ME 323 Midterm Exam. 5/8/08, 100pts total **Name**________________________

True/False. Circle the correct answer. (1pt each, 9pts total)

1. Thermal conductivity k is independent of the direction of heat transfer in any material T F

2. Free convection is caused by external means other than buoyancy forces T F

3. Radiation heat transfer occurs most efficiently in a vacuum. T F

4. If contact region between two surfaces is evacuated then the thermal contact resistance will decrease. T F

5. Convection heat transfer mode is sustained only by the bulk motion of a fluid near a solid surface. T F

6. For one dimensional, steady-state heat conduction, without internal heat generation, through a plane wall, the heat flux is a constant T F

7. In general, the thermal conductivity of fluids is higher than that of solids. T F

8. For steady state heat conduction through a medium, the temperature at a particular location varies with time. T F

9. The lumped capacitance method is for studying 2-D steady state conduction heat transfer. T F

Short Answer. Using heat transfer jargon...(3pts each, 18 pts total)

10. Thermal diffusivity of a material is $\alpha = k/\rho C_p$. Will a material of larger α respond more quickly or slowly to changes in the thermal environment than material of smaller α? Why?

More quickly; a material of larger α has better capability conducting heat than storing heat than a material of smaller α

11. A sleeping bag can keep you warm during night when you camp out. Explain why.

Mainly it is because that the air pockets in the filling material of the sleep bag has high thermal resistance

12. Can radiation heat transfer occur in a vacuum? Explain your answer.

Yes; radiation occurs by electro-magnetic wave which can transfer without a medium.

13. At atomic and molecular level, describe the mechanism of conduction heat transfer.

Heat is transfered from particles of higher thermal energy to those of lower thermal energy due to the interactions between the particles.

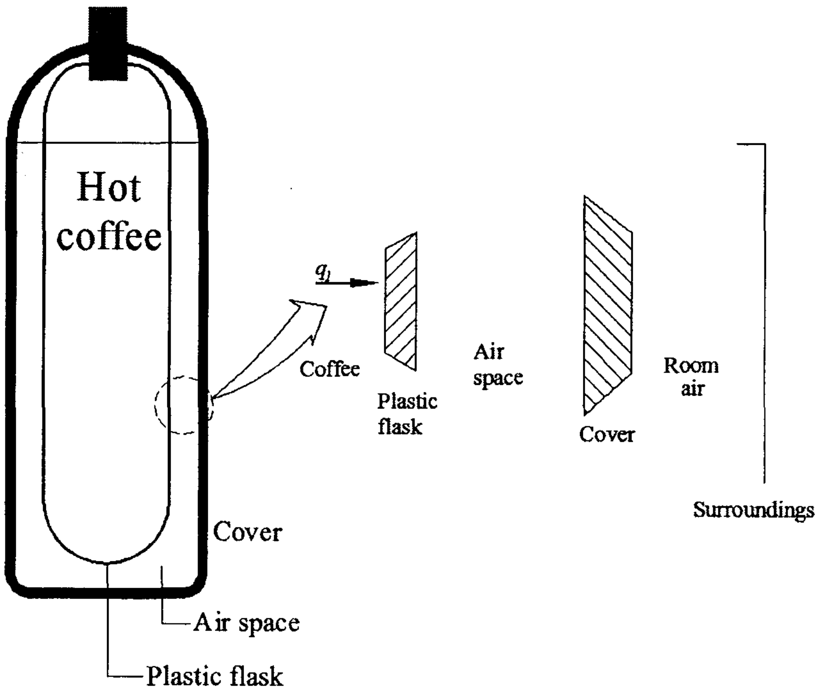
14. Is radiation heat transfer dominant at high temperature? Explain why.

Yes; Heat transfer rate for radiation is proportional to T^4 while it is proportional to T for conduction and convection.

15. What purpose is served by attaching fins to a surface?

It enhances the convection heat transfer by increasing the surface area.

