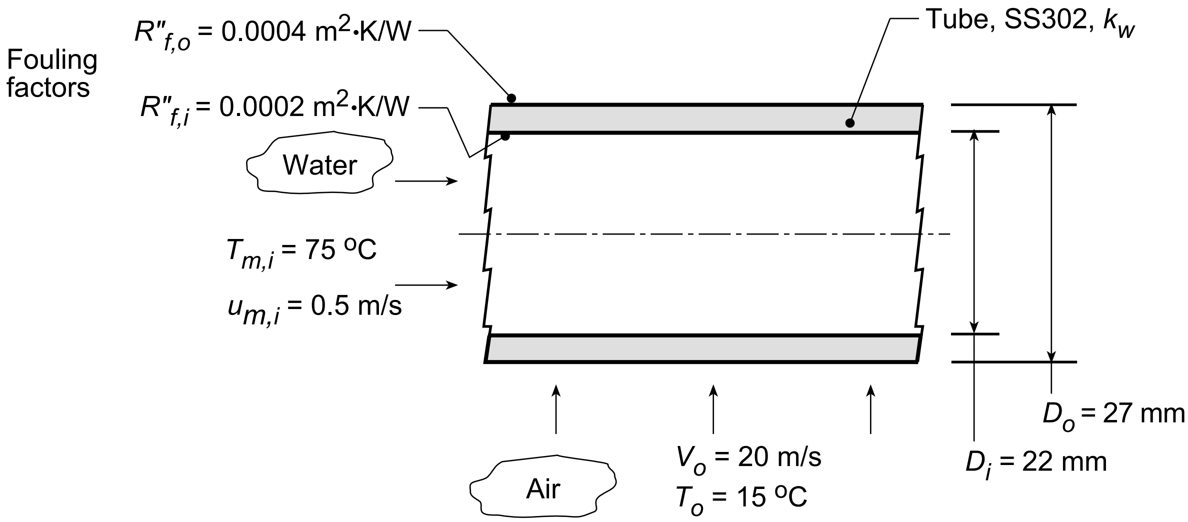
PROBLEM 11.2

KNOWN: Type-302 stainless tube with prescribed inner and outer diameters used in a cross-flow heat exchanger. Prescribed fouling factors and internal water flow conditions.

FIND: (a) Overall coefficient based upon the outer surface, U_o, with air at T_o =15°C and velocity V_o = 20 m/s in cross-flow; compare thermal resistances due to convection, tube wall conduction and fouling; (b) Overall coefficient, U_o, with water (rather than air) at T_o = 15°C and velocity V_o = 1 m/s in cross-flow; compare thermal resistances due to convection, tube wall conduction and fouling; (c) For the water-air conditions of part (a), compute and plot U_o as a function of the air cross-flow velocity for $5 \le V_o \le 30$ m/s for water mean velocities of $u_{m,i}$ = 0.2, 0.5 and 1.0 m/s; and (d) For the water-water conditions of part (b), compute and plot U_o as a function of the water mean velocity for $0.5 \le u_{m,i} \le 2.5$ m/s for air cross-flow velocities of V_o = 1, 3 and 8 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed internal flow,

PROPERTIES: *Table A.1*, Stainless steel, AISI 302 (300 K): k_w = 15.1 W/m·K; *Table A.6*, Water ($\overline{T}_{m,i}$ = 348 K): ρ_i = 974.8 kg/m^3, $\mu_i = 3.746 \times 10^{-4}$ N·s/m^2, k_i = 0.668 W/m·K, Pr_i = 2.354; *Table A.4,* Air (assume $\overline{T}_{f,o}$ = 315K, 1 atm): k_o = 0.02737 W/m·K, $\nu_o = 17.35 \times 10^{-6}$ m^2/s, Pr_o = 0.705.

ANALYSIS: (a) For the water-air condition, the overall coefficient, Eq. 11.1, based upon the outer area can be expressed as the sum of the thermal resistances due to convection (cv), tube wall conduction (w) and fouling (f):

$$1/U_o A_o = R_{tot} = R_{cv,i} + R_{f,i} + R_w + R_{f,o} + R_{cv,o}$$

$$R_{cv,i} = 1/\overline{h}_i A_i \qquad R_{cv,o} = 1/\overline{h}_o A_o$$

$$R_{f,i} = R''_{f,i}/A_i \qquad R_{f,o} = R''_{f,o}/A_o$$

and from Eq. 3.28,

$$R_w = \ln(D_o/D_i)/(2\pi L k_w)$$

The convection coefficients can be estimated from appropriate correlations.

