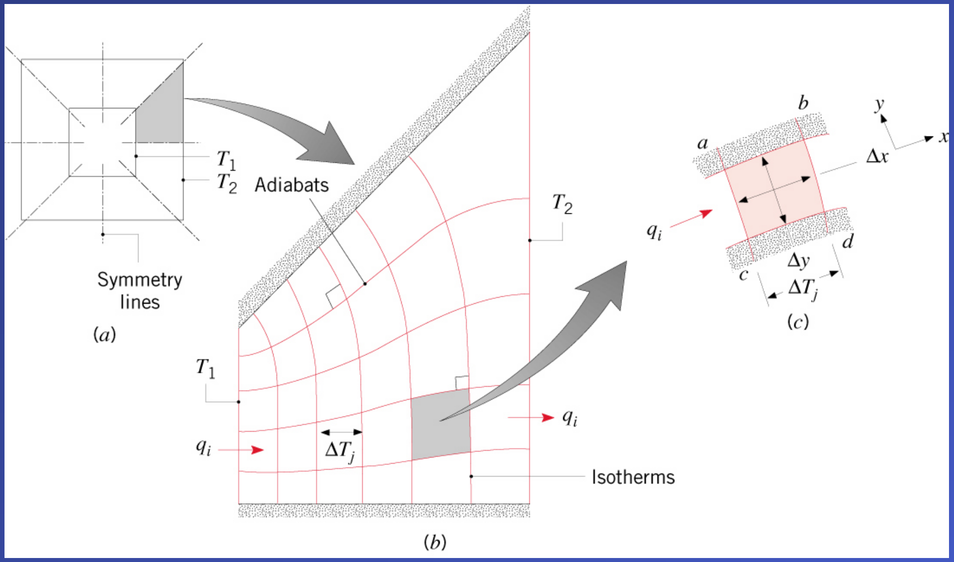
Flux Plots

- Utility: Requires delineation of isotherms and heat flow lines. Provides a quick means of estimating the rate of heat flow.

- Procedure: Systematic construction of nearly perpendicular isotherms and heat flow lines to achieve a network of curvilinear squares.

- Rules:
  - On a schematic of the two-dimensional conduction domain, identify all lines of symmetry, which are equivalent to adiabats and hence heat flow lines.
  - Sketch approximately uniformly spaced isotherms on the schematic, choosing a small to moderate number in accordance with the desired fineness of the network and rendering them approximately perpendicular to all adiabats at points of intersection.
  - Draw heat flow lines in accordance with requirements for a network of curvilinear squares.

Example: Square channel with isothermal inner and outer surfaces.



– Note simplification achieved by identifying lines of symmetry.

– Requirements for curvilinear squares:

- Intersection of isotherms and heat flow lines at right angles
- Approximate equivalence of sums of opposite sides

$$\Delta x \equiv \frac{ab + cd}{2} \approx \frac{ac + bd}{2} \equiv \Delta y \qquad (4.20)$$

– Determination of heat rate:

$$q \approx M q_i \quad \approx M\left[k\left(\Delta y \cdot \ell\right)\frac{\Delta T_j}{\Delta x}\right] \quad \approx \frac{M\,\ell}{N} k \Delta T_{1-2}$$

$$q' \approx \frac{M}{N} k \Delta T_{1-2} \qquad (4.24)$$

The Conductor Shape Factor

- Two-dimensional heat transfer in a medium bounded by two isothermal surfaces at T_1 and T_2 may be represented in terms of a conduction shape factor *S*.

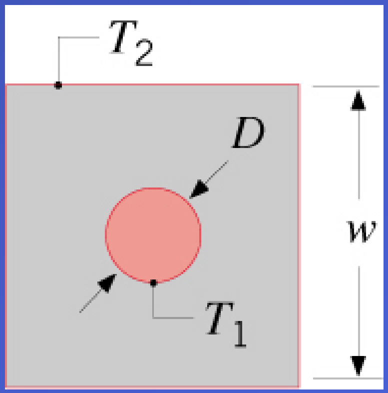
$$q = Sk(T_1 - T_2) \qquad (4.25)$$

- For a flux plot,

$$S \approx \frac{M\ell}{N} \qquad (4.26)$$

- Exact and approximate results for common two-dimensional systems are provided in Table 4.1. For example,

Case 6. Long ($L >> w$) circular cylinder centered in square solid of equal length

