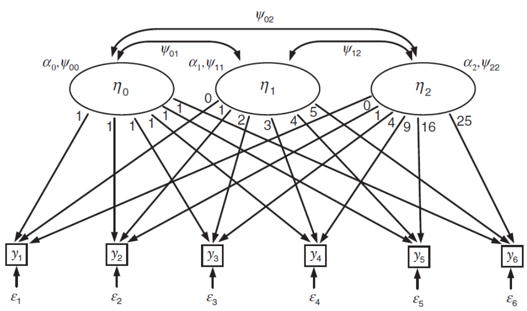
# Advanced Topics and Further Reading

## Longitudinal Structural Equation Models-General

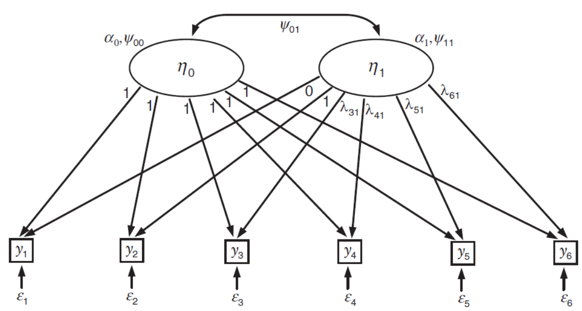
Little, T. D. (2024). *Longitudinal structural equation modeling, second edition*. Guilford.
McArdle, J.J., & Nesselroade, J.R. (2014). *Longitudinal data analysis using structural equation models*. Washington, D.C.: APA.
Newsom, J.T. (2024). Longitudinal structural equation modeling: A comprehensive introduction, second edition. Routledge.

## Latent Growth Curve Models

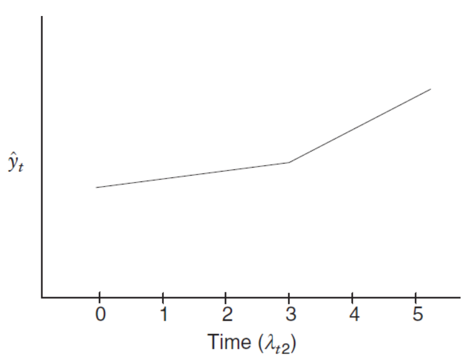
*Quadratic Latent Growth Curve Models.* Additional factors can be added to growth curve models with loadings corresponding to the functional form (e.g., squared time codes for quadratic curves, cubed time codes for cubic curves). This general approach can be modified for log forms or other mathematical functions.

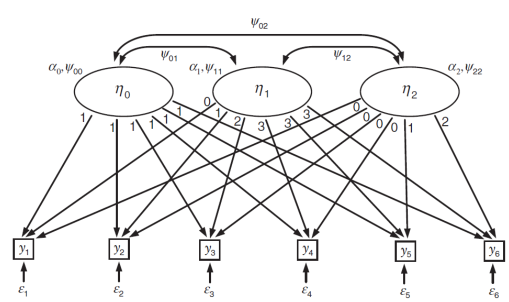


*Latent Basis Growth Curve Models.* The latent basis model fits a free from shape function by estimating loadings for the growth curve factor. As with other nonlinear forms, I recommended ensuring that the curvilinear adds beyond a simple linear trend by comparing this model to a linear model or including a linear slope.



*Piecewise Growth Curve Models.* Piecewise growth curve models specify two slopes, one before and one after an event. These models can be used to evaluate the changes that take place after an intervention or other phenomena, such as a policy change. They can be extended to incorporate nonlinear forms as well. Two growth factors are used with loadings that model changes only up to a chosen knot point and from a chosen knot point.



