

Second-Order Growth Curve Example

Mplus

I recommend effects coding (Little, Slegers, & Card, 2006), as illustrated below, for first-order factor identification to improve interpretability of factor means and slopes. Loadings and measurement intercepts are set equal over time in this example, which should be examined in prior steps.

```
title: Newsom Longitudinal SEM2 Chapter 7, Example 7.6c,  
      Latent Growth Curve Models;  
  
data: file=health.dat; format=free;  
  
variable:  
  names=  
    age  
    srh1 srh2 srh3 srh4 srh5 srh6  
    bmi1 bmi2 bmi3 bmi4 bmi5 bmi6  
    cesdna1 cesdpa1 cesdso1  
    cesdna2 cesdpa2 cesdso2  
    cesdna3 cesdpa3 cesdso3  
    cesdna4 cesdpa4 cesdso4  
    cesdna5 cesdpa5 cesdso5  
    cesdna6 cesdpa6 cesdso6  
    diab1 diab2 diab3 diab4 diab5 diab6;  
  
  usevariables=  
    cesdna1 cesdpa1 cesdso1  
    cesdna2 cesdpa2 cesdso2  
    cesdna3 cesdpa3 cesdso3  
    cesdna4 cesdpa4 cesdso4  
    cesdna5 cesdpa5 cesdso5  
    cesdna6 cesdpa6 cesdso6;  
  
analysis: type=general;  
model: !second order growth curve model with effects coding scaling of 1st order factors;  
!free first loading with * to remove default 1st item referent;  
!use of the same labels across waves sets equality constraints on loadings and intercepts;  
  eta1 by cesdna1* (lambda1)  
  cesdpa1 (lambda2)  
  cesdso1 (lambda3);  
  eta2 by cesdna2* (lambda1)  
  cesdpa2 (lambda2)  
  cesdso2 (lambda3);  
  eta3 by cesdna3* (lambda1)  
  cesdpa3 (lambda2)  
  cesdso3 (lambda3);  
  eta4 by cesdna4* (lambda1)  
  cesdpa4 (lambda2)  
  cesdso4 (lambda3);  
  eta5 by cesdna5* (lambda1)  
  cesdpa5 (lambda2)  
  cesdso5 (lambda3);  
  eta6 by cesdna6* (lambda1)  
  cesdpa6 (lambda2)  
  cesdso6 (lambda3);  
  
[cesdna1] (nu1);  
[cesdpa1] (nu2);  
[cesdso1] (nu3);  
[cesdna2] (nu1);  
[cesdpa2] (nu2);  
[cesdso2] (nu3);  
[cesdna3] (nu1);  
[cesdpa3] (nu2);  
[cesdso3] (nu3);  
[cesdna4] (nu1);  
[cesdpa4] (nu2);  
[cesdso4] (nu3);  
[cesdna5] (nu1);  
[cesdpa5] (nu2);  
[cesdso5] (nu3);
```

```
[cesdna6] (nu1);
[cesdpa6] (nu2);
[cesdso6] (nu3);

i by eta1@1 eta2@1 eta3@1 eta4@1 eta5@1 eta6@1;
s by eta1@0 eta2@1 eta3@2 eta4@3 eta5@4 eta6@5;
i s;
i with s;
[i s];
[eta1-eta6@0];

cesdna1 with cesdna2 cesdna3 cesdna4 cesdna5 cesdna6;
cesdna2 with cesdna3 cesdna4 cesdna5 cesdna6;
cesdna3 with cesdna4 cesdna5 cesdna6;
cesdna4 with cesdna5 cesdna6;
cesdna5 with cesdna6;
cesdpa1 with cesdpa2 cesdpa3 cesdpa4 cesdpa5 cesdpa6;
cesdpa2 with cesdpa3 cesdpa4 cesdpa5 cesdpa6;
cesdpa3 with cesdpa4 cesdpa5 cesdpa6;
cesdpa4 with cesdpa5 cesdpa6;
cesdpa5 with cesdpa6;
cesdso1 with cesdso2 cesdso3 cesdso4 cesdso5 cesdso6;
cesdso2 with cesdso3 cesdso4 cesdso5 cesdso6;
cesdso3 with cesdso4 cesdso5 cesdso6;
cesdso4 with cesdso5 cesdso6;
cesdso5 with cesdso6;

model constraint:
  lambda1 = 3 - lambda2 - lambda3;
  nu1 = 0 - nu2 - nu3;

output: sampstat stdyx;
```