Continued...

Estimating $\bar{h}_i$: For internal flow, characterize the flow evaluating thermophysical properties at $T_{m,i}$ with

$$Re_{D,i} = \frac{u_{m,i} D_i}{\nu_i} = \frac{0.5\,m/s \times 0.022m}{3.746\times10^{-4}\,N\cdot s/m^2 / 974.8\,kg/m^3} = 28,625$$

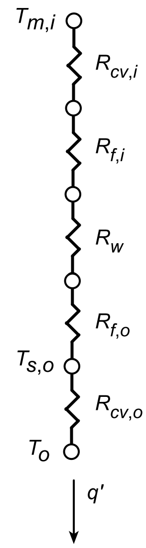
For the turbulent flow, use the Dittus-Boelter correlation, Eq. 8.60,

$$Nu_{D,i} = 0.023\,Re_{D,i}^{0.8}\,Pr_i^{0.4}$$

$$Nu_{D,i} = 0.023(28,625)^{0.8}(2.354)^{0.4} = 119.1$$

$$\bar{h}_i = Nu_{D,i}\,k_i/D_i = 119.1\times0.668\,W/m^2\cdot K/0.022m = 3616\,W/m^2\cdot K$$

Estimating $\bar{h}_o$: For external flow, characterize the flow with



$$Re_{D,o} = \frac{V_o D_o}{\nu_o} = \frac{20\,m/s\times0.027m}{17.35\times10^{-6}\,m^2/s} = 31,124$$

evaluating thermophysical properties at $T_{f,o} = (T_{s,o} + T_o)/2$ when the surface temperature is determined from the thermal circuit analysis result,

$$\left(T_{m,i} - T_o\right)/R_{tot} = \left(T_{s,o} - T_o\right)/R_{cv,o}$$

Assume $T_{f,o}$ = 315 K, and check later. Using the Churchill-Bernstein correlation, Eq. 7.57, find

$$\overline{Nu}_{D,o} = 0.3 + \frac{0.62\,Re_{D,o}^{1/2}\,Pr_o^{1/3}}{\left[1+\left(0.4/Pr_o\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_{D,o}}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_{D,o} = 0.3 + \frac{0.62(31,124)^{1/2}(0.705)^{1/3}}{\left[1+\left(0.4/0.705\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{31,124}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_{D,o} = 102.6$$

$$\bar{h}_o = \overline{Nu}_{D,o}\,k_o/D_o = 102.6\times0.02737\,W/m\cdot K/0.027m = 104.0\,W/m\cdot K$$

Using the above values for $\bar{h}_i$, and $\bar{h}_o$, and other prescribed values, the thermal resistances and overall coefficient can be evaluated and are tabulated below.

$R_{cv,i}$ (K/W)	$R_{f,i}$ (K/W)	R_w (K/W)	$R_{f,o}$ (K/W)	$R_{cv,o}$ (K/W)	U_o ($W/m^2\cdot K$)	R_{tot} (K/W)
0.00436	0.00578	0.00216	0.00236	0.1134	92.1	0.128

The major thermal resistance is due to outside (air) convection, accounting for 89% of the total resistance. The other thermal resistances are of similar magnitude, nearly 50 times smaller than $R_{cv,o}$.

(b) For the water-water condition, the method of analysis follows that of part (a). For the internal flow, the estimated convection coefficient is the same as part (a). For an assumed outer film coefficient, $\bar{T}_{f,o}$ = 292 K, the convection correlation for the outer water flow condition V_o = 1 m/s and T_o = 15°C, find

PROBLEM 11.2 (Cont.)

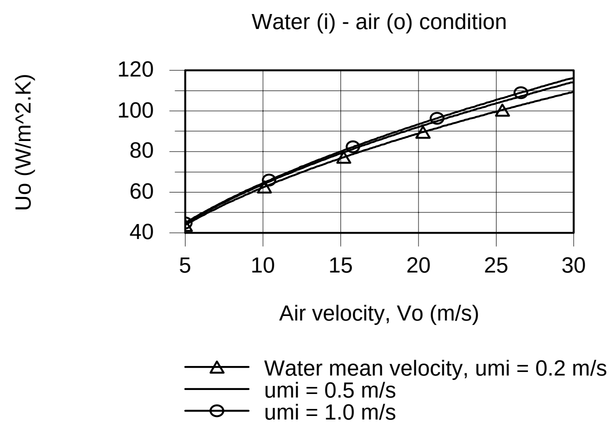
$$Re_{D,o} = 26,260 \qquad Nu_{D,o} = 220.6 \qquad \bar{h}_o = 4914\ W/m^2 \cdot K$$

The thermal resistances and overall coefficient are tabulated below.