16 (8 pts) A closed container filled with hot coffee is in a room whose air and walls are at a fixed temperature. Identify all heat transfer processes that contribute to cooling of the coffee



q_1:

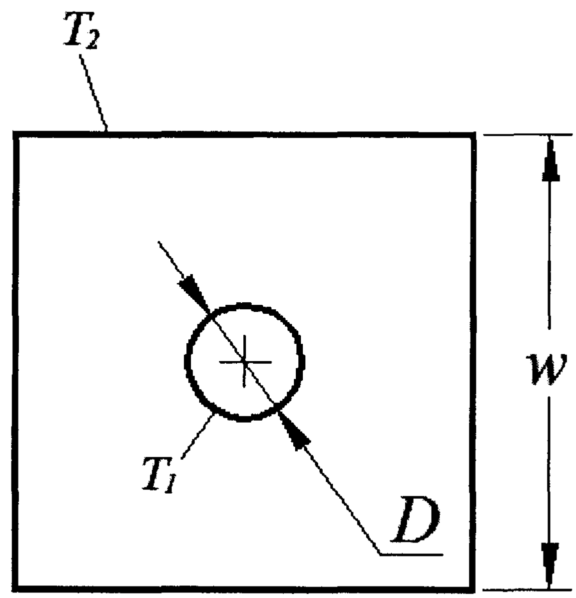
See Example 1.8, p. 40, Incropera & Dewitt Text, 6th. ed.

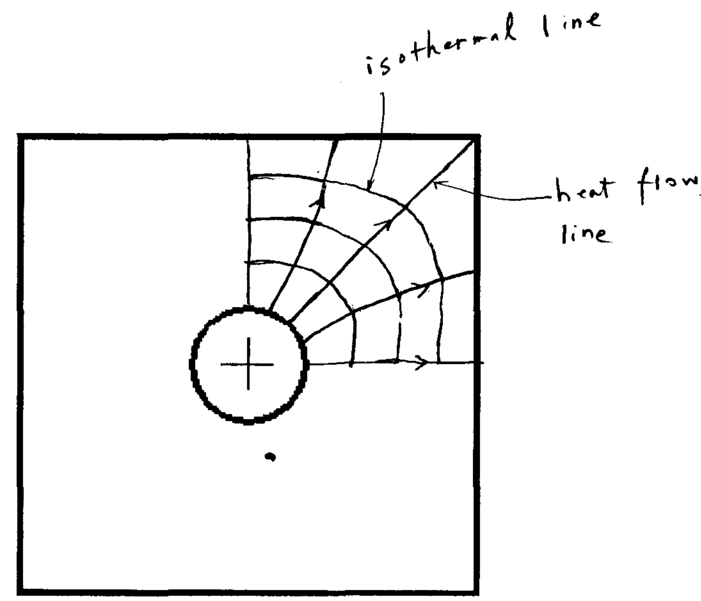
Comment on features that would contribute to a superior container design (bonus points: 3)

17. (8 pts) A circular cylinder of diameter D=2 is buried in a square solid of width w=8 of equal length L»w. The surface temperature of the cylinder (T_1) is higher than that of the surface of the square solid (T_2). In the figure shown, the dimensions are given on the left. On the right, sketch the isothermal lines and heat flow lines between the cylinder and the surface of the square solid and compute the shape factor $S=M/N*L$ where M is the number of the heat flow lanes and N the number of temperature increment that you pick. The shape factor S can also be computed using

$$S = \frac{2\pi L}{\ln(1.08\, w/D)}.$$

Compare your results from those two methods, what is the percentage difference?





① $S_1 = \frac{2\pi L}{\ln(1.08\,\frac{W}{D})} = \frac{2\pi L}{\ln(1.08\cdot\frac{8}{2})}$

$= 4.29L$

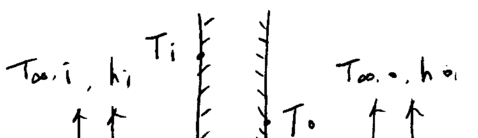
② $S_2 = 4\cdot\frac{4}{4}L = 4L$

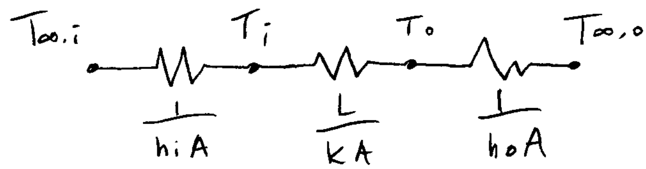
③ $\frac{|S_2-S_1|}{S_1}\times100\% = \frac{|4-4.29|}{4.29}\times100\% = 6.75\%$

18 (10 pts) The rear window of an automobile is defogged by passing warm air over its inner surface. It is found that the warm air is at $T_{\infty,i}$ = 40°C and the corresponding convection coefficient is h_i=30W/m^2•K; the inner surface temperature is 7.7 °C; the outside ambient air temperature is $T_{\infty,o}$ = -10°C. The thickness of the glass is 4mm and the thermal conductivity of the glass is 1.4W/ m•K. What is the outer surface temperature? What is the convection coefficient associated with the outer surface?