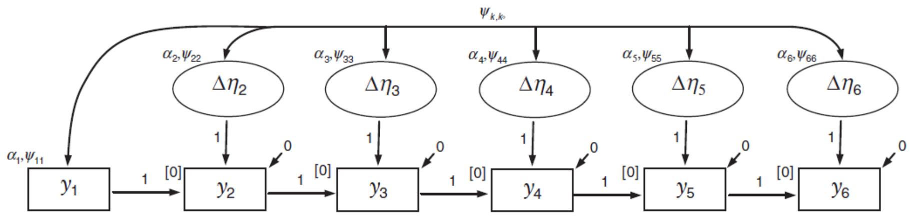



Bollen, K. A., & Curran, P. J. (2006). *Latent curve models: A structural equation perspective* (Vol. 467). New York: John Wiley & Sons.
Duncan, T. E., Duncan, S.C., Strycker, L.A. (2006). *An introduction to latent variable growth curve modeling: Concepts, issues, and applications, second edition*. Mahwah, NJ: Erlbaum.
Grimm, K. J., Ram, N., & Estabrook, R. (2016). Growth modeling: Structural equation and multilevel modeling approaches. New York: Guilford Publications.
Newsom, J.T. (2024). Chapters 7 and 8 in Longitudinal Structural Equation Modeling: A Comprehensive Introduction, Second edition. New York: Routledge.
Preacher, K.J., Wichman, A.L., MacCallum, R.C., & Briggs, N.E. (2008). *Latent growth curve modeling*. Thousand Oaks, CA: Sage.

Singer, J. D., & Willett, J. B. (2003). *Applied longitudinal data analysis: Modeling change and event occurrence*. Oxford, England: Oxford university press. (covers multilevel regression approach to growth curve models, latent growth curve models, and survival analysis).

### Latent Change Score Model

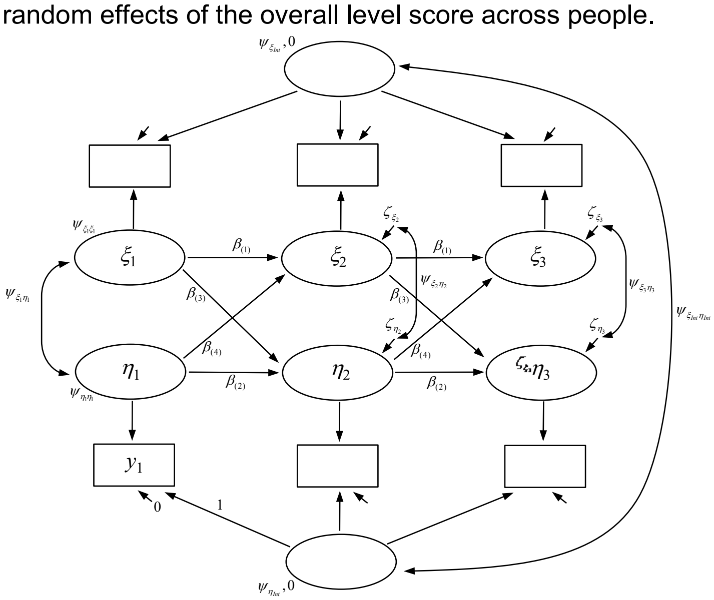
Also referred to as the latent difference score model. Latent variables are set up to capture the mean difference between each consecutive pair of time points. Loading and intercept constraints are placed on single indicator factors to capture the difference scores. The variances of the difference scores (random effects) are captured in the latent variable variances for the difference factors.




Hamagami, F., & McArdle, J. J. (2001). Advanced studies of individual differences linear dynamic models for longitudinal data analysis. In G. Marcoulides & R. Schumacker (Eds.), New developments and techniques in structural equation modeling New developments and techniques (pp. 203–246). Mahwah, NJ: Erlbaum.
Hamagami, F., & McArdle, J. J. (2007). Dynamic extensions of latent difference score models. In S.M. Boker & M.J. Wenger, Data analytic techniques for dynamic systems. Mahwah, NJ: Erlbaum.
Kievit, R. A., Brandmaier, A. M., Ziegler, G., van Harmelen, A. L., de Mooij, S. M., Moutoussis, M.,Goodyear, I.M., Bullmore, E., Jones, JB,, Fonagy, P, NSPN Consortium, Lindenberger,U., & Dolan, R.J.. (2018). Developmental cognitive neuroscience using latent change score models: A tutorial and applications. Developmental Cognitive Neuroscience, 33, 99-117.
Klopack, E. T., & Wickrama, K. (2020). Modeling latent change score analysis and extensions in Mplus: A practical guide for researchers. Structural Equation Modeling: A Multidisciplinary Journal, 27, 97-110.

### Random Intercept Cross-Lagged Panel Model (RI-CLPM)

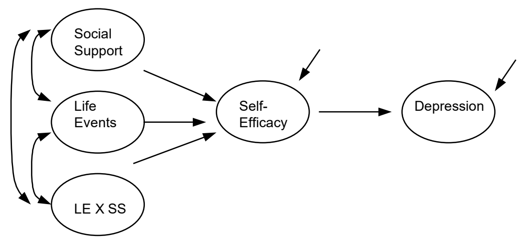
The RI-CLPM attempts to capture within-person autoregressive and cross-lagged effects as well as capture random effects of the overall level score across people.




