Some Matrix Algebra Properties

$\mathbf{S}+\mathbf{T}=\mathbf{T}+\mathbf{S}$ Commutative property

$(\mathbf{S}+\mathbf{T})+\mathbf{U}=\mathbf{S}+(\mathbf{T}+\mathbf{U})$ Associative property

$\mathbf{ST}\neq\mathbf{TS}$ Not commutative for multiplication (except for special circumstances)

$\mathbf{ST}(\mathbf{U})=\mathbf{S}(\mathbf{TU})$ Associative

$\mathbf{S}(\mathbf{T}+\mathbf{U})=\mathbf{ST}+\mathbf{SU}$ Distributive

$c(\mathbf{S}+\mathbf{T})=c\mathbf{S}+c\mathbf{T}$, where c is a constant

$(\mathbf{S}')'=\mathbf{S}$

$(\mathbf{S}+\mathbf{T})'=\mathbf{S}'+\mathbf{T}'$ Transpose if distributive for addition

$(\mathbf{ST})'=\mathbf{T}'\mathbf{S}'$, if conformable

$(\mathbf{STU})'=\mathbf{U}'\mathbf{T}'\mathbf{S}'$

$\mathrm{tr}(\mathbf{S})=\mathrm{tr}(\mathbf{S}')$

$\mathrm{tr}(\mathbf{ST})=\mathrm{tr}(\mathbf{TS})$, if conformable

$\mathrm{tr}(\mathbf{S}+\mathbf{T})=\mathrm{tr}(\mathbf{S})+\mathrm{tr}(\mathbf{T})$

$\mathbf{IS}=\mathbf{SI}=\mathbf{S}$

$\mathbf{I}=\mathbf{I}'$

$|\mathbf{S}'|=|\mathbf{S}|$

$|\mathbf{ST}|=|\mathbf{S}||\mathbf{T}|$

$|\mathbf{I}|=1$

$|\mathbf{A}^{-1}|=\frac{1}{|\mathbf{A}|}$

$|\mathbf{D}|=\prod d_{ii}$, determinant of a diagonal matrix is equal to the product of the diagonal elements

$\mathbf{SS}^{-1}=\mathbf{S}^{-1}\mathbf{S}=\mathbf{I}$

$(\mathbf{ST})^{-1}=\mathbf{T}^{-1}\mathbf{S}^{-1}$

$(\mathbf{STU})^{-1}=\mathbf{U}^{-1}\mathbf{T}^{-1}\mathbf{S}^{-1}$

$(\mathbf{S}')^{-1}=(\mathbf{S}^{-1})'$

Sources and Further Reading

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