

Factorial Invariance Example¹

To keep it simple, I illustrate a weak invariance test (loadings only) and I have requested no mean structure. The model is a one-factor CFA with five positive affect items, and widowed participants are compared to those who are not widowed. Degrees of freedom for a multigroup model are: $df = g[v(v+1)/2] - p$, where g is the number of groups, v is the number of measured variables, and p is the number of freely estimated parameters. So, for the free model (loadings allowed to be freely estimated in the two groups), with five measured variables and two groups, $df = 2[5(5+1)/2] - 16 = 2(15) - 20 = 10$. The 16 free parameters include 5 loadings in each group (10) and 5 residual variances in each group (10). Equality constraints are imposed by using the same label behind the loading in Mplus [e.g., (ly1)] and by referring to the same label for the two groups in parentheses preceding each variable in lavaan [e.g., (ly1)*]. When the five loadings are set equal across groups, four degrees of freedom are gained because a scaling constraint still has to be imposed to estimate the factor variance.

For invariance tests, I prefer to use the effects coding method of factor scaling (Little, Slegers, & Card, 2006), which sets the average loading to 1 rather than setting either the first loading (referent or marker approach) or setting the factor variances.² This makes it possible to inspect the differences in all loading values and factor variance across groups. To set the loadings to an average of 1, model constraints must be used, requiring that the first loading equal the number of loadings minus all of the other loadings, $\lambda_{11} = 5 - \lambda_{21} - \lambda_{31} - \lambda_{41} - \lambda_{51}$.

I also used listwise deletion in this example to keep the computation of the chi-square difference test straightforward, but I would usually recommend missing data estimation and robust estimation (e.g., MLR; the same size may be too small in the separate groups too). If robust adjustments are used, chi-square difference tests would need to weight by the scaling correction factor using the Bryant-Satorra spreadsheet discussed earlier (see the handout “Examples of Chi-square Difference Tests with Nonnormal and Categorical Variables” for details) or the DIFFTEST process in Mplus if categorical indicators are used.

All Parameters Free Across Groups

```
title: Factorial invariance example;

data: file=C:\Jason\mplus\semclass\stack1.dat; format=11f1.0;
      listwise=on;

variable: names = widow panas1 panas2 panas3 panas4 panas5
           panas6 panas7 panas8 panas9 panas10 ;
           grouping is widow (0=notwidow,1=widow);

           missing = blank;

           usevariables = panas1-panas5;

analysis: type=general; iterations = 200;
           model=nomeanstructure; information=expected;

model: posaff by panas1* panas2-panas5;

! Note: by default in Mplus, measurement errors and factor correlations are not
! constrained to be equal across groups;

Model notwidow:
  posaff by panas1* (ly1)
           panas2 (ly2)
           panas3 (ly3)
           panas4 (ly4)
           panas5 (ly5);
posaff;
```

¹ Note: I only illustrate some of the tests that may be needed, but there are other possible tests that may be desirable (e.g., all parameters equal vs. all parameters free, factor variance comparisons, or other specific paths). For brevity sake, I do not illustrate comparison of variances, covariances, or mean and intercept comparisons. See the handout “Invariance Tests in Multigroup SEM” for an overview. By default in Mplus Version 6 and later, analyses with mean structures set the intercepts to zero in the first group and allow them to be freely estimated in the second group.

² When means are compared, an analogous set of effects coding constraints can be used for the measurement intercepts. See the “Mean Structures” handout for this class.

Model widow:

```
posaff by panas1* (ly6)
        panas2 (ly7)
        panas3 (ly8)
        panas4 (ly9)
        panas5 (ly10);
posaff;
```

Model constraint:

```
ly1 = 5 - ly2 - ly3 -ly4 - ly5;
ly6 = 5 - ly7 - ly8 -ly9 - ly10;
```

output: stdyx ;

INPUT READING TERMINATED NORMALLY

Factorial invariance example ;

SUMMARY OF ANALYSIS

Number of groups	2
Number of observations	
Group NOTWIDOW	161
Group WIDOW	41
Total sample size	202
Number of dependent variables	5
Number of independent variables	0
Number of continuous latent variables	1