Hamaker, E. L., Kuiper, R. M., & Grasman, R. P. (2015). A critique of the cross-lagged panel model. Psychological methods, 20, 102-116.
Lüdtke, O., & Robitzsch, A. (2022). A comparison of different approaches for estimating cross-lagged effects from a causal inference perspective. Structural Equation Modeling: A Multidisciplinary Journal, 1-20.
Usami, S., Murayama, K., & Hamaker, E. L. (2019). A unified framework of longitudinal models to examine reciprocal relations. Psychological methods, 24(5), 637.

## Latent Variable Interactions

There are two general approaches to testing moderator (i.e., interaction) hypotheses in SEM. The most common and simplest approach is by using multigroup SEM. In this approach, the moderator must be a categorical variable that moderates the relationships tested in the model. In regression analysis, moderation is tested by computing a product variable where the two predictors hypothesized to interact are multiplied together. The two predictors and the product variable are used to predict the dependent variable[1]. If the two interacting predictors are measured variables, the exact same procedure as that used in regression can be used to test for moderation. An analogous procedure also can be used for latent variables, but latent variable interactions have faced a number of issues, including dealing with multiple product indicators and standard error accurate estimate in the face of nonnormality of interaction terms. Mplus (Version 3 and above) will automatically handle latent variable interactions using a modified version of the Klein and Moosbrugger latent moderated structural equations (2000; LMS) approach (using the XWITH keyword). The chapters by Marsh and colleagues (2012) and Kelava & Brandt (2023) are good overviews and introductions to some of the issues.