$R_{cv,i}$ (K/W)	$R_{f,i}$ (K/W)	R_w (K/W)	$R_{f,o}$ (K/W)	$R_{cv,o}$ (K/W)	R_{tot} (K/W)	U_o ($W/m^2 \cdot K$)
0.00436	0.00579	0.00216	0.00236	0.00240	0.0171	691

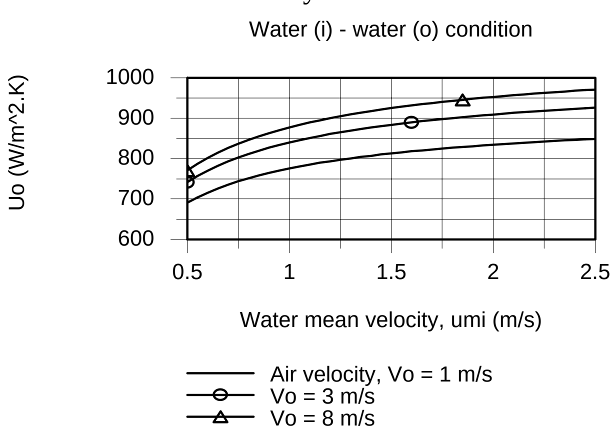
Note that the thermal resistances are of similar magnitude. In contrast with the results for the water-air condition of part (a), the thermal resistance of the outside convection process, $R_{cv,o}$, is nearly 50 times smaller. The overall coefficient for the water-water condition is 7.5 times greater than that for the water-air condition.

(c) For the water-air condition, using the IHT workspace with the analysis of part (a), U_o was calculated as a function of the air cross-flow velocity for selected mean water velocities.



The effect of increasing the cross-flow air velocity is to increase U_o since the $R_{cv,o}$ is the dominant thermal resistance for the system. While increasing the water mean velocity will increase $\bar{h}_i$, because $R_{cv,i} << R_{cv,o}$, this increase has only a small effect on U_o.

(d) For the water-water condition, using the IHT workplace with the analysis of part (b), U_o was calculated as a function of the mean water velocity for selected air cross-flow velocities.



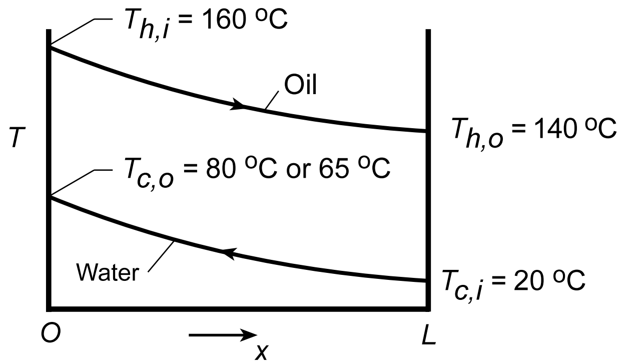
Because the thermal resistances for the convection processes, $R_{cv,i}$ and $R_{cv,o}$, are of similar magnitude according to the results of part (b), we expect to see U_o significantly increase with increasing water mean velocity and air cross-flow velocity.

PROBLEM 11.16

KNOWN: Inner tube diameter (D = 0.02 m) and fluid inlet and outlet temperatures corresponding to design conditions for a concentric tube heat exchanger. Overall heat transfer coefficient (U = 500 $W/m^2\cdot K$) and desired heat rate (q = 3000 W). Cold fluid outlet temperature after three years of operation.

FIND: (a) Required heat exchanger length, (b) Heat rate, hot fluid outlet temperature, overall heat transfer coefficient, and fouling factor after three years.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to the surroundings and kinetic and potential energy changes, (2) Negligible tube wall conduction resistance, (3) Constant properties.