A. Draw the schematic of the problem, make appropriate assumptions, and sketch the thermal circuit of the problem

Assumptions:
1. 1-D
2. steady state
3. Constant thermal conductivity
4. negligible radiation





B. Outer surface temperature?

Heat flux: $q'' = h_i (T_{\infty,i} - T_i)$

$= 30 \times (40 - 7.7)$

$= 969 \ \frac{W}{m^2}$

Also, $q'' = \dfrac{T_i - T_o}{\frac{L}{k}} = \dfrac{7.7 - T_o}{\frac{4 \times 10^{-3}}{1.4}} \Rightarrow T_o = 7.7 - 969 \dfrac{4 \times 10^{-3}}{1.4}$

$= 4.93 \ °C$

C. The convection coefficient associated with the outer surface?

Knowing T_o, $T_{\infty,o}$ we can find h_o by using

$q'' = h_o (T_o - T_{\infty,o})$

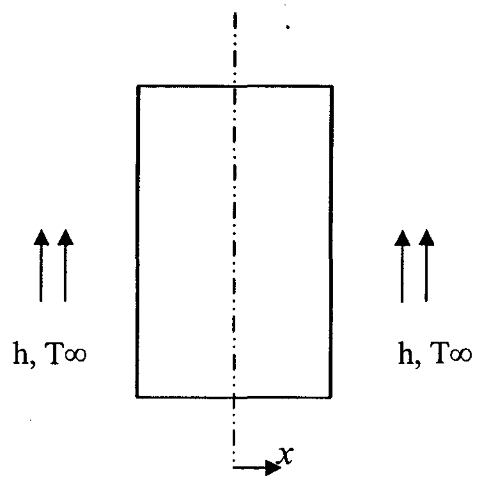
$\Rightarrow h_o = \dfrac{q''}{T_o - T_{\infty,o}}$

$= \dfrac{969}{4.93 + 10}$

$= 64.9 \ \frac{W}{m^2 \cdot K}$

19. (12 pts) A 100-mm-thick plate of large surface area that is initially at a uniform temperature of 300 °C and is heated on both sides in a gas-fired furnace for which T_∞ =700 °C and h=500 W/m^2•K. How long will it take for a minimum temperature of 550 °C to be reached in the plate?

The thermophysical properties of the plate material are density ρ = 7800kg/m^3, conductivity k = 45 W/mK, and c_p = 500 J/kgK.



Assumptions: ① 1-D
② Symmetric heating
③ Constant heating
④ Negligible radiation
⑤ Fo > 0.2

Firstly. calculate Bi number.

$$Bi = \frac{h L_c}{k} = \frac{500 \times 50 \times 10^{-3}}{45} = 0.55 > 0.1$$

So lumped capacitance method can not be used.

Instead, we use 1-term approximation. Also, note that the minimum temperature occurs at the mid plane of the plate, i.e. $x^* = 0$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = C_1 \exp(-\zeta_1^2 Fo)$$

$$\frac{550 - 700}{300 - 700} = C_1 \exp(-\zeta_1^2 Fo)$$

$$C_1 = \frac{1.07 + 1.081}{2} = 1.0755; \quad \zeta_1 = \frac{0.6533 + 0.7051}{2} = 0.6792$$

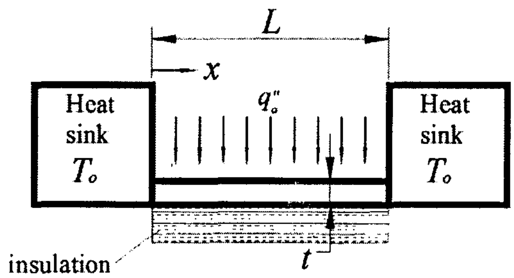
$$Fo = -\frac{1}{0.6792^2} \ln \frac{0.375}{1.0755} = 2.28$$

$$Fo = \frac{\alpha t}{L_c^2} \Rightarrow t = \frac{Fo\, L_c^2}{\alpha}, \quad \alpha = \frac{k}{\rho c_p} = 1.15 \times 10^{-5}$$

$$\therefore t = \frac{2.28 \times (50 \times 10^{-3})^2}{1.15 \times 10^{-5}} = 495.6 \text{ s.}$$

Note: Fo = 2.28 > 0.2, assumption ⑤ is correct.