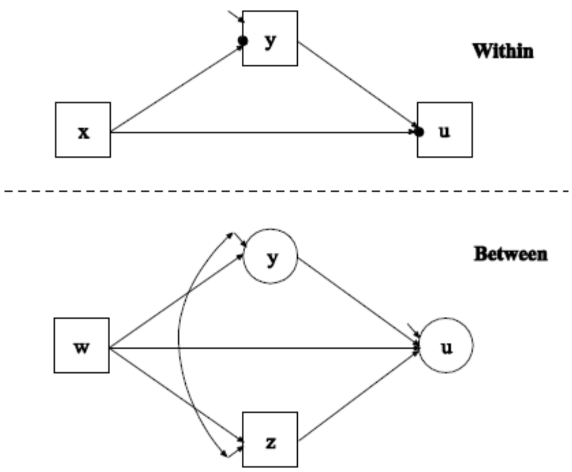



Algina, J., & Moulder, B.C. (2001). A note on estimating the Joreskog-Yang model for latent variable interaction using LISREL 8.3. *Structural Equation Modeling, 8*, 40-52.
Bollen, K.A., & Paxton, P. (1998). Interactions of latent variables in structural equation models. *Structural Equation Modeling, 5*, 267-293.
Cham, H., West, S. G., Ma, Y., & Aiken, L. S. (2012). Estimating latent variable interactions with nonnormal observed data: A comparison of four approaches. *Multivariate behavioral research, 47*(6), 840-876.
Cham, H., West, S. G., Ma, Y., & Aiken, L. S. (2012). Estimating latent variable interactions with nonnormal observed data: A comparison of four approaches. *Multivariate Behavioral Research, 47*(6), 840-876.
Cham, H., Reshetnyak, E., Rosenfeld, B., & Breitbart, W. (2017). Full Information Maximum Likelihood Estimation for Latent Variable Interactions With Incomplete Indicators. *Multivariate Behavioral Research, 52*, 12-3.
Coenders, G., Batista-Foguet, J. M., and Saris, W. E. (2008) Simple, Efficient and Distribution-Free Approach to Interaction Effects in Complex Structural Equation Models, *Quality & Quantity, 42(3)* 369-396.
Jaccard, J., & Wan, C.K. (1996). *Lisrel approaches to interaction effects in multiple regression*. Newbury Park, CA: Sage.
Joreskog, K.G., & Yang, F. (1996). Nonlinear structural equation models: The Kenny-Judd model with interaction effects. In G.A. Marcoulides & R.E. Schumacker (Eds.), *Advanced structural equation modeling: Issues and techniques*. Mahwah, NJ: Erlbaum.
Kelava, A., & Brandt, H. (2023). Latent interaction effects. In R.H. Hoyle, *Handbook of structural equation modeling, second edition* (pp. 427-446). Guilford.
Lin, G. C., Wen, Z., Marsh, H. W., & Lin, H. S. (2010). Structural equation models of latent interactions: Clarification of orthogonalizing and double-mean-centering strategies. *Structural Equation Modeling, 17*, 374-391.
Klein, A., & Moosbrugger, H. (2000). Maximum likelihood estimation of latent interaction effects with the LMS method. *Psychometrika, 65, 457-474.*
Marsh, H.W, Wen, Z., Hau, K-T, Little, T.D, Bovaird, J,A, & Widaman, K.F. (2007). Unconstrained structural equation models of latent interactions: Contrasting residual- and mean-centered approaches. *Structural Equation Modeling, 14,* 570-580.
Marsh, H.W., Wen, Z., & Hau, K-T. (2004). Structural equation models of latent interactions: Evaluation of alternative estimation strategies and indicator construction. *Psychological Methods, 9*, 275-300.
Marsh, H.W., Wen, Z., Hau, K-T., Nagengast, B. (2013). Structural equation models of latent interaction and quadratic effects. In G.R. Hancock & R.O. Mueller (Eds.), *Structural equation modeling: A second course, 2nd Edition* (pp. 267-308). Charlotte, NC: Information Age Publishing.
Marsh, H.W., Wen, Z., Nagengast, B., & Hau, K.-T. (2012). *Structural equation models of latent interaction*. In R. H. Hoyle (Ed.), *Handbook of Structural Equation Modeling* (pp. 436-458). New York: Guilford.
Ping, R.A. (1996). Latent variable interaction and quadratic effect estimation: A two-step technique using structural equation analysis. *Psychological Bulletin, 119*, 166-175.
Schumacker, R.E., & Marcoulides, G.A. (Eds.). (1998). *Interaction and nonlinear effects in structural equation modeling*. Mahway, NJ: Erlbaum.

[1] The two variables should be "centered" first where the mean of the variable is subtracted from the variable (the result is sometimes called a *deviation score*). The centered variables are used to compute the product variable. Two texts provide good descriptions of the details of the moderated regression approach: Aiken, L.S., & West, S.G. (1991). Multiple regression: Testing and interpreting interactions. Newbury Park, CA: Sage. Jaccard, J., Turrisi, R., & Wan, C.K. (1990). Interaction effects in multiple regression. Newbury Park, CA: Sage.

## Multilevel SEM

Hierarchical linear modeling (multilevel regression) can be extended to latent variable models as well. When data are hierarchically structured, as is the case with students nested within schools or patients within hospitals, assumptions about the independence of observations are violated in regression and SEM. An older approach used a multigroup analysis to model between-group variance (level-2 variance) and within group variance (level-1 variance; Muthen & Satorra, 1994). Multilevel SEM is not available in most SEM packages. Mplus is perhaps the most widely used, with special features that facilitate analysis of multilevel models with two levels (see Heck & Thomas, 2020), but xxM package in R (Mehta, 2013) and OpenMx (Boker et al., 2020) also estimate multilevel SEMs. Advantages of this approach over HLM include the use of latent variables, the ability to test multilevel measurement hypotheses, and the ability to estimate correlated errors and test various measurement error assumptions. Good introductions are found in Heck and Thomas (2020) and Hox (2017).

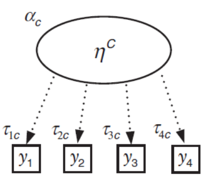


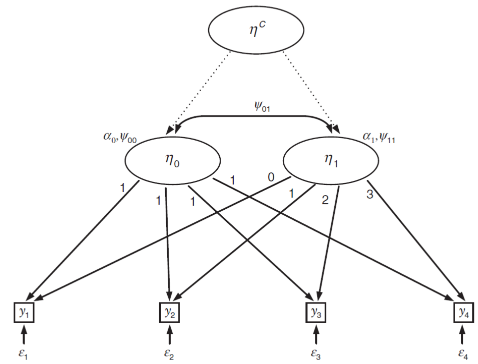

From Muthén, L.K. and Muthén, B.O. (1998-2017). Mplus User's Guide. Eighth Edition. Los Angeles, CA: Muthén & Muthén

