ELECTRIC POTENTIAL-ENERGY \( (U) \) and ELECTRIC POTENTIAL \( (V) \)

\[ U \quad \text{in units of Joules} \]
\[ V \quad \text{in units of Joules/coulomb} = \text{volt} \]

Potential difference \( V_a - V_b \) between two points A and B

Diagram of a battery with terminals labeled + and −.
A useful interpretation of the electric potential-energy

- Change \( Q \) creates an electric field
  
  The charge \( q_0 \) is, initially, far away (at the infinite)

- We want to bring \( q_0 \) from "infinite" to a distance \( r \) from charge \( Q \)

  How much work does an external force \( F_{\text{ext}} \) have to do to bring \( q_0 \) from infinite to a distance \( r \) from \( Q \) at constant velocity?

  \[ W_{\text{ext}} = ? \quad (+) \]

Notice that the requirement to transport \( q_0 \) at a constant velocity implies \( |F| = |F_{\text{ext}}| \)
$$W_{\text{ext}}(\infty \to r) = \int_{\infty}^{r} \vec{F}_{\text{ext}} \cdot d\vec{s}$$

$$= \int_{\infty}^{r} \left( \frac{1}{4\pi \varepsilon_0} \frac{Q q_0}{r^2} \right) (-dr) \cos 0^\circ$$

$$= \int_{\infty}^{r} -\frac{Q q_0}{4\pi \varepsilon_0} \frac{dr}{r^2} = \frac{Q q_0}{4\pi \varepsilon_0} \frac{1}{r} \bigg|_{\infty}^{r}$$

Energy deposited by the external force into the system formed by \(Q\) and \(q_0\)

$$W_{\text{ext}}(\infty \to r) = \frac{1}{4\pi \varepsilon_0} \frac{Q q_0}{r} \quad \text{unit of work: Joules}$$

$$U = W_{\text{ext}}(\infty \to r) \quad \text{unit of } U : \text{Joules}$$
Definition of the electric potential $V$

For the particular case of a point-charge $Q$, we have:

$$V = \frac{U}{q_0}$$

Units of $V = \frac{\text{Joule}}{\text{coulomb}} = \text{Volt}$
\[ \frac{W_{\text{ext}}(\infty \to r)}{q_0} = \frac{U}{q_0} = V \]

**Electric Potential**

Produced by the charge \( Q \)

**Check Point**

\[
\begin{array}{c|c|c|c}
Q & q_0 & W_{\text{ext}}(\infty \to r) = U & V_q \\
\hline
+e & +e & \text{positive} & + \\
\hline
+e & -e & \text{negative} & - \\
\hline
-e & +e & \text{negative} & - \\
\hline
-e & -e & \text{positive} & - \\
\end{array}
\]
POTENTIAL DIFFERENCE

\[ V_A = \frac{W_{\text{ext}} (\infty \to A)}{q_0} = \frac{1}{q_0} \int_{\infty}^{A} \vec{F}_{\text{ext}} \cdot d\vec{s} \]

\[ V_B = \frac{W_{\text{ext}} (\infty \to B)}{q_0} = \frac{1}{q_0} \int_{\infty}^{B} \vec{F}_{\text{ext}} \cdot d\vec{s} \]

**Notice:**

\[ V_B - V_A = \frac{1}{q_0} \int_{A}^{B} \vec{F}_{\text{ext}} \cdot d\vec{s} = \frac{W_{\text{ext}} (A \to B)}{q_0} \]
Relationship between ELECTRIC POTENTIAL and ELECTRIC FIELD

\[
V_B - V_A = \frac{W_{ext} (A \rightarrow B)}{q_0} = \frac{1}{q_0} \int_A^B \vec{F}_{ext} \cdot d\vec{s}
\]

External work needed to take the charge \( q_0 \) from A to B at constant velocity (\( V \approx 0 \))

Therefore \( |\vec{F}_{ext}| = |q_0 \vec{E}| \)

Actually \( \vec{F}_{ext} = -q_0 \vec{E} \)

\[
V_B - V_A = \frac{1}{q_0} \int_A^B \vec{E} \cdot d\vec{s}
\]

Potential difference between the points A and B

\[
V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}
\]
The potential difference between A and B does not depend on the particular path that joins A and B.

\[ V_B - V_A = \frac{q}{4\pi\varepsilon_0} \left( \frac{1}{r_B} - \frac{1}{r_A} \right) \]

This result is independent of the particular path that joins A and B.
\[ V_B - V_A = - \int_A^B \mathbf{E} \cdot d\mathbf{s} \]
is the same whether we choose path I, II or III.

