Chapter 12  ANOVA
Analysis of Variance

Used to compare the means of more than 2 populations

$H_0: \mu_1 = \mu_2 = \ldots = \mu_k$

$H_1: \text{The population means are not all equal}$

### Example:

<table>
<thead>
<tr>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Sample 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>70</td>
<td>40</td>
<td>100</td>
</tr>
</tbody>
</table>

$H_0: \mu_1 = \mu_2 = \mu_3$

$H_1: \text{The pop. means are not all equal}$

**Step 1:** Collect $n, \bar{x}, s^2$ for each group

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1 = 2$</td>
<td>$n_2 = 3$</td>
<td>$n_3 = 2$</td>
</tr>
<tr>
<td>$\bar{x}_1 = 60$</td>
<td>$\bar{x}_2 = 40$</td>
<td>$\bar{x}_3 = 80$</td>
</tr>
<tr>
<td>$s^2_1 = 200$</td>
<td>$s^2_2 = 100$</td>
<td>$s^2_3 = 800$</td>
</tr>
</tbody>
</table>
Step 2: Compute $SSE$ (Sum of squares due to error)

$$SSE = \sum_{i=1}^{3} (n_i-1)S_i^2$$

$$= (2-1)200 + (3-1)100 + (2-1)800$$

$$= 1200$$

Step 3: Compute $\bar{x}$ (overall sample mean)

$$\bar{x} = \frac{\frac{1}{n} \sum_{i=1}^{3} n_i \bar{x}_i}{n}$$

$$n = \text{Combined sample size}$$

$$= \frac{2(60) + 3(40) + 2(80)}{7}$$

$$= 57.143$$

Step 4: Compute $SSTR$ (Sum of squares due to treatment)

$$SSTR = \sum n_i \bar{x}_i^2 - n \bar{x}^2$$

$$= 2(60^2) + 3(40^2) + 2(80^2) - 7(57.143^2)$$

$$= 1942.743$$
**ANOVA table**

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRT</td>
<td>1942.743</td>
<td>2</td>
<td>971.37</td>
<td>3.24</td>
</tr>
<tr>
<td>ERR</td>
<td>1200</td>
<td>4</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Tot</td>
<td>3142.743</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ MS = \frac{SS}{df} \quad F = \frac{MSTR}{MSE} \]

\( P_{val} = 0.1458 \)

**F table**

\( df_{TRT} \) \hspace{1cm} \( df_{6} \)

\( df = 2.4 \)

\( \alpha = 0.05 \)

6.8443

**Fail to reject H0. We were unable to find sufficient evidence that the population means differ.**
Example:

\( n_1 = 8 \) \hspace{1cm} \( n_2 = 10 \) \hspace{1cm} \( n_3 = 10 \)

\( \bar{x}_1 = 11.19 \) \hspace{1cm} \( \bar{x}_2 = 1.07 \) \hspace{1cm} \( \bar{x}_3 = 1.00 \)

\( s_1^2 = .014686 \) \hspace{1cm} \( s_2^2 = .096449 \) \hspace{1cm} \( s_3^2 = .017378 \)

\( s_1 = .1212 \) \hspace{1cm} \( s_2 = .0982 \) \hspace{1cm} \( s_3 = .1318 \)

H_0: \mu_1 = \mu_2 = \mu_3

H_1: The pop. means differ

\textbf{Step 2:} \text{SSE} = 7(.014686) + 9(.096449) + 9(.017378)

\[ = .3460 \]

\textbf{Step 3:} \[ \bar{x} = \frac{8(1.19) + 10(1.07) + 10(1.00)}{28} \]

\[ = 1.079286 \]

\textbf{Step 4:} \text{SSTR} = 8(1.19)^2 + 10(1.07)^2 + 10(1.00)^2 - \frac{28}{28}(1.079286)^2

\[ = .161768 \]

<table>
<thead>
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<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRT</td>
<td>.161768</td>
<td>2</td>
<td>.080884</td>
<td>5.84</td>
</tr>
<tr>
<td>ERR</td>
<td>.3460</td>
<td>25</td>
<td>.01384</td>
<td></td>
</tr>
<tr>
<td>TOT</td>
<td>.507768</td>
<td>27</td>
<td></td>
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</tr>
</tbody>
</table>

Source: Chapter 7
Reject H0. We were able to show that population means are not all equal.

HW p.652 # 12, 13, 14

Quiz #4 H.T. for 1 variance, 2 variances, \( \chi^2 \) GOF, \( \chi^2 \) independence