Boker, S.M. et al. (2020). *OpenMx 2.17.4 User Guide*. https://vipbg.vcu.edu/vipbg/OpenMx2/docs//OpenMx/latest/OpenMxUserGuide.pdf
Croon, M.A., & van Veldhoven, M.J. P.M. (2007). Predicting Group-Level Outcome Variables From Variables Measured at the Individual Level: A Latent Variable Multilevel Model. *Psychological Methods, 12*, 45-57.
Curran, P.J., & Bauer, D.J. (2007). Building path diagrams for multilevel models. *Psychological Methods, 12*, 283-297.
du Toit, S. H., & Du Toit, M. (2008). Multilevel structural equation modeling. In J. de Leeuw & E. Meijer (Eds.), *Handbook of multilevel analysis* (pp. 435-478). Springer New York.
Heck, R.H. (2001). Multilevel modeling with SEM. In G.A. Marcoulides & R.E. Schumacker, *New Developments and techniques in structural equation modeling*. Mahway, NJ: Erlbaum.
Heck, R.H., & Reid, (2023). Multilevel structural equation modeling. In R.H. Hoyle, *Handbook of structural equation modeling, second edition* (pp. 481-499). Guilford.
Heck, R.H., & Thomas, S.L. (2020). *An Introduction to Multilevel Modeling Techniques: MLM and SEM Approaches Using Mplus, Fourth Edition.* New York: Routledge.
Hox, J. J., Moerbeek, M., & van de Schoot, R. (2017). Chapters 14 & 15, *Multilevel analysis: Techniques and applications* (Third Edition). New York: Routledge.
Kaplan, D., & Elliott, P.R. (1997). A didactic example of multilevel structural equation modeling applicable to the study of organizations. *Structural Equation Modeling, 4*, 1-24.
Li, F., Duncan, T.E., Harmer, P., Acock, A., & Stoolmiller, M. (1998). Analyzing measurement models of latent variables through multilevel confirmatory factor analysis and hierarchical linear modeling approaches. *Structural Equation Modeling, 5*, 3, 294-306.
Mehta, P. (2013). N-level structural equation modeling. In Y. Petscher, C. Schatschneider, & D. L. Compton (Eds.) *Applied quantitative analysis in education and social sciences* (pp. 329–362). New York: Routledge. https://xxm.times.uh.edu/
Muthen, B. (1994). Multilevel covariance structure analysis. In J. Hox &I. Kreft (eds.), Multilevel Modeling, a special issue of *Sociological Methods & Research, 22*, 376-398.
McArdle, JJ., & Hamagami, F. (1996). Multilevel models from a multiple group structural equation perspective. In G.A. Marcoulides & R.E. Schumacker (eds.), *Advanced Structural Equation Modeling: Issues and Techniques* (pp. 89-124). Mahway, NJ: Erlbaum.
Preacher, K. J., Zyphur, M. J., & Zhang, Z. (2010). A general multilevel SEM framework for assessing multilevel mediation. *Psychological methods, 15*, 209.
Preacher, K. J., Zhang, Z., & Zyphur, M. J. (2016). Multilevel structural equation models for assessing moderation within and across levels of analysis. *Psychological methods, 21*, 189.
Roesch, S.C, Aldridge, A.A., Stocking, S. N., Villodas, F., Leung, Q., Bartley, C.E., & Black, L.J. (2010). Multilevel factor analysis and structural equation modeling of daily diary coping data: Modeling trait and state variation. *Multivariate Behavioral Research, 45*, 767-789.
Shiyko, M. P., Ram, N., & Grimm, K. J. (2012). An overview of growth mixture modeling: A simple nonlinear application in OpenMx. In R. H. Hoyle (Ed.), *Handbook of structural equation modeling* (pp. 532–546). New York: Guilford Press.

## Latent Class Analysis and Growth Mixture Models

Latent class analysis tests categorical latent variables. Used when the latent variables are assumed to be categorical (2 or more classes). Akin to a latent variable approach to discriminant analysis in which individuals are classified according to probability of membership in classes. Mplus, Lisrel, EQS, and Amos all have features for latent class analysis and “mixture modeling,” the term used to refer to combining continuous and categorical latent variables in a single model. Use of latent classes to categorize trajectories from growth models is referred to as "growth mixture models".