Even though we have obtained this result for the particular case of a punctual charge \( q \), we can demonstrate that it is valid for arbitrary charge distributions. This is done below.

\[ \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \ldots \]

\( Q \) is decomposed into several "punctual" charges 1, 2, 3, ...
\[ \int_{A}^{B} \vec{E} \cdot d\vec{s} = \int_{A}^{B} \vec{E}_1 \cdot d\vec{s} + \int_{A}^{B} \vec{E}_2 \cdot d\vec{s} + \ldots \]

since each term is independent of the path that goes from A to B,

therefore

the term on the left side of the equality is also independent of the path that joins A and B
ELECTRIC POTENTIAL ENERGY $U$

STORED IN A SYSTEM OF POINT CHARGES

A) \[ W_{\text{ext}} (\infty \rightarrow r_{12}) = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r_{12}} = U_{12} \]

B) \[ U = U_{12} + U_{13} + U_{23} \]

\[ U = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi \varepsilon_0} \frac{q_2 q_3}{r_{23}} \]

C) And so on...

Assignment: Problems 36 and 37; page 586

Question: If the system had $N$ point charges, how many terms will there be in the expression for $U$?

\[ U = \sum_{i \neq j} \frac{q_i q_j}{r_{ij}} \]
Exercise 5.  4 point charges

\[ u = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} + \ldots + \ldots + \]

How many terms?  6

\[ r_A \]

Exercise 6.  5 point charges

\[ u = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} + \ldots + \ldots + \]

How many terms?

Exercise 7.  8 point charges

\[ u = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} + \ldots + \ldots + \]

How many terms?

Exercise 8.  1000 point charges

\[ u = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} + \ldots + \ldots + \]

How many terms?
Electric Potential

Established by a system of point charges

- Here we do not worry about the energy necessary to form the system of charges.

- We rather concentrate in the external work \( W_{\text{ext}} \) necessary to bring a test charge \( q_0 \) from far away (infinite) to a given position \( A \)

\[
W_{\text{ext}} (\infty \to A)
\]

- The electric potential \( V \) established by the system of charges is defined as

\[
V_A = \frac{W_{\text{ext}} (\infty \to A)}{q_0}
\]
ELECTRIC POTENTIAL

ESTABLISHED BY A SYSTEM OF POINT CHARGES

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{1A}} \]

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{1A}} + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_{2A}} \]

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{1A}} + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_{2A}} + \frac{1}{4\pi\varepsilon_0} \frac{q_3}{r_{3A}} \]
**Example.** Electric Potential $V$ on the axis of a charged ring.

We want to find the electric potential $V$ at point $A$, produced by the charged ring.

First, we notice that the ring is made out of small charges $\Delta Q$.

Each $\Delta Q$ produces a electric potential \[ V = \frac{1}{4\pi \epsilon_0} \frac{\Delta Q}{\sqrt{R^2 + z^2}} \]

The total electric potential produced by all the changes will be

\[ V = \frac{1}{4\pi \epsilon_0} \frac{1}{\sqrt{R^2 + z^2}} \int dQ \]
If you don't like integrals:

\[
V = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_{1A}} + \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_{2A}} + \frac{1}{4\pi \varepsilon_0} \frac{q_3}{r_{3A}} + \cdots
\]

But notice \( r_{1A} = r_{2A} = r_{3A} = \cdots = \sqrt{R^2 + z^2} \)

\[
V = \frac{1}{4\pi \varepsilon_0} \frac{1}{r_{1A}} \left( \frac{q_1 + q_2 + q_3 + \cdots}{Q} \right)
\]

\[
V = \frac{1}{4\pi \varepsilon_0} \frac{Q}{\sqrt{R^2 + z^2}}
\]
**Example:** Electric Potential $V$ on the axis of a uniformly charged Disk