ANALYSIS: (a) The tube length needed to achieve the prescribed conditions may be obtained from Eqs. 11.14 and 11.15 where $\Delta T_1 = T_{h,i} - T_{c,o} = 80°C$ and $\Delta T_2 = T_{h,o} - T_{c,i} = 120°C$. Hence, $\Delta T_{1m} = (120 - 80)°C/\ln(120/80) = 98.7°C$ and

$$L = \frac{q}{(\pi D) U \Delta T_{1m}} = \frac{3000\,W}{(\pi \times 0.02\,m) 500\,W/m^2\cdot K \times 98.7°C} = 0.968\,m$$ <

(b) With $q = C_c(T_{c,o} - T_{c,i})$, the following ratio may be formed in terms of the design and 3 year conditions.

$$\frac{q}{q_3} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_c\left(T_{c,o} - T_{c,i}\right)_3} = \frac{60°C}{45°C} = 1.333$$

Hence,

$$q_3 = q/1.33 = 3000\,W/1.333 = 2250\,W$$ <

Having determined the ratio of heat rates, it follows that

$$\frac{q}{q_3} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_h\left(T_{h,i} - T_{h,o}\right)_3} = \frac{20°C}{160°C - T_{h,o(3)}} = 1.333$$

Hence,

$$T_{h,o(3)} = 160°C - 20°C/1.333 = 145°C$$ <

With $\Delta T_{lm,3} = (125 - 95)/\ln(125/95) = 109.3°C$,

$$U_3 = \frac{q_3}{(\pi D L)\Delta T_{lm,3}} = \frac{2250\,W}{\pi(0.02\,m)0.968\,m\left(109.3°C\right)} = 338\,W/m^2\cdot K$$ <

Continued...

PROBLEM 11.16 (Cont.)

With $U = \left[(1/h_i) + (1/h_o)\right]^{-1}$ and $U_3 = \left[(1/h_i) + (1/h_o) + R''_{f,c}\right]^{-1}$,

$$R''_{f,c} = \frac{1}{U_3} - \frac{1}{U} = \left(\frac{1}{338} - \frac{1}{500}\right) m^2 \cdot K/W = 9.59 \times 10^{-4} m^2 \cdot K/W$$ <

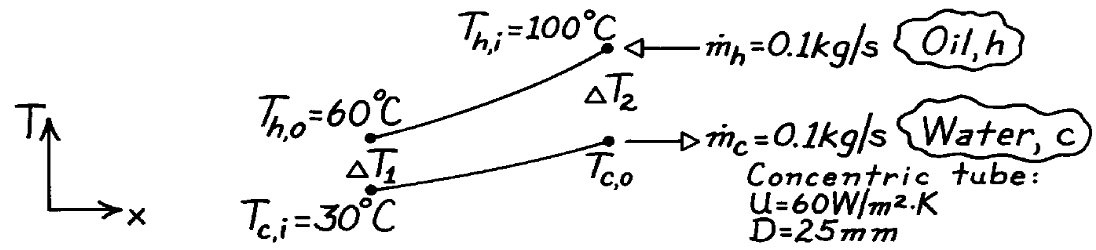
COMMENTS: Over time fouling will always contribute to a degradation of heat exchanger performance. In practice it is desirable to remove fluid contaminants and to implement a regular maintenance (cleaning) procedure.

PROBLEM 11.20

KNOWN: Counterflow concentric tube heat exchanger.

FIND: (a) Total heat transfer rate and outlet temperature of the water and (b) Required length.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance due to tube wall thickness.

PROPERTIES: (given):

	ρ (kg/m^3)	c_p (J/kg·K)	ν (m^2/s)	k (W/m·K)	Pr
Water	1000	4200	7×10^{-7}	0.64	4.7
Oil	800	1900	1×10^{-5}	0.134	140

ANALYSIS: (a) With the outlet temperature, $T_{c,o}$ = 60°C, from an overall energy balance on the hot (oil) fluid, find

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.1\,kg/s \times 1900\,J/kg \cdot K\,(100-60)°C = 7600\ W. \qquad <$$

From an energy balance on the cold (water) fluid, find

$$T_{c,o} = T_{c,i} + q/\dot{m}_c c_c = 30°C + 7600\,W/0.1\,kg/s \times 4200\,J/kg \cdot K = 48.1°C. \qquad <$$