Abar, B., & Loken, E. (2012). Consequences of fitting nonidentified latent class models. *Structural Equation Modeling, 19*, 1-15.
Asparouhov, T., & Muthén, B. (2014) Auxiliary Variables in Mixture Modeling: Three-Step Approaches Using Mplus. *Structural Equation Modeling: A Multidisciplinary Journal, 21,* 329-341.
Bray, B. C., Lanza, S. T., & Tan, X. (2015). Eliminating bias in classify-analyze approaches for latent class analysis. *Structural equation modeling: a multidisciplinary journal, 22*, 1-11.
Collins, L. M., & Lanza, S. T. (2009). *Latent class and latent transition analysis: With applications in the social, behavioral, and health sciences* (Vol. 718). John Wiley & Sons.
Clogg, C. C. (1995). Latent class models. In G. Arminger, C. C. Clogg, & M. E. Sobel (Eds.), *Handbook of statistical modeling for the social and behavioral sciences* (pp. 311-359). New. York: Plenum.
Flaherty, B.Pl, & Kiff, C.J. (2012). Latent class and latent profile models. In H. Cooper, P.M Camic, D.L. Long, P.A. Panter, D. Rindskopf, & K. Sher. *APA handbook of research methods in psychology, Vol 3: Data analysis and research publication.* (pp. 391-404). Washington, DC, US: American Psychological Association; US.
Grimm, K. J., Ram, N., & Estabrook, R. (2016). Growth modeling: Structural equation and multilevel modeling approaches. New York: Guilford Publications.
McCutheon, A.L. (1987). *Latent Class Analysis*. Newbury Park, CA: Sage.
Muthen, B.O. (2001). Latent variable mixture modeling. In G.A. Marcoulides & R.E. Schumacker, *New Developments and techniques in structural equation modeling*. Mahway, NJ: Erlbaum.
Nagin, Daniel S. (2005). *Group-Based Modeling of Development*. Cambridge, MA: Harvard University Press.
Nagin, D. S. and R. E. Tremblay. (2001). Analyzing Developmental Trajectories of Distinct but Related Behaviors: A Group-Based Method. *Psychological Methods 6*: 18-34.
Newsom, J.T. (2024). Chapter 10, Latent transition and growth mixture models. *Longitudinal Structural Equation Modeling: A Comprehensive Introduction, Second edition.* New York: Routledge.
Nylund-Gibson, K., Grimm, R., Quirk, M., & Furlong, M. (2014). A latent transition mixture modeling using the three-step specification. *Structural Equation Modeling, 21*, 439-454.
Rindskopf, D. (2009) Latent class analysis. In R.E. Millsap & A. Maydeu-Olivares (Eds.) *The Sage handbook of quantitative methods in psychology* (pp. 199-215). Thousand Oaks, CA: Sage.
Shockey, J.W. (1988). Latent class analysis: An introduction to discrete data models with unobserved variables. In J.S. Long, *Common problems/proper solutions: Avoiding error in quantitative research*. Newbury Park, CA: Sage.
von Eye, A., & Clogg, C. C. (1995, Editors). *Latent variables analysis.* Thousand Oaks, CA: Sage Publications.

## Causal Indicators

One can specify “latent” variables so that the indicators predict the latent variable (arrows go in the opposite direction from usual). Such indicators are sometimes called “causal indicators” or “formative indicators”. The resulting aggregate variables are not really latent variables, because they do not estimate measurement error. They are more related to linear composites such as that found in principal components analysis. The Edwards and Bagozzi article provides an excellent overview and link to the literature.

Bollen, K., & Lennox, R. (1991). Conventional wisdom on measurement: A structural equation perspective. *Psychological bulletin, 110*(2), 305.
Edwards, J.R., Bagozzi, R.P. (2000). On the nature and direction of relationships between constructs and measures. *Psychological Methods, 5*, 155-174.
MacCallum, R. C., & Browne, M. W. (1993). The use of causal indicators in covariance structure models: Some practical issues. *Psychological Bulletin, 114*(3), 533.
See also the special section on formative indicators in the 2007 *Psychological Methods*, Issue 2, Howe et al. Reconsidering formative measurement; Bollen, Interpretational confounding is due to misspecification, not to type of indicator: Comment on Howell, Breivik, and Wilcox