**Disk of radius** "$R$$^1$** breaking down into rings of thickness** $dr$ (**"strip"**)

**Amount of charge in this ring:**

$$\Delta q = \sigma \times \text{area of the strip}$$

$$= \sigma \times 2\pi r \, dr$$

**Potential at point** "A" **established by a "strip" of radius** $r$

$$\Delta V = \frac{1}{4\pi \varepsilon_0} \frac{\text{charge in the strip}}{\sqrt{r^2 + z^2}}$$

$$\Delta V = \frac{1}{4\pi \varepsilon_0} \frac{\sigma 2\pi r \, dr}{\sqrt{r^2 + z^2}}$$
The total potential is obtained adding all the potentials established by all the strips, from \( r = 0 \) to \( r = R \).

\[
V = \int_{r=0}^{R} \frac{1}{4\pi \varepsilon_0} \frac{\varphi 2\pi r dr}{\sqrt{r^2 + z^2}}
\]

\[
= \frac{\varphi}{2\pi \varepsilon_0} \int_{r=0}^{R} \frac{r dr}{\sqrt{r^2 + z^2}}
\]

**Notice:**

\[
\frac{d}{dr} \left( \sqrt{r^2 + z^2} \right)^{1/2} = \frac{1}{2} (r^2 + z^2)^{-1/2} \cdot 2r
\]

\[
= \frac{r}{(r^2 + z^2)^{1/2}}.
\]

\[
V(\omega) = \left. \frac{\varphi}{2\pi \varepsilon_0} \sqrt{r^2 + z^2} \right|_{r=0}^{r=R} = \frac{\varphi}{2\pi \varepsilon_0} \left( \sqrt{R^2 + z^2} - \sqrt{z^2} \right)
\]

Electric potential at a distance "\( \omega \)" from the disk and along the axis that passes through its center and perpendicular to it.
**Electric Potential Energy:**

**Electric Potential:** \( V \) [volts]

- First, let's recall

\[ W = \int F \cdot dr \]  

Work done on the particle of mass \( m \) by the force \( F \)

\[ W = \int F \cdot dr = \int m \, \vec{a} \cdot d\vec{r} = \int m \, \frac{d\vec{v}}{dt} \cdot d\vec{r} \]

\[ = \int m \, d\vec{r} \cdot \vec{v} = \int m \, \frac{1}{2} \, d(v^2) = \int \left( \frac{1}{2}mv^2 \right) \]

Kinetic Energy \( K \)

\[ W = K_f - K_i \]

Work done by the force \( F \)

\[ K = \frac{1}{2}mv^2 \]
\[ W = \frac{K_f - K_i}{\text{work}} \]

change in kinetic energy

Force \( \vec{F} \) does work \( W \) on the particle of mass \( m \).

The amount of work on the particle manifests in the change of kinetic energy:

\[ \Delta K = K_f - K_i. \]

In this chapter \( \vec{F} \) has an electrostatic origin.

\[ \text{TEST charge} \]

\[ q_0 \]

\[ \text{displacement} \]

\[ \vec{R} \]

\[ \text{vector position} \]
On the other hand, we know that

\[ \vec{F} = q_0 \vec{E} \]

and we have obtained previously the following result

\[ V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s} \]

\[ \Rightarrow q_0 (V_f - V_i) = - \int_i^f \frac{q_0}{\epsilon} \vec{E} \cdot d\vec{s} = - \int_i^f \vec{E} \cdot d\vec{s} \]

according to expression 1

this is equal to

\[ W \]

\[ q_0 (V_f - V_i) = - W \]

From 2 and 3 we obtain:

\[ K_f + q_0 V_f = K_i + q_0 V_i \]

Conservation of Energy
Example: A ring of radius $R=4\text{ cm}$ carries a charge $Q=8\text{ nC}$ uniformly distributed.

A small particle of mass $m=6\text{ mg}=6\times10^{-6}\text{ kg}$ and charge $q_0=5\text{ nC}$ is placed at $z=3\text{ cm}$. Find the speed of the particle when it is at a great distance from the ring.