Observed dependent variables

Continuous					
PANAS1	PANAS2	PANAS3	PANAS4	PANAS5	

Continuous latent variables

POSAFF

Variables with special functions

Grouping variable WIDOW

Estimator	ML
Information matrix	EXPECTED
Maximum number of iterations	200
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20

Input data file(s)

C:\Jason\mplus\semclass\stack1.dat

Input data format

(11F1.0)

MODEL FIT INFORMATION

Number of Free Parameters 20

Loglikelihood

H0 Value	-921.811
H1 Value	-901.383

Information Criteria

Akaike (AIC)	1883.622
Bayesian (BIC)	1949.787
Sample-Size Adjusted BIC	1886.423
(n* = (n + 2) / 24)	

Chi-Square Test of Model Fit

Value	40.855
Degrees of Freedom	10
P-Value	0.0000

Chi-Square Contribution From Each Group

NOTWIDOW	31.578
WIDOW	9.277

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.175	
90 Percent C.I.	0.121	0.232
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.969
TLI	0.938

Chi-Square Test of Model Fit for the Baseline Model

Value	1018.190
Degrees of Freedom	20
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.028
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MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group NOTWIDOW				
POSAFF BY				
PANAS1	0.907	0.039	23.134	0.000
PANAS2	1.061	0.054	19.703	0.000
PANAS3	0.967	0.037	25.918	0.000
PANAS4	1.048	0.037	28.394	0.000
PANAS5	1.017	0.044	22.943	0.000
Variances				
POSAFF	0.669	0.080	8.400	0.000
Residual Variances				
PANAS1	0.179	0.025	7.310	0.000
PANAS2	0.414	0.052	8.002	0.000
PANAS3	0.152	0.023	6.716	0.000
PANAS4	0.138	0.023	6.074	0.000
PANAS5	0.249	0.033	7.470	0.000
Group WIDOW				
POSAFF BY				
PANAS1	1.022	0.039	26.396	0.000
PANAS2	1.172	0.062	18.856	0.000
PANAS3	0.970	0.054	17.890	0.000
PANAS4	0.988	0.056	17.786	0.000
PANAS5	0.848	0.073	11.557	0.000
Variances				
POSAFF	1.116	0.255	4.383	0.000
Residual Variances				
PANAS1	0.042	0.022	1.913	0.056
PANAS2	0.228	0.059	3.847	0.000
PANAS3	0.157	0.041	3.851	0.000
PANAS4	0.168	0.043	3.871	0.000
PANAS5	0.325	0.076	4.295	0.000

STANDARDIZED MODEL RESULTS

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
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Group NOTWIDOW

POSAFF	BY				
PANAS1		0.868	0.022	38.769	0.000
PANAS2		0.803	0.031	26.185	0.000
PANAS3		0.897	0.019	47.818	0.000
PANAS4		0.917	0.016	56.261	0.000
PANAS5		0.858	0.024	36.066	0.000
Variances					
POSAFF		1.000	0.000	999.000	999.000
Residual Variances					
PANAS1		0.246	0.039	6.321	0.000
PANAS2		0.355	0.049	7.214	0.000
PANAS3		0.195	0.034	5.799	0.000
PANAS4		0.159	0.030	5.301	0.000
PANAS5		0.265	0.041	6.486	0.000

Group WIDOW

POSAFF	BY				
PANAS1		0.982	0.010	98.103	0.000
PANAS2		0.933	0.022	41.705	0.000
PANAS3		0.933	0.022	41.541	0.000
PANAS4		0.931	0.023	40.606	0.000
PANAS5		0.844	0.047	18.073	0.000
Variances					
POSAFF		1.000	0.000	999.000	999.000
Residual Variances					
PANAS1		0.035	0.020	1.772	0.076
PANAS2		0.130	0.042	3.105	0.002
PANAS3		0.130	0.042	3.108	0.002
PANAS4		0.133	0.043	3.124	0.002
PANAS5		0.288	0.079	3.654	0.000

R-SQUARE

Group NOTWIDOW

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
PANAS1	0.754	0.039	19.384	0.000
PANAS2	0.645	0.049	13.092	0.000
PANAS3	0.805	0.034	23.909	0.000
PANAS4	0.841	0.030	28.131	0.000
PANAS5	0.735	0.041	18.033	0.000