(Portions of this output have been omitted)

SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	5335

Number of dependent variables	18
Number of independent variables	0
Number of continuous latent variables	8

Estimator	ML
Information matrix	OBSERVED
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20

Input data file(s)
health.dat

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters	78
---------------------------	----

Chi-Square Test of Model Fit

Value	815.215
Degrees of Freedom	111
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.034	
90 Percent C.I.	0.032	0.037
Probability RMSEA <= .05	1.000	

CFI/TLI

CFI	0.981
TLI	0.974

Chi-Square Test of Model Fit for the Baseline Model

Value	38014.882
Degrees of Freedom	153
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.045
-------	-------

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
ETA1 BY				
CESDNA1	1.361	0.012	115.421	0.000
CESDPA1	0.765	0.008	101.739	0.000
CESDSO1	0.875	0.010	87.579	0.000
ETA2 BY				
CESDNA2	1.361	0.012	115.421	0.000
CESDPA2	0.765	0.008	101.739	0.000
CESDSO2	0.875	0.010	87.579	0.000
ETA3 BY				
CESDNA3	1.361	0.012	115.421	0.000
CESDPA3	0.765	0.008	101.739	0.000
CESDSO3	0.875	0.010	87.579	0.000
ETA4 BY				
CESDNA4	1.361	0.012	115.421	0.000
CESDPA4	0.765	0.008	101.739	0.000
CESDSO4	0.875	0.010	87.579	0.000
ETA5 BY				
CESDNA5	1.361	0.012	115.421	0.000
CESDPA5	0.765	0.008	101.739	0.000
CESDSO5	0.875	0.010	87.579	0.000
ETA6 BY				
CESDNA6	1.361	0.012	115.421	0.000
CESDPA6	0.765	0.008	101.739	0.000
CESDSO6	0.875	0.010	87.579	0.000
I BY				
ETA1	1.000	0.000	999.000	999.000
ETA2	1.000	0.000	999.000	999.000
ETA3	1.000	0.000	999.000	999.000
ETA4	1.000	0.000	999.000	999.000
ETA5	1.000	0.000	999.000	999.000
ETA6	1.000	0.000	999.000	999.000
S BY				
ETA1	0.000	0.000	999.000	999.000
ETA2	1.000	0.000	999.000	999.000
ETA3	2.000	0.000	999.000	999.000
ETA4	3.000	0.000	999.000	999.000
ETA5	4.000	0.000	999.000	999.000
ETA6	5.000	0.000	999.000	999.000
I WITH				
S	-0.004	0.001	-5.708	0.000

CESDNA1 WITH				
CESDNA2	-0.019	0.004	-4.967	0.000
CESDNA3	-0.014	0.004	-3.562	0.000
CESDNA4	-0.015	0.004	-3.908	0.000
CESDNA5	-0.007	0.004	-1.730	0.084
CESDNA6	-0.009	0.004	-2.296	0.022
CESDNA2 WITH				
CESDNA3	-0.007	0.004	-1.790	0.073
CESDNA4	-0.012	0.004	-3.194	0.001
CESDNA5	-0.019	0.004	-4.962	0.000
CESDNA6	-0.007	0.004	-1.958	0.050
CESDNA3 WITH				
CESDNA4	0.008	0.004	1.912	0.056
CESDNA5	-0.003	0.004	-0.730	0.465
CESDNA6	-0.013	0.004	-3.260	0.001
CESDNA4 WITH				
CESDNA5	0.000	0.004	-0.009	0.993
CESDNA6	-0.009	0.004	-2.162	0.031
CESDNA5 WITH				
CESDNA6	-0.010	0.004	-2.179	0.029
CESDPA1 WITH				
CESDPA2	0.036	0.002	15.588	0.000
CESDPA3	0.030	0.002	12.313	0.000
CESDPA4	0.032	0.002	13.672	0.000
CESDPA5	0.029	0.002	12.623	0.000
CESDPA6	0.025	0.002	11.165	0.000
CESDPA2 WITH				
CESDPA3	0.035	0.002	13.943	0.000
CESDPA4	0.031	0.002	13.030	0.000
CESDPA5	0.031	0.002	13.208	0.000
CESDPA6	0.031	0.002	13.023	0.000
CESDPA3 WITH				
CESDPA4	0.040	0.003	15.417	0.000
CESDPA5	0.039	0.003	15.369	0.000
CESDPA6	0.031	0.003	12.257	0.000
CESDPA4 WITH				
CESDPA5	0.040	0.002	16.216	0.000
CESDPA6	0.038	0.002	15.444	0.000
CESDPA5 WITH				
CESDPA6	0.041	0.002	16.887	0.000
CESDSO1 WITH				
CESDSO2	0.103	0.005	21.253	0.000
CESDSO3	0.107	0.005	20.918	0.000
CESDSO4	0.096	0.005	18.911	0.000
CESDSO5	0.099	0.005	19.477	0.000
CESDSO6	0.094	0.005	18.973	0.000
CESDSO2 WITH				
CESDSO3	0.116	0.005	23.092	0.000
CESDSO4	0.100	0.005	20.254	0.000
CESDSO5	0.112	0.005	22.215	0.000
CESDSO6	0.091	0.005	18.674	0.000
CESDSO3 WITH				
CESDSO4	0.117	0.005	21.989	0.000
CESDSO5	0.117	0.005	21.904	0.000
CESDSO6	0.105	0.005	20.264	0.000
CESDSO4 WITH				
CESDSO5	0.132	0.005	24.247	0.000
CESDSO6	0.113	0.005	21.682	0.000
CESDSO5 WITH				
CESDSO6	0.135	0.005	25.224	0.000

