Use $1/4\pi\varepsilon_0 = 9 \times 10^{-9} \text{N.m}^2/\text{C}^2$, $g = 10 \text{m/s}^2$

1. The figure shows the cross section of a family of parallel equipotential surfaces and three paths along which we (as an external agent) shall move an electron from one surface to another (at constant velocity).

1A Use the graph above to draw the electric field lines everywhere in between the surfaces.

1B The external work done in each case (P, S, U) is, respectively

   a) +, +, +
   b) +, +, -
   c) -, -, -
   d) -, -, +
   e) NA

2. The figure below shows a circular plastic rod of diameter $D=6 \text{cm}$, lying in the $XY$-plane and with its center at the origin of the systems of coordinates. The rod has a uniformly distributed positive charge $+Q=5 \mu\text{C}$,
2A How much external work is needed to bring an electron (at constant velocity) from infinitely far away to the point P of coordinates (0, 0, 4 cm). Give your answer in units of \(10^{-15} \text{J}\)

a) 6 \hspace{1cm} b) -0.24 \hspace{1cm} c) 72 \hspace{1cm} d) -144 \hspace{1cm} e) NA

2B At \(t=0\) an electron is placed at point P, and released with zero initial velocity. What is the kinetic energy of the electron when it passes through the origin of coordinates. Give your answer in units of \(10^{-15} \text{J}\).

a) 72 \hspace{1cm} b) 0.36 \hspace{1cm} c) 96 \hspace{1cm} d) 0.48 \hspace{1cm} e) NA

3. A capacitor-1 of capacitance \(C_1 = 3.55 \mu\text{F}\) is charged by a 6.3 V battery. Then the battery is removed. Subsequently the capacitor is connected to an uncharged capacitor-2 of capacitance \(C_2 = 8.95 \mu\text{F}\), as shown in the figure.

\[
\begin{align*}
&\left(\begin{array}{c}
C_1 \\
= 3.55 \mu\text{F}
\end{array}\right) \\
&\left(\begin{array}{c}
C_2 \\
= 8.95 \mu\text{F}
\end{array}\right)
\end{align*}
\]

3A After the switch is closed, the charges \(q_1\) and \(q_2\) in the capacitors are, respectively (in \(\mu\text{C}\))

a) 4.1 and 6.3 \hspace{1cm} b) 6.3 and 16 \hspace{1cm} c) 3.5 and 8.8 \hspace{1cm} d) 4.3 and 10.7 \hspace{1cm} e) NA

3B After the switch is closed, the electric potential across the capacitors is equal to (in Volts)

a) 1.8 \hspace{1cm} b) 3.6 \hspace{1cm} c) 0.9 \hspace{1cm} d) 4.1 \hspace{1cm} e) NA

4. The figure below shows a parallel-plate capacitor. In what follows, assume for simplicity that the electric field produced by the plates is equivalent to the electric field produced by infinite plates charged uniformly all over their surface.

In the figures, \(Q\) stands for the initial charge accumulated in the top plate, \(E\) is the electric field in between the plates, and \(d\) is the distance between the plates.
4A  The capacitor is initially charged and then the battery is disconnected. Subsequently, an external agent (not shown in the figure) increases the distance between the plates.

Which expression describes correctly what happens as the distance between the plates is increased?

a) The electric field remains constant.
b) The electric potential between the plates decreases.
c) The electric potential energy stored in the capacitor decreases.
d) The magnitude of the charge in each plate decreases.
e) All the expressions above are incorrect.

4B In this case the capacitor is charged by a battery and while keeping the battery connected to the plates, an external agent (not shown in the figure) increases the distance between the plates.

Which expression describes correctly what happens as the distance between the plates is increased?

a) The electric field increases.
b) The magnitude of the charge in each plate increases.
c) The electric field remains constant.
d) The energy stored in the capacitor decreases.
e) All the expressions above are incorrect.
5. The figure shows a parallel plate capacitor with a plate area $A = 5.56 \text{ cm}^2$ and separation $d = 5.56 \text{ mm}$. (1 mm = $10^{-3} \text{ m}$). The left half of the gap is filled with material of dielectric constant $\kappa_1 = 7$; the right half is filled with material of dielectric constant $\kappa_2 = 12$. The capacitor is charged with a battery; subsequently the battery is disconnected.

