Geometric Transformation Based on Ground Control Points

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Introduction

- Several methods to transform satellite image to projected map for analysis
- Using ground control points (GCPs) one way
- Empirical or nonparametric: do not need orbital data, etc.
Limitations

- “Easy” but labor intensive
- Must have “enough” GCPs for statistically reasonable transformation
- Limited GCPs in “boring” topographies: desert, ocean
- Map must “match” scale & area covered
- Transformation can be complicated by heterogenous terrain

Requirements

- Suitable vector map with sufficient GCPs
- Mapping software (GIS) to display & transform image
- Good hand-eye coordination
- Digitizing tablet or on-screen marking
Basic Set-up

Least Squares Regression

Regress or plot \((r,c)\) values versus \((x,y)\) values

Generic equations:

\[ x = f(c,r); \quad y = f(c,r); \quad c = f(x,y); \quad r = f(x,y) \]

Want to minimize sum of square of residuals i.e. difference in real values versus estimated values

Software solves all 4 equations simultaneously
Least Squares Regression

Simplest is linear regression:
\[ X = a_0 + a_1 R \]

Most common geometric transformation is bivariate, affine or first-order least squares function:
\[
\begin{align*}
X &= a_0 + a_1 R + a_2 C \\
Y &= b_0 + b_1 R + b_2 C \\
R &= d_0 + d_1 X + d_2 Y \\
C &= f_0 + f_1 X + f_2 Y
\end{align*}
\]
Least Squares Regression

- First-order usually OK for modest resolution on relatively flat area
- Can accomplish scaling, rotation, shearing & reflection
- May need higher-order functions for oblique angles and/or rough terrain
GCPs

- Best are pinpoint, permanent features
- Need 10-15 for first-order fit, and image area up to 1024 x 1024 pixels
- Need more for relief or wide areas that induce distortion from nadir
- Need to be spread out to cover all of area
- Keep some in reserve to validate transformation

Geometric Transformation

- Once equations known:
  - Calculate X,Y coordinates of 4 corners to form bounding rectangle of transformed image
  - Then calculate X,Y coordinate of the center of each pixel
  - To get pixel values image has to be re-sampled (later)
### Root-Mean-Square Error

<table>
<thead>
<tr>
<th>GCP</th>
<th>R</th>
<th>C</th>
<th>X</th>
<th>Y</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>134</td>
<td>230</td>
<td>3098</td>
<td>12</td>
<td>-18.9</td>
</tr>
<tr>
<td>2</td>
<td>1304</td>
<td>304</td>
<td>4449</td>
<td>23</td>
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<td>3</td>
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<td>3245</td>
<td>2345</td>
<td>213</td>
<td>302.3</td>
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<td>534</td>
<td>645</td>
<td>1235</td>
<td>324</td>
<td>15.5</td>
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<td>1287</td>
<td>3456</td>
<td>250</td>
<td>-12.3</td>
</tr>
</tbody>
</table>
RMSE

- Check each GCP for outlier
- Can try different models to minimize total RMSE
- Use other GCPs to validate transformation
Questions ?