Solution:

At point $A$:
\[ U_A = q_0 V = 5\times10^{-9}\text{ C} \cdot \frac{8\times10^{-9}}{4\pi \varepsilon_0 \sqrt{(0.04\text{ m})^2 + (0.03\text{ m})^2}} \]
\[ = 7.2\times10^{-6}\text{ J} \]
\[ k_A = 0 \]

At a point far away:
\[ U_\infty = 0 \]
\[ k_\infty = \frac{1}{2}mv^2 \]
\[ U_A + K_A = U_\infty + K_\infty \]

\[ 7.2 \times 10^6 J + 0 = 0 + \frac{1}{2} m v^2 \]

Solving for \( v \):

\[ v = 1.55 \text{ m/s} \]
IMPORTANT: Notice that the electric potential $V$ is an **scalar** quantity.

Also, remember that the electric field $\vec{E}$ is a **vector**

**Connection between $V$ and $\vec{E}$**

As the charge $q_0$ moves along the path, the electric potential $V$, in general, varies.

Distribution of changes that produce the electric field $\vec{E}$
\[ \Delta W = \vec{F} \cdot d\vec{r} \quad \text{work done by the electric force} \]

\[ = q_0 \vec{E} \cdot d\vec{r} \]

\[ \frac{\Delta W}{q_0} = \vec{E} \cdot d\vec{r} \]

But, we know that

\[ K + U = \text{const} \]

Therefore

\[ \Delta K + \Delta U = 0 \]

\[ \Delta U = -\frac{\Delta K}{\Delta W} \]

So,

\[ \Delta W = -\Delta U \]

\[ -\frac{\Delta U}{q_0} = \vec{E} \cdot d\vec{r} \]

In terms of the definition of Electric Potential \( V \), we obtain

\[ \Delta V = -\vec{E} \cdot d\vec{r} \]
\[ \Delta V = - \mathbf{E} \cdot d\mathbf{R} \]

\[ \mathbf{E} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z} \quad \text{electric field} \]

\[ d\mathbf{R} = dx \hat{x} + dy \hat{y} + dz \hat{z} \quad \text{displacement vector} \]

**Case 1**

\[ d\mathbf{R} = dx \hat{x} \]

Then

\[ \Delta V = - \mathbf{E} \cdot d\mathbf{R} = - E_x dx \]

\[ \Rightarrow E_x = - \frac{\Delta V}{dx} \]

**Case 2**

\[ d\mathbf{R} = dy \hat{y} \]

Then

\[ \Delta V = - \mathbf{E} \cdot d\mathbf{R} = - E_y dy \Rightarrow E_y = - \frac{\Delta V}{dy} \]

**Case 3**

\[ d\mathbf{R} = dz \hat{z} \]

\[ \Rightarrow E_z = - \frac{\Delta V}{dz} \]

So,

\[ \mathbf{E} = - \frac{\partial V}{\partial x} \hat{x} - \frac{\partial V}{\partial y} \hat{y} - \frac{\partial V}{\partial z} \hat{z} \]

\[ = - \nabla V \]

*If you know V you can find E*
Our previous result is telling us that:

If you happen to know the electric potential \( V = V(x, y, z) \), then we can find the electric field \( \vec{E} \) by evaluating the derivatives of \( V \).

Lo first, the \( x \)-component of the vector \( \vec{E} \) can be found from

\[
E_x = -\frac{2V}{\partial x}
\]

Lo the \( y \)-component of \( \vec{E} \) is given by

\[
E_y = -\frac{2V}{\partial y}
\]

Lo similarly for the \( z \)-component

\[
E_z = -\frac{2V}{\partial z}
\]
In summary

→ If we know \( V \)
we can find the electric field \( \vec{E} \)
using \( \vec{E} = -\nabla V \)

This is typically useful when we have complicated distribution of charges. In such cases it is simpler to evaluate just one quantity (the electric potential \( V \)) and then, through evaluation of derivatives, we find the vector \( \vec{E} \)

→ If we know \( \vec{E} \)
we can find the electric potential \( V \)
using
\[
\Delta V = - \vec{E} \cdot d\vec{r}
\]
or
\[
V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{r}
\]

This is typically useful when we can evaluate
for symmetric distribution of charges (aided, for example, by Gauss' law) and all that matters is the electric potential difference.