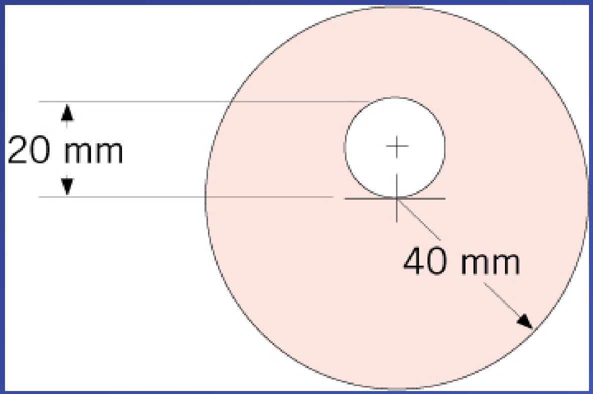


$$S = \frac{2\pi L}{\ln(1.08w/D)}$$

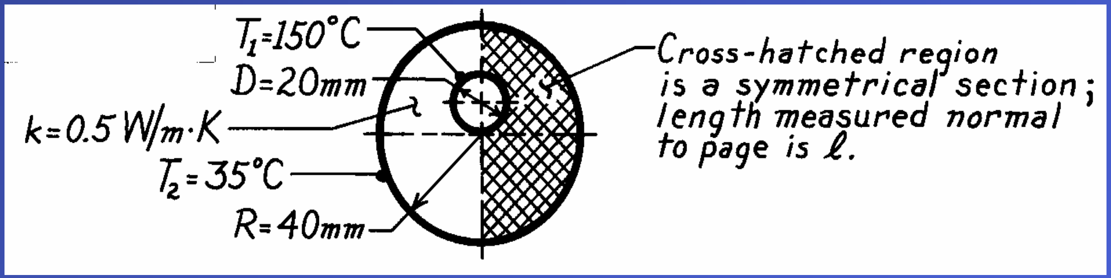
- Two-dimensional conduction resistance:

$$R_{cond(2D)} = (Sk)^{-1} \qquad (4.27)$$

Problem 4.6: Heat transfer from a hot pipe embedded eccentrically in a solid rod.

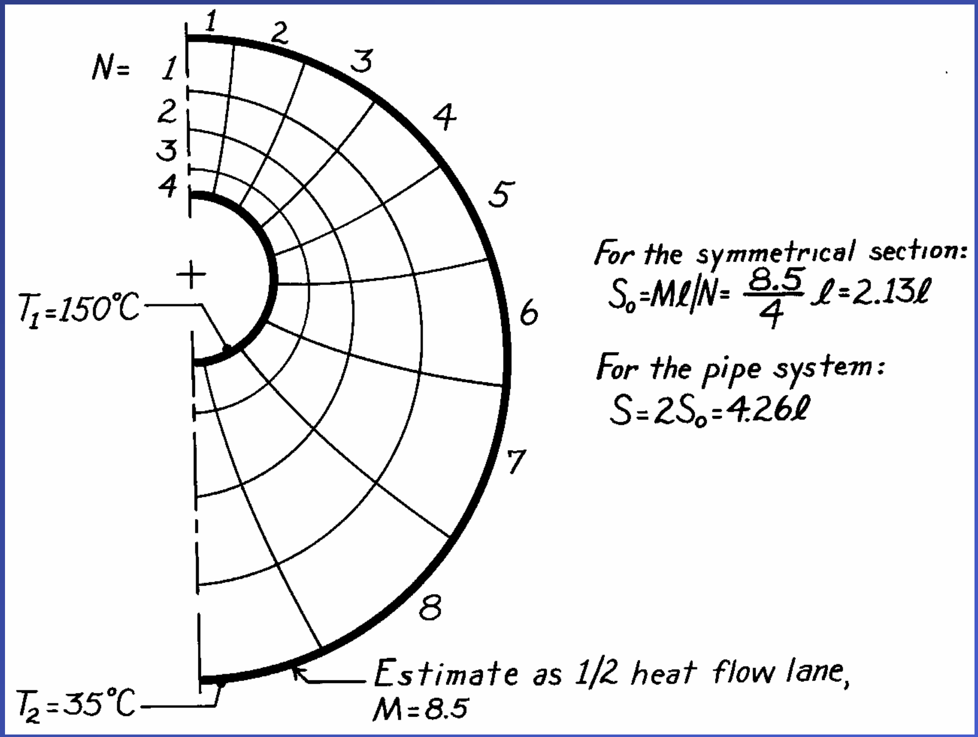


Schematic



ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Length ℓ >> diametrical dimensions, (4) Constant thermal conductivity.

ANALYSIS: For the symmetrical section and four temperature increments (N = 4), the flux plot is:



For the pipe the heat rate per unit length is

$$q' = \frac{q}{\ell} = kS(T_1 - T_2) = 0.5\frac{W}{m \cdot K} \times 4.26(150 - 35)^{\circ}\,C = 245\ W/m.$$

COMMENTS: Because the curvilinear squares are irregular in the lower, right-hand quadrant of the flux plot a finer network would be needed to obtain a more accurate estimate of the shape factor. Determine the error associated with the flux plot by using a result from Table 4.1 to compute the actual value of the shape factor.