Flux Plots

• **Utility:** Requires delineation of isotherms and heat flow lines. Provides a quick means of estimating the rate of heat flow.

• **Procedure:** Systematic construction of nearly perpendicular isotherms and heat flow lines to achieve a network of curvilinear squares.

• **Rules:**
  - On a schematic of the two-dimensional conduction domain, identify all lines of symmetry, which are equivalent to adiabats and hence heat flow lines.
  - Sketch approximately uniformly spaced isotherms on the schematic, choosing a small to moderate number in accordance with the desired fineness of the network and rendering them approximately perpendicular to all adiabats at points of intersection.
  - Draw heat flow lines in accordance with requirements for a network of curvilinear squares.
Example: Square channel with isothermal inner and outer surfaces.

- Note simplification achieved by identifying lines of symmetry.
- Requirements for curvilinear squares:
  - Intersection of isotherms and heat flow lines at right angles
  - Approximate equivalence of sums of opposite sides

\[
\Delta x \equiv \frac{ab + cd}{2} \approx \frac{ac + bd}{2} \equiv \Delta y
\]  

(4.20)

- Determination of heat rate:

\[
q \approx Mq_i \approx M k \left( \Delta y \cdot \ell \right) \frac{\Delta T_i}{\Delta x} \approx \frac{M \ell}{N} k \Delta T_{1-2}
\]

\[
q' \approx \frac{M}{N} k \Delta T_{1-2}
\]  

(4.24)
The Conductor Shape Factor

• Two-dimensional heat transfer in a medium bounded by two isothermal surfaces at $T_1$ and $T_2$ may be represented in terms of a conduction shape factor $S$.

$$q = Sk(T_1 - T_2)$$  \hspace{1cm} (4.25)

• For a flux plot,

$$S \approx \frac{M \ell}{N}$$  \hspace{1cm} (4.26)

• Exact and approximate results for common two-dimensional systems are provided in Table 4.1. For example,

Case 6. Long ($L>>w$) circular cylinder centered in square solid of equal length

$$S = \frac{2\pi L}{\ln(1.08w/D)}$$

• Two-dimensional conduction resistance:

$$R_{\text{cond}(2D)} = (Sk)^{-1}$$  \hspace{1cm} (4.27)
Problem 4.6: Heat transfer from a hot pipe embedded eccentrically in a solid rod.

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Length $\ell >>$ diametrical dimensions, (4) Constant thermal conductivity.
**ANALYSIS:** For the symmetrical section and four temperature increments \((N = 4)\), the flux plot is:

\[
q' = \frac{q}{\ell} = kS(T_1 - T_2) = 0.5 \frac{W}{m \cdot K} \times 4.26 (150 - 35)^\circ C = 245 \ W/m.
\]

**COMMENTS:** Because the curvilinear squares are irregular in the lower, right-hand quadrant of the flux plot a finer network would be needed to obtain a more accurate estimate of the shape factor. Determine the error associated with the flux plot by using a result from Table 4.1 to compute the actual value of the shape factor.