Group WIDOW

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
PANAS1	0.965	0.020	49.052	0.000
PANAS2	0.870	0.042	20.853	0.000
PANAS3	0.870	0.042	20.771	0.000
PANAS4	0.867	0.043	20.303	0.000
PANAS5	0.712	0.079	9.037	0.000

Loadings Only Constrained Equal Across Groups
 Only unstandardized loadings³ shown to save space

```

title: Factorial invariance example ;

data: file=C:\Jason\mplus\semclass\stack1.dat; format=11f1.0;
      listwise=on;

variable: names = widow panas1 panas2 panas3 panas4 panas5
           panas6 panas7 panas8 panas9 panas10 ;
           grouping is widow (0=notwidow,1=widow);
  
```

³ It is the unstandardized loadings that need to be compared. Caution is warranted when comparing standardized values across groups. Standardized values can be misleading in some cases because of differences in variances across groups (Kim & Ferree, 1981).

```

missing = blank;

usevariables = panas1-panas5;

analysis: type=general; iterations = 200;
          model=nomeanstructure; information=expected;

model: posaff by panas1* panas2-panas5;

! Note: by default in Mplus, measurement errors and factor correlations are not
! constrained to be equal across groups;

Model notwidow:
  posaff by panas1* (ly1)
            panas2 (ly2)
            panas3 (ly3)
            panas4 (ly4)
            panas5 (ly5);
  posaff;

Model widow:
  posaff by panas1* (ly1)
            panas2 (ly2)
            panas3 (ly3)
            panas4 (ly4)
            panas5 (ly5);
  posaff;

Model constraint:
  ly1 = 5 - ly2 - ly3 -ly4 - ly5;

output: stdyx ;

```

Factorial invariance example ;

SUMMARY OF ANALYSIS

Number of groups	2
Number of observations	
Group NOTWIDOW	161
Group WIDOW	41
Total sample size	202

Number of dependent variables	5
Number of independent variables	0
Number of continuous latent variables	1

Observed dependent variables

Continuous					
PANAS1	PANAS2	PANAS3	PANAS4	PANAS5	

Continuous latent variables

POS AFF

Variables with special functions

Grouping variable	WIDOW
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Estimator	ML
Information matrix	EXPECTED
Maximum number of iterations	200
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20

MODEL FIT INFORMATION

Number of Free Parameters	16
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Loglikelihood

H0 Value	-926.082
H1 Value	-901.383

Information Criteria

Akaike (AIC)	1884.164
Bayesian (BIC)	1937.096
Sample-Size Adjusted BIC	1886.405
(n* = (n + 2) / 24)	

Chi-Square Test of Model Fit

Value	49.397
Degrees of Freedom	14
P-Value	0.0000

Chi-Square Contribution From Each Group

NOTWIDOW	35.010
WIDOW	14.388

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.158	
90 Percent C.I.	0.112	0.207
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.965
TLI	0.949

Chi-Square Test of Model Fit for the Baseline Model

Value	1018.190
Degrees of Freedom	20
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.065
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MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group NOTWIDOW				
POSAFF BY				
PANAS1	0.960	0.027	35.266	0.000
PANAS2	1.100	0.040	27.695	0.000
PANAS3	0.959	0.030	31.654	0.000
PANAS4	1.018	0.031	33.315	0.000
PANAS5	0.963	0.038	25.269	0.000
Variances				
POSAFF	0.671	0.080	8.426	0.000
Residual Variances				
PANAS1	0.172	0.024	7.051	0.000
PANAS2	0.402	0.051	7.900	0.000
PANAS3	0.152	0.022	6.770	0.000
PANAS4	0.147	0.023	6.393	0.000
PANAS5	0.260	0.034	7.686	0.000
Group WIDOW				
POSAFF BY				
PANAS1	0.960	0.027	35.266	0.000
PANAS2	1.100	0.040	27.695	0.000
PANAS3	0.959	0.030	31.654	0.000
PANAS4	1.018	0.031	33.315	0.000
PANAS5	0.963	0.038	25.269	0.000
Variances				
POSAFF	1.176	0.267	4.410	0.000
Residual Variances				
PANAS1	0.052	0.022	2.353	0.019
PANAS2	0.229	0.059	3.865	0.000