(b) Using the LMTD method, the length of the CF heat exchanger follows from

$$q = UA\Delta T_{lm,CF} = U(p\,DL)\Delta T_{lm,CF} \qquad L = q/U(p\,D)\Delta T_{lm,CF}$$

where

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(60-30)℃ - (100-48.1)℃}{\ln(30/51.9)} = 40.0°C$$

$$L = 7600\,W/60\,W/m^2 \cdot K\,(p \times 0.025m) \times 40.0℃ = 40.3m. \qquad <$$

COMMENTS: Using the ε-NTU method, find $C_{min} = C_h$ = 190 W/K and $C_{max} = C_c$ = 420 W/K. Hence

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 190\ W/K(100-30)K = 13{,}300\ W$$

and $\varepsilon = q/q_{max}$ = 0.571. With $C_r = C_{min}/C_{max}$ = 0.452 and using Eq. 11.30b,

$$NTU = \frac{UA}{C_{min}} = \frac{1}{C_r - 1}\ln\left(\frac{e-1}{eC_r - 1}\right) = \frac{1}{0.452-1}\ln\left(\frac{0.571-1}{0.571 \times 0.452 - 1}\right) = 1.00$$

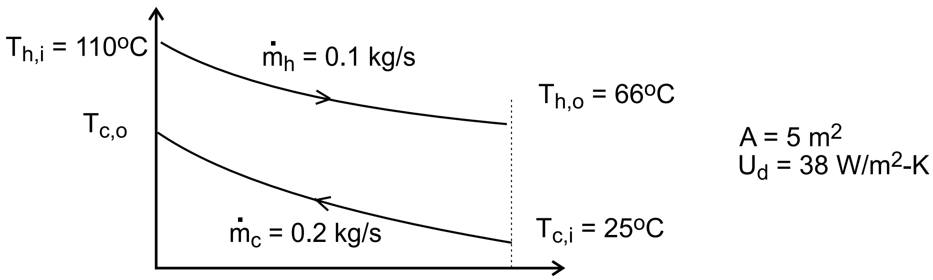
so that with A = πDL, find L = 40.3 m.

PROBLEM 11.21

KNOWN: Counterflow, concentric tube heat exchanger undergoing test after service for an extended period of time; surface area of 5 m^2 and design value for the overall heat transfer coefficient of U_d = 38 $W/m^2 \cdot K$.

FIND: Fouling factor, if any, based upon the test results of engine oil flowing at 0.1 kg/s cooled from 110°C to 66°C by water supplied at 25°C and a flow rate of 0.2 kg/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-5*, Engine oil ($\overline{T}_h$ = 361 K): c = 2166 J/kg·K; *Table A-6*, Water $\left(\overline{T}_c = 304 \text{ K, assuming } T_{c,o} = 36^\circ C\right)$: c = 4178 J/kg·K.

ANALYSIS: For the CF conditions shown in the Schematic, find the heat rate, q, from an energy balance on the hot fluid (oil); the cold fluid outlet temperature, $T_{c,o}$, from an energy balance on the cold fluid (water); the overall coefficient U from the rate equation; and a fouling factor, R, by comparison with the design value, U_d.

Energy balance on hot fluid

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.1 \text{ kg/s} \times 2166 \text{ J/kg} \cdot \text{K} \left(110 - 66\right)\text{K} = 9530 \text{ W}$$

Energy balance on the cold fluid

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right), \qquad \text{find} \quad T_{c,o} = 36.4^\circ \text{C}$$

Rate equation

$$q = UA\Delta T_{\ell n,CF}$$

$$\Delta T_{\ell n,CF} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ell n\left[\left(T_{h,i} - T_{c,o}\right) / \left(T_{h,o} - T_{c,i}\right)\right]} = \frac{(110 - 36.4)^\circ \text{C} - (66 - 25)^\circ \text{C}}{\ell n[73.6 / 41.0]} = 55.7^\circ \text{C}$$

$$9530 \text{ W} = U \times 5 \text{ m}^2 \times 55.7^\circ \text{C}$$

$$U = 34.2 \text{ W/m}^2 \cdot \text{K}$$

Overall resistance including fouling factor

$$U = 1/\left[1/U_d + R''_f\right]$$

$$34.2 \text{ W/m}^2 \cdot \text{K} = 1/\left[1/38 \text{ W/m}^2 \cdot \text{K} + R''_f\right]$$

$$R''_f = 0.0029 \text{ m}^2 \cdot \text{K/W}$$ <