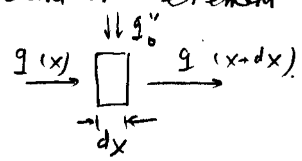
20 (15pts) A thin flat plate of length L, thickness t, and width $W \gg L$ is thermally joined to two large heat sinks that are maintained at a temperature T_o. The bottom of the plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of q_o''.



a) Make appropriate assumptions and derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.

Assumptions : (1) steady state. (2) 1-D. (3) Constant properties
(4) uniform heat flux. (5) Adiabatic bottom
(6) Negligible contact resistance.

Take an differential element of the thin plate.



Heat balance: $q(x+dx) = q(x) + q_o'' W dx$

Using Taylor expansion:

$$q(x+dx) = q(x) + \frac{dq}{dx}dx$$

$$\Rightarrow q_o'' W dx = dq$$

$$q = -kWt\frac{dT}{dx} \Rightarrow \frac{dq}{dx} = -kWt\frac{d^2T}{dx^2}$$

$$\Rightarrow q_o'' W = \frac{dq}{dx} = -kWt\frac{d^2T}{dx^2}$$

b) Solve the forgoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

$$\frac{d^2T}{dx^2} = -\frac{q_o''}{kt}, \quad \text{B.C.s} \quad T = T_o \ @ \ x=0, \quad T = T_o \ @ \ x = L$$

$$\Rightarrow \frac{dT}{dx} = -\frac{q_o''}{kt}x + C_1$$

$$T = -\frac{1}{2}\frac{q_o''}{kt}x^2 + C_1 x + C_2$$

$T = T_o$, @ $x = 0$ $\Rightarrow$ $C_2 = T_o$

$T = T_o$ @ $x = L$ $\Rightarrow$ $-\frac{1}{2}\frac{q_o''}{kt}L^2 + C_1 L + T_o = T_o \Rightarrow C_1 = \frac{1}{2}\frac{q_o''}{kt}L$

$$\therefore T = -\frac{1}{2}\frac{q_o''}{kt}x(x-L) + T_o.$$

H T rate @ $x=0$: $q(0) = -kWt\left.\frac{dT}{dx}\right|_{x=0} = -\frac{1}{2}q_o'' WL$

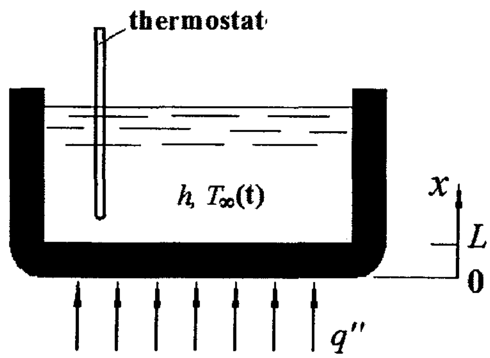
@ $x=L$: $q(L) = -kWt\left.\frac{dT}{dx}\right|_{x=L} = \frac{1}{2}q_o'' WL$

Total heat transfer rate:

$$q_{tot} = -q(0) + q(L) = q_o'' WL.$$

21. (20pts) To determine the performance of a pan of thickness L and diameter D, the pan is used to boil water by placing it on a stove. Heat is transferred from the stove at a fixed flux q". A thermostat is dipped into the water to measure the variation of the water temperature with time from its initial temperature T_i to the boiling point as heat is transformed from the pan. The time history of the water temperature is recorded as $T_\infty = T_\infty(t)$. Assuming constant heat transfer coefficient h. The conductivity, density, and specific heat of the pan material are k, ρ, and c_p respectively. The full 3-D heat equation in Cartesian coordinates is

$$\rho c_p \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q} \qquad (1)$$



A. What heat transfer mode is responsible for the heat transferred from the pan bottom to the water?

Convection

B. To determine the variation of temperature with position and time, T(x,t), in the pan bottom, make appropriate assumptions and simplify eqn. (1). DO NOT SOLVE

Assumptions: (1) 1-D
(2) $\dot{q} = 0$
(3) Constant properties.