Which of the following expressions is correct:

a) The electric field at “P” is greater than the electric field at “S”.

b) The electric field at “P” is equal to the electric field at “S”.

c) The electric field at “S” is greater than the electric field at “P”.

d) Since the electromotive force of the battery is unknown, there is not enough information to compare the values of the electric field at “P” and at “S”.

e) All the expressions above are incorrect.

5A The capacitance is equal to (in units of $10^{-12} \text{ F}$)

a) 14  b) 8.4  c) 48  d) 1.7  e) NA

6. The figure shows a parallel plate capacitor with a plate area $A = 7.89 \text{ cm}^2$ and separation $d = 4.62 \text{ mm}$. (1 mm = $10^{-3} \text{ m}$). The top half of the gap is filled with material of dielectric constant $\kappa_1 = 11$; the bottom half is filled with material of dielectric constant $\kappa_2 = 12$. The capacitor is charged by a 2V battery, which remains connected.
6A Which of the following expressions is correct:

a) The electric field at “P” is greater than the electric field at “U”.

b) The electric field at “P” is equal to the electric field at “U”.

c) The electric field at “U” is greater than the electric field at “P”.

d) Despite knowing the electromotive force of the battery, there is not enough information to compare the values of the electric field at “P” and at “U”.

e) All the expressions above are incorrect.

6B The capacitance is equal to (in units of $10^{-12}$ F):

a) 17

b) 0.7

c) 14

d) 144

e) NA

7. 7A The figure shows three situations in which a positively charged particle moves at velocity $\mathbf{v}$ through a uniform magnetic field $\mathbf{B}$ and experiences a magnetic force. In each situation, determine whether the orientation of the vectors are physically reasonable.

I

II

III

a) T, F, T

b) T, T, F

c) F, F, T

d) T, F, F

e) NA

7B The figure shows four directions for the velocity vector $\mathbf{v}$ of a negatively charged particle moving through a uniform magnetic field $\mathbf{B}$ pointing to the left. (Of course, the magnetic fields exist all over the space). Rank the directions according to the magnitude of the net force on the particle, greatest first

a) 1, 2, 3, 4

b) 1 tie with 4, 2, 3

d) 2, 3 tie with 4, 1

e) 3, 2, 1 tie with 4

e) NA
8. **8A** A graph of the electric potential as a function of x is given in the figure below. Plot (on scale) the electric field component $E_x$ as a function of x (use the right-side diagram). Ignore the points on the graph where the slope changes discontinuously.

![Graph of electric potential and electric field](image)

**8B** How much external work would be needed take an electron from C to H (at constant velocity)? Give your answer in units of $10^{19}$J.

- a) 4.8
- b) -3.2
- c) -4.8
- d) 1.6
- e) NA

9. An electron with kinetic energy $1.5 \times 10^3 \text{ eV}$ circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25 cm. Mass of electron: $9.109 \times 10^{-31}$ kg

![Electron orbit](image)

**9A** An approximate value of the electron’s speed, and the direction it is circling, are:

- a) $1.5 \times 10^{15}$ m/s, counterclockwise
- b) $2 \times 10^7$ m/s, counterclockwise
- c) $5 \times 10^3$ m/s, counterclockwise
- d) $1 \times 10^2$ m/s, clockwise
- e) NA
The magnitude of the magnetic field is (approximately):

a) \(6 \times 10^{-15} \text{ T}\)  

b) \(6 \text{ T}\)  
c) \(5 \text{ Gauss}\)  
d) \(17 \text{ Gauss}\)  
e) NA

Note: \(1 \text{ Tesla} = 10^4 \text{ Gauss}\)

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**BONUS QUESTION (3 points)**

**B1.** Derive an expression for the external work required to set-up the four-charge configuration shown in the figure, assuming the charges are initially infinitely apart.