PANAS3	0.155	0.041	3.778	0.000
PANAS4	0.166	0.044	3.737	0.000
PANAS5	0.340	0.081	4.197	0.000

lavaan

All Parameters Free Across Groups (some output deleted)

```
> #all parameters free across groups
> model1 = '
+           posaff =~ c(NA,NA)*panas1 + c(1y1,1y6)*panas1 + c(1y2,1y7)*panas2 + c(1y3,1y8)*panas3 + c(1y4
+ ,1y9)*panas4 + c(1y5,1y10)*panas5
+   panas1~~panas1
+     panas2~~panas2
+     panas3~~panas3
+     panas4~~panas4
+     panas5~~panas5
+     posaff~~posaff
+
+     #complex constraints
+     1y1 == 5 - 1y2 - 1y3 - 1y4 - 1y5
+     1y6 == 5 - 1y7 - 1y8 - 1y9 - 1y10
+
+
+
+
+ #I resort back to listwise deletion to simplify examples and LR tests
> fit1 <- sem(model=model1, data = stack1, missing="listwise", estimator="ML", group="widow", meanstructure=FALSE)
> summary(object=fit1,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)
lavaan 0.6.15 ended normally after 37 iterations
```

Estimator	ML		
Optimization method	NLMINB		
Number of model parameters	22		
Number of equality constraints	2		
Number of observations per group:	Used	Total	
0	161	162	
1	41	43	
Model Test User Model:			
Test statistic	40.855		
Degrees of freedom	10		
P-value (Chi-square)	0.000		
Test statistic for each group:			
0	31.578		
1	9.277		
Model Test Baseline Model:			
Test statistic	1018.190		
Degrees of freedom	20		
P-value	0.000		
User Model versus Baseline Model:			
Comparative Fit Index (CFI)	0.969		
Tucker-Lewis Index (TLI)	0.938		
Loglikelihood and Information Criteria:			
Loglikelihood user model (H0)	-921.811		
Loglikelihood unrestricted model (H1)	-901.383		
Akaike (AIC)	1883.622		
Bayesian (BIC)	1949.787		
Sample-size adjusted Bayesian (SABIC)	1886.423		
Root Mean Square Error of Approximation:			
RMSEA	0.175		
90 Percent confidence interval - lower	0.121		
90 Percent confidence interval - upper	0.232		
P-value H_0: RMSEA <= 0.050	0.000		
P-value H_0: RMSEA >= 0.080	0.997		
Standardized Root Mean Square Residual:			
SRMR	0.028		
Parameter Estimates:			
Standard errors	Standard		
Information	Expected		
Information saturated (h1) model	Structured		

Group 1 [0]:

Latent Variables:

		Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
posaff =~							
panas1	(1y1)	0.907	0.039	23.134	0.000	0.742	0.868
panas2	(1y2)	1.061	0.054	19.703	0.000	0.867	0.803

panas3	(1y3)	0.967	0.037	25.918	0.000	0.791	0.897
panas4	(1y4)	1.048	0.037	28.394	0.000	0.857	0.917
panas5	(1y5)	1.017	0.044	22.943	0.000	0.832	0.858

Variiances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
.panas1	0.179	0.025	7.310	0.000	0.179	0.246
.panas2	0.414	0.052	8.002	0.000	0.414	0.355
.panas3	0.152	0.023	6.716	0.000	0.152	0.195
.panas4	0.138	0.023	6.074	0.000	0.138	0.159
.panas5	0.249	0.033	7.470	0.000	0.249	0.265
posaff	0.669	0.080	8.400	0.000	1.000	1.000

R-Square:

	Estimate
panas1	0.754
panas2	0.645
panas3	0.805
panas4	0.841
panas5	0.735

Group 2 [1]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
posaff =~						
panas1 (1y6)	1.022	0.039	26.396	0.000	1.080	0.982
panas2 (1y7)	1.172	0.062	18.856	0.000	1.238	0.933
panas3 (1y8)	0.970	0.054	17.890	0.000	1.025	0.933
panas4 (1y9)	0.988	0.056	17.786	0.000	1.043	0.931
panas5 (1y10)	0.848	0.073	11.557	0.000	0.896	0.844