Means				
I	0.301	0.006	53.655	0.000
S	0.005	0.001	4.109	0.000
Intercepts				
CESDNA1	-0.127	0.005	-27.483	0.000
CESDPA1	-0.072	0.004	-20.109	0.000
CESDSO1	0.199	0.005	40.331	0.000
CESDNA2	-0.127	0.005	-27.483	0.000
CESDPA2	-0.072	0.004	-20.109	0.000
CESDSO2	0.199	0.005	40.331	0.000
CESDNA3	-0.127	0.005	-27.483	0.000
CESDPA3	-0.072	0.004	-20.109	0.000
CESDSO3	0.199	0.005	40.331	0.000
CESDNA4	-0.127	0.005	-27.483	0.000
CESDPA4	-0.072	0.004	-20.109	0.000
CESDSO4	0.199	0.005	40.331	0.000
CESDNA5	-0.127	0.005	-27.483	0.000
CESDPA5	-0.072	0.004	-20.109	0.000
CESDSO5	0.199	0.005	40.331	0.000
CESDNA6	-0.127	0.005	-27.483	0.000
CESDPA6	-0.072	0.004	-20.109	0.000
CESDSO6	0.199	0.005	40.331	0.000
ETA1	0.000	0.000	999.000	999.000
ETA2	0.000	0.000	999.000	999.000
ETA3	0.000	0.000	999.000	999.000
ETA4	0.000	0.000	999.000	999.000
ETA5	0.000	0.000	999.000	999.000
ETA6	0.000	0.000	999.000	999.000
Variances				
I	0.096	0.003	29.014	0.000
S	0.002	0.000	9.133	0.000
Residual Variances				
ETA1	0.074	0.003	23.480	0.000
ETA2	0.072	0.003	26.284	0.000
ETA3	0.063	0.003	23.607	0.000
ETA4	0.060	0.003	23.310	0.000
ETA5	0.059	0.003	22.541	0.000
ETA6	0.067	0.003	21.645	0.000

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.105E-07
 (ratio of smallest to largest eigenvalue)

STANDARDIZED MODEL RESULTS

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
ETA1 BY				
CESDNA1	0.905	0.008	107.357	0.000
CESDPA1	0.660	0.008	83.123	0.000
CESDSO1	0.540	0.009	63.011	0.000
ETA2 BY				
CESDNA2	0.889	0.008	105.061	0.000
CESDPA2	0.640	0.008	80.324	0.000
CESDSO2	0.543	0.009	63.388	0.000
ETA3 BY				
CESDNA3	0.837	0.009	92.097	0.000
CESDPA3	0.596	0.008	74.049	0.000
CESDSO3	0.509	0.008	60.895	0.000
ETA4 BY				

