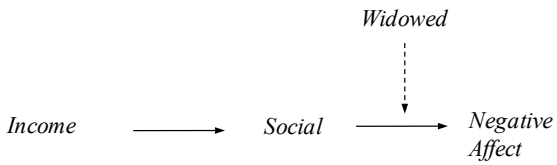
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Effects from INCOME to NEGAFF				
Total	0.009	0.017	0.507	0.612
Total indirect	-0.016	0.006	-2.450	0.014

```
 Specific indirect 1
    NEGAFF
    SOCIAL
    INCOME            -0.016      0.006     -2.450      0.014

  Direct
    NEGAFF
    INCOME             0.024      0.017      1.411      0.158


Effects from WIDOW to NEGAFF

  Total               -0.003      0.123     -0.027      0.979
  Total indirect       0.037      0.031      1.214      0.225

  Specific indirect 1
    NEGAFF
    SOCIAL
    WIDOW              0.037      0.031      1.214      0.225

  Direct
    NEGAFF
    WIDOW             -0.040      0.120     -0.337      0.736


Effects from WIDXINC to NEGAFF

  Total               -0.030      0.043     -0.696      0.487
  Total indirect       0.005      0.010      0.466      0.641

  Specific indirect 1
    NEGAFF
    SOCIAL
    WIDXINC            0.005      0.010      0.466      0.641

  Direct
    NEGAFF
    WIDXINC           -0.034      0.042     -0.822      0.411

CONFIDENCE INTERVALS OF TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS


                  Lower .5%  Lower 2.5%    Lower 5%    Estimate    Upper 5%  Upper 2.5%   Upper .5%

Effects from INCOME to NEGAFF

  Total             -0.034      -0.023      -0.018       0.009       0.038       0.044       0.056
  Total indirect    -0.036      -0.030      -0.027      -0.016      -0.007      -0.006      -0.003

  Specific indirect 1
    NEGAFF
    SOCIAL
    INCOME          -0.036      -0.030      -0.027      -0.016      -0.007      -0.006      -0.003

  Direct
    NEGAFF
    INCOME          -0.016      -0.009      -0.003       0.024       0.052       0.059       0.072


Effects from WIDOW to NEGAFF

  Total             -0.318      -0.238      -0.207      -0.003       0.195       0.236       0.314
  Total indirect    -0.032      -0.013      -0.006       0.037       0.093       0.105       0.137

  Specific indirect 1
    NEGAFF
    SOCIAL
    WIDOW           -0.032      -0.013      -0.006       0.037       0.093       0.105       0.137

  Direct
    NEGAFF
    WIDOW           -0.364      -0.282      -0.236      -0.040       0.154       0.184       0.275
```

```
Effects from WIDXINC to NEGAFF

  Total             -0.144      -0.114      -0.097      -0.030       0.040       0.056       0.072
  Total indirect    -0.022      -0.016      -0.012       0.005       0.020       0.023       0.033

  Specific indirect 1
    NEGAFF
    SOCIAL
    WIDXINC         -0.022      -0.016      -0.012       0.005       0.020       0.023       0.033

  Direct
    NEGAFF
    WIDXINC         -0.142      -0.113      -0.101      -0.034       0.037       0.048       0.064
```

Moderated Mediation with *Z* as Moderator of *M-Y* Relationship

```
   title:  Moderator examples ;

     data:  file=C:\Jason\mplus\semclass\moderator.dat;
     format=free;

     define:
           center widow(grandmean);
           center income(grandmean);
           center social(grandmean);
           widxsoc = widow*social;

     variable:  names = widow income social posaff negaff
              pos21 pos22 pos23 pos25
              panas1 panas2 panas3 panas4 panas5
              panas6 panas7 panas8 panas9 panas10 ;
              !  grouping is widow (0=notwidow,1=widow);

                missing = all (-99);

                usevariables = widow income social negaff
     widxsoc;

       analysis:  type=general; estimator=mlr;
         !bootstrap estimation of indirect effects is
     also possible
         !but a sepearte run withou estimator=mlr is
     needed for the indirect test;
      !  bootstrap = 1000;

       !follows the second example with moderation of M
     predicting Y;
       model:  negaff on widow income social widxsoc;
               social on income;

        ! model indirect: negaff ind income;

       output:  stdyx; ! cinterval(bootstrap);
```