Eqn (1) simplifies to

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{d^2 T}{dx^2}$$

C. Write the appropriate initial and boundary conditions for your simplified equation. Discuss how the boundary conditions are determined.

I.C. $T(x,0) = T_i$

B.C.s. Applying heat balance at the bottom $x=0$, gives

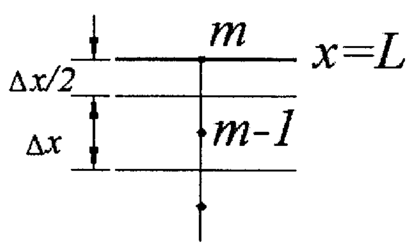
$$-k \left.\frac{dT}{dx}\right|_{x=0} = q''$$

Applying heat balance at the upper side of the pan bottom. $x=L$

$$-k \left.\frac{dT}{dx}\right|_{x=L} = h(T_L - T_\infty).$$

D. We want to use numerical computation to predict T(x,t). In order to do that, we must correctly discretize the boundary conditions. A sketch of the boundary nodal point is shown below. Because the temperature is unknown at the boundary nodes, the finite-difference equations for the boundary nodes should be obtained by applying energy balance, i.e. (q_{in} + q_g - q_{out} = q_{st}) to the nodes. Do this for the boundary node m at x=L using explicit method.

$$q_{in} + q_g - q_{out} = q_{st}.$$



We know that $q_g = 0$

$\therefore$ $q_{in} - q_{out} = q_{st}$

i.e. $$kA \frac{T_{m-1}^p - T_m^p}{\Delta x} - hA(T_m^p - T_\infty^p) = \rho c_p \frac{T_m^{p+1} - T_m^p}{\Delta t} A \frac{\Delta x}{2}$$

$$\Rightarrow k\frac{T_{m-1}^p - T_m^p}{\Delta x} - h(T_m^p - T_\infty^p) = \frac{\rho c_p \Delta x}{2\Delta t}(T_m^{p+1} - T_m^p)$$

$$\Rightarrow \frac{2k}{\rho c_p}\frac{\Delta t}{\Delta x^2}(T_{m-1}^p - T_m^p) - \frac{2h}{\rho c_p}\frac{\Delta t}{\Delta x}(T_m^p - T_\infty^p) = T_m^{p+1} - T_m^p$$

$$\Rightarrow T_m^{p+1} = T_m^p + \frac{2k}{\rho c_p}\frac{\Delta t}{\Delta x^2}(T_{m-1}^p - T_m^p) - \frac{2h}{\rho c_p}\frac{\Delta t}{\Delta x^2}(T_m^p - T_\infty^p)$$

$$= T_m^p(1 - 2Fo - 2Bi\,Fo) + 2Fo\,T_{m-1}^p + 2Bi\,Fo\,T_\infty^p$$

where $Fo = \frac{\alpha \Delta t}{\Delta x^2}$ $Bi = \frac{h}{k}\Delta x$.

22. (bonus: 10 pts) For the preceding problem, once the water comes to boil, its temperature remains at a fixed value, $T_\infty = T_b$, as heating continues. The surface of the pan in contact with the water achieves a known and constant temperature T_L, where $T_L > T_b$.

A. To determine the variation of temperature with position, $T(x)$, in the pan bottom, make appropriate assumptions and simplify eqn. (1). DO NOT SOLVE

Assumptions ① 1-D , ② steady-state
③ $\dot{q} = 0$.

Eqn (1) now simplifies to $\dfrac{d^2T}{dx^2} = 0$.

B. Write the appropriate boundary conditions for your simplified equation.

B.C.s. $-k \left.\dfrac{dT}{dx}\right|_{x=0} = q''$

$\left.T\right|_{x=L} = T_L$

C. Solve your simplified equation with the boundary conditions.

$$\frac{d^2T}{dx^2} = 0 \Rightarrow \frac{dT}{dx} = C_1 \Rightarrow T = C_1 x + C_2$$

$$-k \left.\frac{dT}{dx}\right|_{x=0} = q'' \Rightarrow C_1 = -\frac{q''}{k}$$

$$\left.T\right|_{x=L} = T_L \Rightarrow C_2 = T_L + \frac{q''}{k} L$$

$$\therefore T = -\frac{q''}{k} x + T_L + \frac{q''}{k} L$$

D. Sketch the temperature distribution in the pan bottom.