Kinematics

Equations and Relations:

**Trigonometry:**
- \( \sin \theta = \frac{o}{h} \)
- \( \cos \theta = \frac{a}{h} \)
- \( \tan \theta = \frac{o}{a} \)
- \( a^2 + o^2 = h^2 \)

**Kinematics**
- Average Speed = distance traveled / time elapsed
- Instantaneous Velocity = slope of tangent to x-t graph
- Instantaneous Acceleration = slope of tangent to v-t graph
- Average Velocity:
  \[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{\Delta t} \]
- Average Acceleration:
  \[ \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{\Delta t} \]

**Motion with Constant Acceleration:**
- \( v = v_0 + at \)
- \( x = x_0 + v_0 t + \frac{1}{2} at^2 \)
- \( v^2 = v_0^2 + 2a(x - x_0) \)
- \( \bar{v} = \frac{(v + v_0)}{2} \)
- \( g = 9.80 \text{ m/s}^2 \approx 10 \text{ m/s}^2 \)

1. Smoky the cat is relaxing on the arm of a couch, one meter above the ground, when he is startled by something and jumps straight up in the air with initial speed 4 m/s. Coming down, he misses the couch and lands on the ground. You can neglect air resistance in your answers below.

   (a) What is Smoky’s acceleration…
      (i) …just after his paws leave the couch and he is on his way up?
      (ii) …at the exact instant when he is at his maximum height?
      (iii) …just before he hits the ground on his way back down?

   (b) What is Smoky’s maximum height above the ground during his motion?
   (c) What is Smoky’s velocity just before he hits the ground?
   (d) How long is Smoky in the air?

   ➢ Sketch the velocity versus time graph for Smoky’s motion.

   ➢ Sketch the velocity versus time graph for an object that is simply dropped from some height to the ground. What does this graph have in common with the previous one?

   ➢ Is it possible to analyze Smoky’s up and down motion without breaking it up into an upward part and then a downward part? Why or why not?
2. Shawn and Maria are playing darts. Shawn is a beginner, and he starts with his first dart at exactly the same height as the bullseye, aims it horizontally (right at the bullseye) and throws it. He releases the dart with speed 10 m/s, a horizontal distance of 2 m from the dartboard.

(a) Make a sketch of this situation, including a coordinate system. Then write down the known quantities in terms of your chosen coordinate system.
(b) Find the time that Shawn's dart is in the air before it hits the dartboard.
(c) How far below the bullseye does Shawn’s dart hit?
(d) Now Maria throws. She also starts the dart level with the bullseye and gives it an initial horizontal velocity of 10 m/s, but in addition she gives the dart just enough initial vertical velocity so that she hits the bullseye perfectly. How much vertical velocity did she give it?
(e) Express the initial velocity of Maria’s dart as a magnitude and direction.

Did Maria throw her dart significantly harder than Shawn threw his? What was the main adjustment she made?

3. At the end of a long day of skiing, you are coasting down an easy slope at constant velocity. Your mass is \( M \), and the slope makes an angle \( \theta \) above the horizontal. There are no numbers in this problem, so write all of your equations in terms of the "known" quantities \( M \), \( \theta \) and \( g \).

(a) Make a FBD for the skier, including a convenient coordinate system.
(b) Write out Newton's 2nd Law in each direction.
(c) In terms of \( M \), \( \theta \) and \( g \), find an expression for the combined force of friction and air resistance that opposes the motion down the slope.
(d) In terms of \( M \), \( \theta \) and \( g \), find an expression for the normal force between the skier and the slope.

Under what conditions is the normal force on an object equal to the object’s weight?
In practice, how does a skier remain moving at constant speed?

4. A box slides down an incline with uniform acceleration. It starts from rest and attains a speed of 2.7 m/s in 5.0 s. Find
(a) the acceleration
(b) the distance moved in the first 10.0 s.
5. A body is projected downward at an angle of $30^\circ$ with the horizontal from the top of a building 100 m high. Its initial speed is 10 m/s.
(a) How long will it take before striking the ground?
(b) How far from the foot of the building will it strike?
(c) At what angle with the horizontal will it strike?

6. Lamborghini advertises that its Diablo model can go from 0-62 mph in 4.1 seconds. Given that $1 \text{m/s} \approx 2.2 \text{ mph}$, what is the average acceleration of the car for these 4.1 seconds in SI units?

- You are finding the average acceleration of this car. What do you suppose its actual acceleration looks like as a function of time?
- What do you think is responsible for making the car go forward?

7. The following graph represents velocity versus time graph for the motion of an object over a particular one minute time interval:

![Graph](image)

Please start by redrawing this graph on a blank sheet of paper. Draw it large!

(a) Make up a story that describes motion that is consistent with this graph. Be creative! Think about the speeds and times involved and make your story realistic.
(b) Find the average acceleration of this object for the interval from 0-6 seconds.
(c) What is the average acceleration of this object for the entire one-minute interval?

- How could you determine how far the object went from the velocity graph? How could you determine this distance using equations?
- Sketch the acceleration versus time graph for this object.
Additional Questions

1. Give an example of an object that is moving upward but which has a downward acceleration, or state why this is impossible.

2. In choosing a coordinate system to use along with a free-body diagram, what should your primary considerations be?

3. Can something be accelerating even though it has zero velocity? If so, give some examples. If not, state why not.

4. You are jogging around a ¼-mile track. Timing yourself, you find that you complete a lap in exactly 2 minutes.
   (a) What is your average speed for this lap, in miles per hour?
   (b) What is your displacement after completing exactly one lap?
   (c) What is your average velocity for this lap?