(2007); Bagozzi, On the meaning of formative measurement and how it differs from reflective measurement: Comment on Howell, Breivik, and Wilcox (2007); Howell, Is formative measurement really measurement? Reply to Bollen (2007) and Bagozzi (2007)

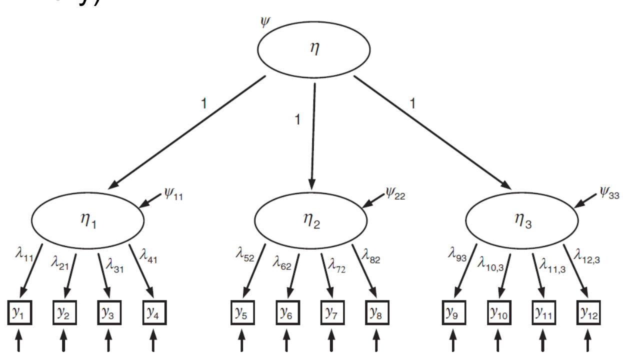
## Multitrait-Multimethod Matrix (MTMM) Models

MTMM models use structural equation modeling to expand upon an approach to testing construct validity by Campbell & Fiske (1959). The object is to separate out variation in the latent variables that is due to method variance. Method variance is variance that is due to the method of measurement rather than the substantive content of the measure. For instance, latent variables based on self-reported volunteerism items reflect true volunteerism and self-report method bias. Using multiple methods to measure a construct (e.g., observation, self-report, archival data) and multiple indicators for each method, one can separate out method and "trait" (the substantive construct being measured) variance. These models often run into problems with empirical underidentification unless there are at least three traits measured and three methods of measurement (see Eid, 2000, a practical approach that seems to work well).

Bagozzi, R. P., Yi, Y., & Phillips, L. W. (1991) Assessing construct validity in organizational research. *Administrative Science Quarterly, 36*, 421-458.
Byrne, B & Goffin, R.D. (1993). Modeling MTMM data from additive and multiplicative covariance structures: An audit of construct validity concordance. *Multivariate Behavioral Research, 28*, 67-96.
Eid, M. (2000). A multitrait-multimethod model with minimal assumptions.*Psychometrika*, *65*, 241-261.
Kenny, D. A., & Kashy, D. A. (1992). Analysis of the multitrait-multimethod matrix by confirmatory factor analysis. *Psychological Bulletin, 112*,165-172.
Marsh, H. W., & Grayson, D. (1995). Latent-variable models of multitrait-multimethod data. In R. H. Hoyle (Ed.), *Structural equation modeling: Issues and applications* (pp. 177-198). Newbury, CA,. Sage.
Wothke, W. (1996). Models for multitrait-multimethod matrix analysis. In G. A. Marcoulides & R. E. Schumacher (Eds.) *Advanced Structural Equation Modelling*. Mahwah, NJ: Erlbaum.
Various authors (2009). Special issue on multitrait-multimethod analysis. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences. 5*, 99-111.

## Latent State-Trait Models

Another longitudinal data approach involves an attempt to use SEM to partition variance that remains stable over time and variance that fluctuates from wave to wave. For instance, some cities have a chronically low or high crime rate over time, but there are yearly fluctuations that may depend on migration or economic factors. The latent trait-state models attempt to separate out the stable aspect of a variable from the fluctuating aspect, and each can be used as a predictor or a predicted variable in a larger model. There are several major approaches that have been proposed (see Newsom, 2024, for a summary).




Ciesla, J.A, Cole, D.A., Steiger, J.H. (2007). Extending the Trait-State-Occasion model: How important is within-wave measurement equivalence? *Structural Equation Modeling,14,* 77-97.
Cole, D. A., Liu, Q. (2023). Latent trait-state models. In R. H. Hoyle (Ed.), *Handbook of Structural Equation Modeling, second edition* (pp. 615–633). New York: Guilford.
Dumenci, L, & Windle, M. (1996). A latent trait-state model of adolescent depression using the center for epidemiologic studies-depression scale. *Multivariate Behavioral Research, 31*, 313-330.
Dumenci, L, & Windle, M. (1998). A multitrait-multioccassion generalization of the latent trait-state model: Description and application. *Structural Equation Modeling, 5*, 391-410.
Eid, M. (2000). A multitrait-multimethod model with minimal assumptions. *Psychometrika*, *65*, 241-261.
Geiser, C., & Lockhart, G. (2012). A comparison of four approaches to account for method effects in latent state–trait analyses. *Psychological methods*, *17*, 255.
Kenny, D.A., & Zautra, A. (1995). The trait-state-error model for multiwave data. *Journal of Consulting and Clinical Psychology, 63*, 52-59.
Hamaker, E.L., Nesselroade, J.R., Molenaar, P.C.M. (2007). The integrated trait--state model. *Journal of Research in Personality, 41*, 295-315.