CESDNA4	0.845	0.009	93.122	0.000
CESDPA4	0.607	0.008	76.981	0.000
CESDSO4	0.503	0.008	60.865	0.000
ETA5 BY				
CESDNA5	0.835	0.009	93.609	0.000
CESDPA5	0.632	0.008	78.019	0.000
CESDSO5	0.508	0.009	59.489	0.000
ETA6 BY				
CESDNA6	0.876	0.009	100.340	0.000
CESDPA6	0.645	0.008	79.913	0.000
CESDSO6	0.536	0.009	62.472	0.000
I BY				
ETA1	0.752	0.010	75.392	0.000
ETA2	0.769	0.011	72.923	0.000
ETA3	0.795	0.012	64.895	0.000
ETA4	0.799	0.013	60.135	0.000
ETA5	0.790	0.014	54.802	0.000
ETA6	0.750	0.015	50.483	0.000
S BY				
ETA1	0.000	0.000	999.000	999.000
ETA2	0.104	0.006	18.048	0.000
ETA3	0.214	0.012	18.003	0.000
ETA4	0.323	0.018	18.092	0.000
ETA5	0.426	0.023	18.149	0.000
ETA6	0.506	0.028	18.134	0.000
I WITH S				
	-0.278	0.036	-7.766	0.000
CESDNA1 WITH				
CESDNA2	-0.253	0.063	-4.022	0.000
CESDNA3	-0.154	0.049	-3.138	0.002
CESDNA4	-0.167	0.049	-3.413	0.001
CESDNA5	-0.072	0.044	-1.633	0.102
CESDNA6	-0.105	0.050	-2.113	0.035
CESDNA2 WITH				
CESDNA3	-0.073	0.043	-1.686	0.092
CESDNA4	-0.130	0.045	-2.880	0.004
CESDNA5	-0.196	0.046	-4.273	0.000
CESDNA6	-0.086	0.047	-1.828	0.068
CESDNA3 WITH				
CESDNA4	0.069	0.034	2.021	0.043
CESDNA5	-0.026	0.036	-0.716	0.474
CESDNA6	-0.125	0.042	-2.966	0.003
CESDNA4 WITH				
CESDNA5	0.000	0.036	-0.009	0.993
CESDNA6	-0.087	0.043	-2.017	0.044
CESDNA5 WITH				
CESDNA6	-0.088	0.043	-2.029	0.042
CESDPA1 WITH				
CESDPA2	0.271	0.015	17.900	0.000
CESDPA3	0.207	0.015	13.418	0.000
CESDPA4	0.231	0.015	15.118	0.000
CESDPA5	0.216	0.016	13.863	0.000
CESDPA6	0.189	0.016	12.033	0.000
CESDPA2 WITH				
CESDPA3	0.233	0.015	15.457	0.000
CESDPA4	0.218	0.015	14.253	0.000
CESDPA5	0.227	0.016	14.585	0.000
CESDPA6	0.221	0.015	14.335	0.000
CESDPA3 WITH				
CESDPA4	0.257	0.015	17.345	0.000
CESDPA5	0.261	0.015	17.388	0.000

CESDPA6	0.206	0.015	13.311	0.000
CESDPA4 WITH				
CESDPA5	0.280	0.015	18.677	0.000
CESDPA6	0.264	0.015	17.528	0.000
CESDPA5 WITH				
CESDPA6	0.298	0.015	19.879	0.000
CESDSO1 WITH				
CESDSO2	0.337	0.013	25.511	0.000
CESDSO3	0.329	0.013	24.808	0.000
CESDSO4	0.293	0.014	21.652	0.000
CESDSO5	0.304	0.013	22.540	0.000
CESDSO6	0.296	0.014	21.768	0.000
CESDSO2 WITH				
CESDSO3	0.369	0.013	28.630	0.000
CESDSO4	0.316	0.013	23.664	0.000
CESDSO5	0.353	0.013	26.977	0.000
CESDSO6	0.293	0.014	21.440	0.000
CESDSO3 WITH				
CESDSO4	0.348	0.013	26.625	0.000
CESDSO5	0.348	0.013	26.547	0.000
CESDSO6	0.320	0.013	23.805	0.000
CESDSO4 WITH				
CESDSO5	0.387	0.013	30.653	0.000
CESDSO6	0.342	0.013	26.032	0.000
CESDSO5 WITH				
CESDSO6	0.408	0.012	32.812	0.000
Means				
I	0.970	0.025	39.232	0.000
S	0.119	0.030	4.015	0.000