EXAMPLES

a) Electric potential due to one point charge.

\[ V = \frac{1}{\frac{4\pi\varepsilon_0}{r}} \]

If \( r = 2 \text{ cm} \), what is the potential at point \( P \)?

\[ V = \]

Question: What other points (surrounding the charge \( q_1 \)) are at the same potential than \( P \)?
A set of points that are at the same electric potential $V$ form a surface, thus called EQUIPOTENTIAL SURFACE.
What would happen if the points A and B lie along a path that runs perpendicular to the orientation of the electric field?

What is the potential difference between A and B?

$V_B - V_A = ? = 0$

<table>
<thead>
<tr>
<th>$V_B - V_A$</th>
<th>$V_B - V_C$</th>
<th>$V_A - V_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
</tr>
</tbody>
</table>
Example 2: Electric potential of 2 punctual charges

\[ V = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2} \]

- If \( q_1 = 1 \mu C \) and \( q_2 = -2 \mu C \), what is the electric potential at a point midway between the charges (\( q_1 \) and \( q_2 \) are separated 4 cm between each other)? \( V = \) 

- If \( q_1 = +1 \mu C \) and \( q_2 = -1 \mu C \), how would the equipotential surfaces look like? (check page 605)
Example: Calculating the potential knowing how is the electric field

\[ \Delta V = - \vec{E} \cdot d\vec{r} \]

\[ V_f - V_i = - \int_{i}^{f} \vec{E} \cdot d\vec{r} \]

A charge \( q \) is distributed throughout a non-conducting sphere of radius \( R \).

The electric field is radially directed and has magnitude

\[ E(r) = \frac{q}{4 \pi \varepsilon_0 R^2} r \] for \( r < R \)

\[ E(r) = \frac{q}{4 \pi \varepsilon_0} \frac{1}{r^2} \] for \( r > R \)

Setting \( V = 0 \) at infinity, find the potential \( V(r) \) for \( r < R \) and for \( r > R \)

For \( r > R \)

\[ V(\infty) - V(r) = - \int_{r}^{\infty} E(r) \, dr = - \int_{r}^{\infty} \frac{q}{4 \pi \varepsilon_0} \frac{1}{r^2} \, dr = \frac{q}{4 \pi \varepsilon_0} \frac{1}{r} \]

\[ = - \frac{q}{4 \pi \varepsilon_0} \frac{1}{r} \]
Setting \( V(\infty) = 0 \)

\[
V(r) = \frac{q}{4\pi \varepsilon_0} \frac{1}{r}
\]

(\text{In particular})

\[
V(R) = \frac{q}{4\pi \varepsilon_0} \frac{1}{R}
\]

For \( r < R \)

\[
V(R) - V(r) = -\int_r^R E(r') \, dr'
\]

\[
= -\int_r^R \frac{q}{4\pi \varepsilon_0 \, R^3} \, r 
\]

\[
= -\left. \frac{q}{4\pi \varepsilon_0 \, R^3} \left( \frac{R^2}{2} - \frac{r^2}{2} \right) \right|_r^R
\]

So,

\[
V(r) = V(R) + \frac{q}{4\pi \varepsilon_0 \, R^3} \left( \frac{R^2}{2} - \frac{r^2}{2} \right)
\]

(We know from the calculation of \( V(r) \)

for \( r > R \) that \( V(R) = \frac{q}{4\pi \varepsilon_0} \frac{1}{R} \)

\[
V(r) = \frac{q}{4\pi \varepsilon_0} \frac{1}{R} + \frac{q}{4\pi \varepsilon_0 \, R^3} \left( \frac{R^2}{2} - \frac{r^2}{2} \right)
\]

\[
V(r) = \frac{q (3R^2 - r^2)}{8\pi \varepsilon_0 \, R^3} \quad \text{for} \quad r < R
\]
In particular, at $r = 0 \to V(r) = \frac{3q}{4\pi \varepsilon_0 R} \neq 0$

* Assignment: *(In the previous problem we set)
  $V = 0$ at $r = \infty$.
  Solve the same problem, but start setting the potential $V = 0$ at $r = 0$.
  Find $V(r)$ for $r < R$ and $r > R$.

In particular find $V$ when $r = \infty$.
(Hint: Find first $V(r)$ for $r < R$.)