LaGrange, B., & Cole, D.A. (2008). An expansion of the trait-state-occasion model: Accounting for shared method variance. *Structural Equation Modeling, 15*, 241-271.
Newsom, J.T. (2024). Chapter 6, *Longitudinal Structural Equation Modeling: A Comprehensive Introduction, Second edition.* New York: Routledge.
Steyer, R., Geiser, C. & Fiege, C. (2012). Latent State-Trait Models. In H. Cooper (Ed.), *Handbook of Research Methods in Psychology: Vol. 3. Data Analysis and Research Publication.* (pp. 291–308). Washington: American Psychological Association.

## Power Issues and Power Analysis in SEM

There are several important issues regarding statistical power in SEM. One of the primary concerns is how to conduct power analyses to determine sample size and effect sizes in SEM. Greg Hancock's chapter is a nice overview of the topic. Online calculators and macros for SAS and R are available (see https://timo.gnambs.at/research/power-for-sem and http://www.datavis.ca/sasmac/csmpower.html and http://quantpsy.org/rmsea/rmsea.htm). See the handout "Power Analysis for SEM: A Few Basics" for this class for some more detail.

Bandalos, D. L. (2014). Relative performance of categorical diagonally weighted least squares and robust maximum likelihood estimation. *Structural Equation Modeling: A Multidisciplinary Journal*, *21*(1), 102-116.
Davey, A., & Savla, J. (2010). *Statistical power analysis with missing data: A structural equation modeling approach*. NY: Routledge/Taylor & Francis.
Davey, A., & Savla, J. (2010). *Statistical power analysis with missing data: A structural equation modeling approach*. NY: Hancock, G.R. (2006). Power analysis in covariance structure modeling. In G.R. Hancock & R.O. Mueller (Eds.), Structural Equation Modeling: A Second Course (pp. 69-115). Greenwich, CT: Information Age Publishing.
Hancock, G.R. (2013). Power analysis in covariance structural modeling. In G.R. Hancock & R.O. Mueller (Eds.), *Structural equation modeling: A second course, 2nd Edition* (pp. 69-114). Charlotte, NC: Information Age Publishing.
Hermida, R., Luchman, J. N., Nicolaides, V., & Wilcox, C. (2015). The issue of statistical power for overall model fit in evaluating structural equation models. *Computational Methods in Social Sciences, 3*, 25.
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## Assumptions and Diagnostics

In addition to assumptions of multivariate normality, there are several other assumptions in SEM that follow the assumptions from regression analysis, including constant distribution of residuals, linearity, and outliers. In addition, we have not discussed multicollinearity issues in much detail. Although these assumptions can be tested, few researchers do enough to screen their data and evaluate various assumptions about the data. One barrier is the fact that most SEM packages do not have features that make testing various assumptions easy.

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## Sample Weighting and Complex Sampling Designs

Large, population-based studies that use stratified random sampling or cluster sampling can be adjusted for sampling selection biases. Such adjustments provide better estimates of the population parameters and standard errors than the assumption of random sampling used in traditional SEM (i.e., an assumption of all traditional statistical testing). Presently, only Mplus provides capabilities for making such adjustments automatically.

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## Bayesian Structural Equation Modeling

Depaoli, S., Kaplan, D., & Winter, S.D. (2023). Foundations and extensions of Bayesian structural equation modeling. In R.H. Hoyle, *Handbook of structural equation modeling, second edition* (pp. 701-721). Guilford.
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## Keeping Abreast of Developments

(There are several journals that often feature articles on developments in SEM.)

*Structural Equation Modeling: An Interdisciplinary Journal* (published by Lawrence Erlbaum)
*Psychological Methods* (published by the American Psychological Association)
*Psychometrika* (published by the Psychometric Society)
*Sociological Methods and Research* (published by Sage)
*Sociological Methodology* (an annual volume published by Blackwell)
*Multivariate Behavioral Research* (published by Society of Multivariate Experimental Psychology)
*Organizational Research Methods* (Sage)