Lavaan¹

```
names(health1) = c("age", "srh1", "srh2", "srh3", "srh4", "srh5", "srh6", "bmi1",
"bmi2", "bmi3", "bmi4", "bmi5", "bmi6", "cesdna1", "cesdpa1", "cesdso1",
"cesdna2", "cesdpa2", "cesdso2", "cesdna3", "cesdpa3", "cesdso3",
"cesdna4", "cesdpa4", "cesdso4", "cesdna5", "cesdpa5", "cesdso5",
"cesdna6", "cesdpa6", "cesdso6", "diab1", "diab2", "diab3", "diab4", "diab5", "diab6")
```

```
library(lavaan)
```

```
#second order growth curve model with effects coding scaling of 1st order factors;
#free first loading with NA* to remove default 1st item referent;
#use of the same labels across waves sets equality constraints on loadings and intercepts;
model7.6c <- ' #first loading is referent by default;
#equality constraints on loading and intercepts omit for this example;
```

```
eta1 =~ NA*cesdna1 + (lambda1)*cesdna1 + (lambda2)*cesdpa1 + (lambda3)*cesdso1
eta2 =~ NA*cesdna2 + (lambda1)*cesdna2 + (lambda2)*cesdpa2 + (lambda3)*cesdso2
eta3 =~ NA*cesdna3 + (lambda1)*cesdna3 + (lambda2)*cesdpa3 + (lambda3)*cesdso3
eta4 =~ NA*cesdna4 + (lambda1)*cesdna4 + (lambda2)*cesdpa4 + (lambda3)*cesdso4
eta5 =~ NA*cesdna5 + (lambda1)*cesdna5 + (lambda2)*cesdpa5 + (lambda3)*cesdso5
eta6 =~ NA*cesdna6 + (lambda1)*cesdna6 + (lambda2)*cesdpa6 + (lambda3)*cesdso6
```

```
cesdna1 ~ (nu1)*1
cesdpa1 ~ (nu2)*1
cesdso1 ~ (nu3)*1
cesdna2 ~ (nu1)*1
cesdpa2 ~ (nu2)*1
cesdso2 ~ (nu3)*1
cesdna3 ~ (nu1)*1
cesdpa3 ~ (nu2)*1
cesdso3 ~ (nu3)*1
cesdna4 ~ (nu1)*1
cesdpa4 ~ (nu2)*1
cesdso4 ~ (nu3)*1
cesdna5 ~ (nu1)*1
cesdpa5 ~ (nu2)*1
cesdso5 ~ (nu3)*1
cesdna6 ~ (nu1)*1
```

¹ Thanks to Amanuel Zimam Melekin for developing the R code translation from Mplus for the first edition of the book

```

cesdpa6 ~ (nu2)*1
cesdso6 ~ (nu3)*1

cesdna1 ~~ cesdna2 + cesdna3 + cesdna4 + cesdna5 + cesdna6
cesdna2 ~~ cesdna3 + cesdna4 + cesdna5 + cesdna6
cesdna3 ~~ cesdna4 + cesdna5 + cesdna6
cesdna4 ~~ cesdna5 + cesdna6
cesdna5 ~~ cesdna6
cesdpa1 ~~ cesdpa2 + cesdpa3 + cesdpa4 + cesdpa5 + cesdpa6
cesdpa2 ~~ cesdpa3 + cesdpa4 + cesdpa5 + cesdpa6
cesdpa3 ~~ cesdpa4 + cesdpa5 + cesdpa6
cesdpa4 ~~ cesdpa5 + cesdpa6
cesdpa5 ~~ cesdpa6
cesdso1 ~~ cesdso2 + cesdso3 + cesdso4 + cesdso5 + cesdso6
cesdso2 ~~ cesdso3 + cesdso4 + cesdso5 + cesdso6
cesdso3 ~~ cesdso4 + cesdso5 + cesdso6
cesdso4 ~~ cesdso5 + cesdso6
cesdso5 ~~ cesdso6

i =~ 1*eta1 + 1*eta2 + 1*eta3 + 1*eta4 + 1*eta5 + 1*eta6
s =~ 0*eta1 + 1*eta2 + 2*eta3 + 3*eta4 + 4*eta5 + 5*eta6

i ~~ i
s ~~ s
i ~~ s

i ~ 1
s ~ 1

eta1 ~ 0*1
eta2 ~ 0*1
eta3 ~ 0*1
eta4 ~ 0*1
eta5 ~ 0*1
eta6 ~ 0*1

#model constraints;
lambda1 == 3 - lambda2 - lambda1
(nu1) == 0 - (nu2) - (nu3) '

fitmodel7.6c <- growth(model7.6c, data=health1)
summary(fitmodel7.6c, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)

```

I have omitted the lavaan output to save space