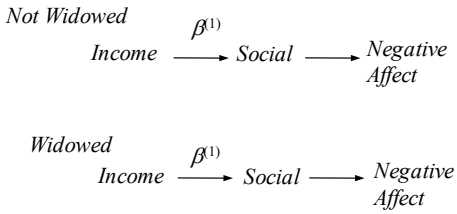

```
  MODEL RESULTS
                                                    Two-Tailed
                      Estimate       S.E.  Est./S.E.    P-Value

   NEGAFF   ON
      WIDOW              0.000      0.111      0.002      0.998
      INCOME             0.028      0.017      1.630      0.103
      SOCIAL            -0.194      0.056     -3.445      0.001
      WIDXSOC           -0.100      0.140     -0.718      0.473

   SOCIAL   ON
      INCOME             0.089      0.018      4.855      0.000

   Intercepts
      SOCIAL             0.000      0.057      0.000      1.000
      NEGAFF             1.732      0.047     37.206      0.000

   Residual Variances
```

```
    SOCIAL             0.816      0.078     10.500      0.000
    NEGAFF             0.528      0.059      8.964      0.000


STANDARDIZED MODEL RESULTS

STDYX Standardization
                                                    Two-Tailed
                   Estimate       S.E.  Est./S.E.    P-Value

 NEGAFF   ON
    WIDOW              0.000      0.063      0.002      0.998
    INCOME             0.112      0.068      1.653      0.098
    SOCIAL            -0.244      0.067     -3.638      0.000
    WIDXSOC           -0.052      0.072     -0.719      0.472

 SOCIAL   ON
    INCOME             0.285      0.057      5.037      0.000

 Intercepts
    SOCIAL             0.000      0.061      0.000      1.000
    NEGAFF             2.311      0.100     23.225      0.000

 Residual Variances
    SOCIAL             0.919      0.032     28.451      0.000
    NEGAFF             0.941      0.032     29.448      0.000


R-SQUARE

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    SOCIAL             0.081      0.032      2.518      0.012
    NEGAFF             0.059      0.032      1.855      0.064
```

Multigroup Comparison of a Mediational Path

Below the test of the X –> M path constrains the income social path to be the same in the two groups. The comparison model would allow the path to be freely estimated in the two groups by removing the (1) equality constraint label. Note that this test of the direct path is of the same hypothesis that the first continuous model tested. The free model, χ^2_{Free} , has df = 0 and so χ^2 = 0, that leads to a chi-square difference (likelihood ratio) test equal, $\chi^2_{\text{Restricted}} - 0 = \chi^2_{\text{Restricted}}$.

Restricted Model




```
   title:  Moderator examples ;

    data:  file=C:\Jason\mplus\semclass\moderator.dat;
    format=free;


    variable:  names = widow income social posaff negaff
             pos21 pos22 pos23 pos25
             panas1 panas2 panas3 panas4 panas5
             panas6 panas7 panas8 panas9 panas10 ;
              grouping is widow (0=notwidow,1=widow);

               missing = all (-99);
               usevariables = widow income social negaff ;

      analysis:  type=general; estimator=mlr;

    !multigroup model comparison of mediational path model;
    !as one illustration, compare the X-M path across groups;

    model:
            negaff on income social;
            social on income;

      model notwidow:
```

```
               negaff on income
               social ;
               social on income (1);

       model widow:
               negaff on income
               social ;
               social on income (1);

       output:  stdyx;

MODEL FIT INFORMATION

Number of Free Parameters                       13

Loglikelihood

          H0 Value                          -608.033
          H0 Scaling Correction Factor        1.1081
            for MLR
          H1 Value                          -607.896
          H1 Scaling Correction Factor        1.1022
            for MLR

Information Criteria

          Akaike (AIC)                      1242.066
          Bayesian (BIC)                    1288.255
          Sample-Size Adjusted BIC          1247.041
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             0.268*
          Degrees of Freedom                    1
          P-Value                           0.6048
          Scaling Correction Factor         1.0257
            for MLR

Chi-Square Contribution From Each Group

          NOTWIDOW                          0.060
          WIDOW                             0.208

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
    for chi-square difference testing in the regular way.  MLM, MLR and WLSM
    chi-square difference testing is described on the Mplus website.  MLMV, WLSMV,
    and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                          0.000
          90 Percent C.I.                   0.000  0.187
          Probability RMSEA <= .05          0.659

CFI/TLI

          CFI                               1.000
          TLI                               1.000

Chi-Square Test of Model Fit for the Baseline Model

          Value                            29.893
          Degrees of Freedom                    6
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                             0.016

MODEL RESULTS

                                                    Two-Tailed
                  Estimate       S.E.  Est./S.E.    P-Value
```