* Assignment: A hollow, uncharged spherical conductor has inner radius $a$ and outer radius $b$.
  A positive point charge $q$ is in the cavity at the center of the sphere.
  Find $V(r)$ assuming $V = 0$ at $r = \infty$.
  Plot $V$ vs $r$ and $E$ vs $r$. 

![Diagram of a hollow sphere with a point charge q at its center]
**SUMMARY**

\[ V_f - V_i = -\int_{i}^{f} E \cdot ds \]

Electric Potential difference

Sometimes, we want to interpret this results in terms of energy

\[ q_0 (V_f - V_i) = W_{ext} (i \rightarrow f) \]

Change in the electric potential energy of the system composed by \( Q \) and \( q_0 \).
\[ V(P) = V(\infty) - \oint_P \vec{E} \cdot d\vec{s} \]

"i : typically taken at \( \infty \) (far away from \( A \))"

"arbitrarily we assign this value to be equal to zero"
If \( q \) were a punctual charge

\[
V(\vec{q}) = V(\infty) - \int \vec{E} \cdot d\vec{s}
\]

\[
= V(\infty) + \frac{1}{4\pi\varepsilon_0} \frac{Q}{|\vec{F} - \vec{F}_a|}
\]

If we place the origin of our coordinates at \( Q \) we obtain

\[
V(\vec{r}) = V(\infty) + \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}
\]

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0:2
0:2
**What is an ELECTRON volt: eV?**

eV is a unit of ENERGY
(eV is not a unit of electric potential)

**Definition:**

One electron volt (eV) is the work necessary to take an elementary charge (e = 1.6 x 10^-19 C) across a 1 volt potential difference.

What is the equivalent of 1eV in terms of joules?
Let's use our general definition:
\[ V_B - V_A = \frac{W_{\text{ext}}(A \rightarrow B)}{q_0} \]

If \( q_0 \) is an elementary charge, 
\[ q_0 = e = 1.6 \times 10^{-19} \text{ C} \]

Also, since \( V_B - V_A = 1 \text{ volt} \) we obtain

\[ 1 \text{ volt} = \frac{W_{\text{ext}}(A \rightarrow B)}{1.6 \times 10^{-19} \text{ C}} = \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ C}} \]

\[ \Rightarrow \]
\[ 1 \text{ eV} = W_{\text{ext}}(A \rightarrow B) = 1 \text{ volt} \times 1.6 \times 10^{-19} \text{ C} \]

\[ 1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules} \]
Does the elementary particle have to be an electron?

Answer: No. 

eV is a unit of energy (so it applies to any particle)
Energy of an electric dipole $\vec{p}$ (the dipole is immersed in an external electric field)

\[ \vec{p} \rightarrow \vec{E}_{\text{external}} \quad U = ? \]

To simplify matters, let's consider a dipole aligned along the electric field

\[ \begin{array}{c}
\rightarrow \quad \vec{p} \\
\downarrow \quad E_{\text{ext}} \\
\end{array} \]

$p$ stands for electric dipole moment

The electric potential established by uniform external electric field is

\[ V = -E_x \]

In general, the energy of a charge $Q$ that is immersed in this electric is

\[ U = QV = -QEx \]
In particular, for the case of the dipole we have:

Energy of the charge $q$: $-qE(x+d)$

Energy of the charge $-q$: $(-q)Ex = qE x$

**Total energy of the dipole**

$$U_{dipole} = -\frac{q}{d}E$$

Dipole oriented along $E$

In the general case:

$$U = -\vec{r} \cdot \vec{E}$$

Scalar product
Notice

\[ \vec{F} \rightarrow \vec{E} \]

The dipole moment \( \vec{p} \) tends to be oriented parallel to \( \vec{E} \).

In other words, since \( U = -\vec{p} \cdot \vec{E} \), the dipole moment tends to go to the state of minimum energy.

The principle that a physical system tends to go to a state of minimum energy is a very general principle in physics.
Exercise A neutral water molecule $H_2O$ in its vapor state has an electric dipole moment of magnitude $6.2 \times 10^{-30}$ C.m.

The dipole is initially oriented parallel to an electric field of magnitude $E = \frac{100 \text{ V}}{\text{cm}}$.

Calculate the energy necessary to flip the dipole to a position anti-parallel to $E$.

Give your answer in eV.

(1p)