Variiances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
.panas1	0.042	0.022	1.913	0.056	0.042	0.035
.panas2	0.228	0.059	3.847	0.000	0.228	0.130
.panas3	0.157	0.041	3.851	0.000	0.157	0.130
.panas4	0.168	0.043	3.871	0.000	0.168	0.133
.panas5	0.325	0.076	4.295	0.000	0.325	0.288
posaff	1.116	0.255	4.383	0.000	1.000	1.000

R-Square:

	Estimate
panas1	0.965
panas2	0.870
panas3	0.870
panas4	0.867
panas5	0.712

Constraints:

	slack
1y1 - (5-1y2-1y3-1y4-1y5)	0.000
1y6 - (5-1y7-1y8-1y9-1y10)	0.000

Loadings Only Constrained Equal Across Groups (some output deleted)

```
> # test of all loadings equal (weak invariance)
> model2 = '
+   posaff =~ NA*panas1 + (1y1)*panas1 + (1y2)*panas2 + (1y3)*panas3 + (1y4)*panas4 + (1y5)*panas5
+   panas1~~panas1
+   panas2~~panas2
+   panas3~~panas3
+   panas4~~panas4
+   panas5~~panas5
+   posaff~~posaff
+   #complex constraints
+   , 1y1 == 5 - 1y2 - 1y3 - 1y4 - 1y5
+
>
> #I resort back to listwise deletion to simplify examples and LR tests
> fit2 <- sem(model=model2, data = stack1, missing="listwise", estimator="ML", group="widow", group.equal=c("loadings"),meanstructure=FALSE)
> summary(object=fit2,fit.measures=TRUE, rsquare=TRUE, standardized=TRUE)
lavaan 0.6.15 ended normally after 33 iterations
```

Estimator	ML	
Optimization method	NLMINB	
Number of model parameters	22	
Number of equality constraints	6	
Number of observations per group:	Used	Total
0	161	162
1	41	43

Model Test User Model:

Test statistic	49.397
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Degrees of freedom	14
P-value (Chi-square)	0.000
Test statistic for each group:	
0	35.010
1	14.387

Model Test Baseline Model:

Test statistic	1018.190
Degrees of freedom	20
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.965
Tucker-Lewis Index (TLI)	0.949

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-926.082
Loglikelihood unrestricted model (H1)	-901.383

Akaike (AIC)	1884.164
Bayesian (BIC)	1937.096
Sample-size adjusted Bayesian (SABIC)	1886.405

Root Mean Square Error of Approximation:

RMSEA	0.158
90 Percent confidence interval - lower	0.112
90 Percent confidence interval - upper	0.207
P-value H ₀ : RMSEA ≤ 0.050	0.000
P-value H ₀ : RMSEA ≥ 0.080	0.996

Standardized Root Mean Square Residual:

SRMR	0.061
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Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Group 1 [0]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
posaff =~						
panas1 (ly1)	0.960	0.027	35.266	0.000	0.786	0.884
panas2 (ly2)	1.100	0.040	27.696	0.000	0.901	0.818
panas3 (ly3)	0.959	0.030	31.654	0.000	0.786	0.896
panas4 (ly4)	1.018	0.031	33.315	0.000	0.834	0.909
panas5 (ly5)	0.963	0.038	25.268	0.000	0.789	0.840

Variances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
.panas1	0.172	0.024	7.051	0.000	0.172	0.218
.panas2	0.402	0.051	7.900	0.000	0.402	0.331
.panas3	0.152	0.022	6.770	0.000	0.152	0.198
.panas4	0.147	0.023	6.393	0.000	0.147	0.174
.panas5	0.260	0.034	7.686	0.000	0.260	0.295
posaff	0.671	0.080	8.426	0.000	1.000	1.000

R-Square:

	Estimate
panas1	0.782
panas2	0.669
panas3	0.802
panas4	0.826
panas5	0.705

Group 2 [1]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
posaff =~						
panas1 (ly1)	0.960	0.027	35.266	0.000	1.041	0.977
panas2 (ly2)	1.100	0.040	27.696	0.000	1.193	0.928
panas3 (ly3)	0.959	0.030	31.654	0.000	1.040	0.935
panas4 (ly4)	1.018	0.031	33.315	0.000	1.105	0.938
panas5 (ly5)	0.963	0.038	25.268	0.000	1.044	0.873

Variances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
.panas1	0.052	0.022	2.353	0.019	0.052	0.046
.panas2	0.229	0.059	3.865	0.000	0.229	0.139
.panas3	0.155	0.041	3.778	0.000	0.155	0.125
.panas4	0.166	0.044	3.737	0.000	0.166	0.120
.panas5	0.340	0.081	4.197	0.000	0.340	0.238
posaff	1.177	0.267	4.410	0.000	1.000	1.000

R-Square:

	Estimate
panas1	0.954
panas2	0.861
panas3	0.875
panas4	0.880
panas5	0.762

Constraints:

	slack
1y1 - (5-1y2-1y3-1y4-1y5)	0.000

Conduct Chi-square Difference Tests

```
> library(psych)
>
> # Chi-square difference test (compare model1 and model2 -- compare fit with single constraint)
> fitMeasures(fit1, c("chisq", "df"))
  chisq  df
40.855 10.000
> fitMeasures(fit2, c("chisq", "df"))
  chisq  df
49.397 14.000
> diffchisq=fitMeasures(fit2, c("chisq", "df"))-fitMeasures(fit1, c("chisq", "df"))
> diffchisq
  chisq  df
 8.542  4.000
> 1-pchisq(diffchisq[2], diffchisq[1])
  df
0.8891296
> rm(diffchisq)
```

Chi-square comparisons

	χ^2	df	Comparison to all-free model	$\Delta\chi^2$	Δdf	w	ΔNCI
All parameters free	40.855	10					
Loadings only equal	49.397	14	8.542***	4	.061	.004	

* p < .05, ** p < .01, *** p < .001. If robust estimates had been used, this test would have used a scaled chi-square comparison that weights by the scaling correction factor instead of the simple subtraction method.

Sample Write-up

A multigroup structural equation modeling approach was used to compare widows and nonwidows on the factor loadings of the positive affect scale. To test for weak factorial invariance (Meredith, 1993) across groups, the chi-square from a model with all parameters allowed to be unequal across groups was compared to the chi-square from a model with only the loadings constrained to be equal across groups. No means or intercepts were estimated in these models. The model with all parameters freely estimated in the two groups, fit the data well, CFI = .969, SRMR = .028, according to fit criteria suggested by Hu and Bentler (1999), although the overall chi-square was significant, $\chi^2(68) = 40.855$ p < .001. The weak invariance model with loadings constrained to be equal across groups had fit that was significantly poorer, $\chi^2(14) = 49.397$, p < .001, $\Delta\chi^2(4) = 8.542$, p < .001, suggesting that the loadings differed significantly across groups. This difference was small in magnitude, however, w = .061, $\Delta NCI = .004$. Further analyses are required to determine which loadings may differ across groups. The findings suggest that the measurement of the one-factor positive affect scale differs for widows and non-widows, and, thus, caution may be warranted in comparing these groups.⁴

References

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, 6, 1-55.

Kim, J. O., & Ferree, G. D., Jr. (1981). Standardization in causal analysis. *Sociological Methods and Research*, 10, 187-210.

Little, T. D., Slegers, D. W., & Card, N. A. (2006). A non-arbitrary method of identifying and scaling latent variables in SEM and MACS models. *Structural Equation Modeling: A Multidisciplinary Journal*, 13, 59-72.

Meredith, W. (1993). Measurement invariance, factor analysis, and factorial invariance. *Psychometrika*, 58, 525-543.

⁴ In an actual analysis, the researcher might stop here if it was concluded that the degree of invariance was significant but trivial. Otherwise, the next step would be identifying the source of the difference (see Cheung and Lau, 2012, for one method of exploring particular loading differences) and consider possible modifications to the scale so that it might have comparable measurement properties.