Because the model with free estimates within groups is saturated and has df = 0, the chi-square difference test is equal to this value for the restricted model

Group NOTWIDOW

NEGAFF ON				
INCOME	0.031	0.019	1.683	0.092
SOCIAL	-0.174	0.060	-2.887	0.004
SOCIAL ON				
INCOME	0.084	0.019	4.384	0.000
Intercepts				
SOCIAL	2.360	0.153	15.435	0.000
NEGAFF	2.042	0.198	10.293	0.000
Residual Variances				
SOCIAL	0.837	0.098	8.541	0.000
NEGAFF	0.506	0.064	7.849	0.000

Group WIDOW

NEGAFF ON				
INCOME	0.003	0.039	0.083	0.934
SOCIAL	-0.240	0.131	-1.823	0.068
SOCIAL ON				
INCOME	0.084	0.019	4.384	0.000
Intercepts				
SOCIAL	2.178	0.144	15.113	0.000
NEGAFF	2.355	0.372	6.331	0.000
Residual Variances				
SOCIAL	0.722	0.106	6.797	0.000
NEGAFF	0.570	0.123	4.640	0.000

STANDARDIZED MODEL RESULTS

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group NOTWIDOW				
NEGAFF ON				
INCOME	0.130	0.076	1.711	0.087
SOCIAL	-0.226	0.074	-3.053	0.002
SOCIAL ON				
INCOME	0.267	0.059	4.547	0.000
Intercepts				
SOCIAL	2.485	0.264	9.431	0.000
NEGAFF	2.793	0.272	10.284	0.000
Residual Variances				
SOCIAL	0.929	0.031	29.692	0.000
NEGAFF	0.948	0.032	29.690	0.000
Group WIDOW				
NEGAFF ON				
INCOME	0.011	0.134	0.083	0.934
SOCIAL	-0.269	0.139	-1.932	0.053
SOCIAL ON				
INCOME	0.255	0.062	4.096	0.000
Intercepts				
SOCIAL	2.478	0.265	9.367	0.000
NEGAFF	3.007	0.443	6.792	0.000

```
 Residual Variances
    SOCIAL             0.935      0.032     29.336      0.000
    NEGAFF             0.929      0.071     13.014      0.000


R-SQUARE

Group NOTWIDOW

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    SOCIAL             0.071      0.031      2.274      0.023
    NEGAFF             0.052      0.032      1.641      0.101

Group WIDOW

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    SOCIAL             0.065      0.032      2.048      0.041
    NEGAFF             0.071      0.071      0.993      0.321
```

Free Model

(excerpt)

```
MODEL FIT INFORMATION
Number of Free Parameters                       14

Loglikelihood

          H0 Value                          -607.896
          H0 Scaling Correction Factor        1.1022
            for MLR
          H1 Value                          -607.896
          H1 Scaling Correction Factor        1.1022
            for MLR

Information Criteria

          Akaike (AIC)                      1243.792
          Bayesian (BIC)                    1293.533
          Sample-Size Adjusted BIC          1249.149
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                               0.000*
          Degrees of Freedom                      0
          P-Value                            0.0000
          Scaling Correction Factor          1.0000
            for MLR

Chi-Square Contribution From Each Group

          NOTWIDOW                            0.000
          WIDOW                               0.000

STANDARDIZED TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS


STDYX Standardization

                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group NOTWIDOW

Effects from INCOME to NEGAFF

  Total               0.066      0.076      0.865      0.387
  Total indirect     -0.064      0.028     -2.310      0.021
```

```
  Specific indirect 1
    NEGAFF
    SOCIAL
    INCOME            -0.064      0.028     -2.310      0.021

  Direct
    NEGAFF
    INCOME             0.130      0.076      1.712      0.087


Group WIDOW

Effects from INCOME to NEGAFF

  Total               -0.042      0.132     -0.321      0.748
  Total indirect      -0.054      0.048     -1.127      0.260

  Specific indirect 1
    NEGAFF
    SOCIAL
    INCOME            -0.054      0.048     -1.127      0.260

  Direct
    NEGAFF
    INCOME             0.011      0.134      0.083      0.934
```

References

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