**ANSWER:**

---

**BONUS QUESTION (3 points)**

**B2.** Consider the arrangement of charges shown in the figure above (question B1). If the magnitude of all the charges were equal to \(1 \mu\text{C}\), and \(a = 1 \mu\text{m}\), the net external work required to bring an elementary charge \((e = -1.6 \times 10^{-19}\text{C})\) from far away (infinity) to the center of the square is equal to (in units of Joules):

a) \(1.44 \times 10^9\)  
b) \(0\)  
c) \(5.6 \times 10^9\)  
d) \(-0.72 \times 10^9\)  
e) NA
Some formulas:

- $\mu = 10^{-6}$    nano = $10^{-9}$
- mass of electron: 9.109 x $10^{-31}$ kg
- 1 eV = 1.602 x $10^{-19}$ J
- **Coulomb's Law:** \[ \vec{F} = \frac{1}{4\pi \varepsilon_0} \frac{q_2 q_1}{r^2} \vec{u} \]

- Electric field, along the z-axis, due to a charge Q distributed uniformly along a thin ring of radius R.: \[ \vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{Q z}{(z^2 + R^2)^{3/2}} \hat{k} \]

- For an infinite uniformly charged sheet: $\vec{E} = \sigma / 2\varepsilon_0$

- **Gauss' Law**
  \[ \Phi = \int_{S} \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}, \quad \text{where } q \text{ is the net charge inside the Gaussian surface} \]

- Definition of Electric Potential \[ V(r) = \frac{W_{\text{ext}}(\infty \rightarrow q_0 \rightarrow r)}{q_0} \]

- Potential difference \[ V_B - V_A = -\int_{A}^{B} \vec{E} \cdot d\vec{s} \]

- Electric potential due to a point charge q: \[ V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} \]

- Relationship between $E$ and $V$: \[ E_x = -\frac{dV}{dx} \]

- **About capacitance**
  \[ Q = C \cdot V \]
  For a parallel-plate capacitor $C = \frac{A\varepsilon_0}{d}$
  \[ U = CV^2 / 2 = \frac{Q^2}{2C} \]
  Capacitors connected in parallel $C_{\text{equiv}} = C_1 + C_2 + C_3$
  Capacitors connected in series $1/C_{\text{equiv}} = (1/C_1) + (1/C_2) + (1/C_3)$

**When filing a capacitor with a dielectric:** Capacitance increases by a factor of $k$
\[ \int \sin \theta \, d\theta = -\cos \theta \quad \int \cos \theta \, d\theta = \sin \theta \]

**MAGNETISM**

- \( \mathbf{F} = q \mathbf{v} \times \mathbf{B} \quad \mathbf{F} = \text{force}, \ q = \text{charge}, \ \mathbf{v} = \text{velocity}, \ \mathbf{B} = \text{magnetic field} \)
- 1 Tesla = \( 10^4 \) gauss
- Magnetic field produced by a charge \( q \) that moves with velocity \( \mathbf{v} \)

\[
\mathbf{B} = \frac{\mu_0 q}{4\pi r^3} \mathbf{v} \times \mathbf{r}
\]

- \( \Phi = LI \quad \Phi = \text{Magnetic flux}, \ I = \text{inductance}, \ i = \text{current} \)
- Hall effect \( BI = nqtV_{\text{Hall}} \)
- Inductive reactance \( X_L = \omega L \)

- \( B = \frac{\mu_0 I}{4} \frac{1}{R} \quad \text{Magnetic field at the center of a semi-circle of radius "R"} \)

- \( B = \frac{\mu_0 I\phi}{4\pi R} \quad \text{Magnetic field at the center of an arc of angle} \ \phi \text{ (in radians)} \ \text{and radius} \ "R". \)

- \( B = \frac{\mu_0 I}{2\pi r} \quad \text{Magnetic field produced by a infinitely long wire at a distance} \ "r" \ \text{from it.} \)

- \( \frac{F}{L} = \frac{\mu_0}{2\pi} \frac{I_a I_b}{d} \quad \text{Force per unit length between two parallel long wires,} \)
- \( \text{carrying currents} \ I_a \ \text{and} \ I_b \ \text{respectively, separated by a} \ \text{distance} \ "d" \)

- Faraday's Law \( \mathcal{E} = -\frac{\partial \Phi}{\partial t} \),
  where \( \Phi = \text{Magnetic flux} \) and \( \mathcal{E} = \text{electromotive force} \)

- Definition of the magnetic dipole moment of a loop of area A, carrying a current I:
  \[ \mu = I \mathbf{A} \]
where \( A = \text{area}, \) \( I = \text{current}, \) \( \mathbf{n} = \text{unit vector perpendicular to the loop} \)