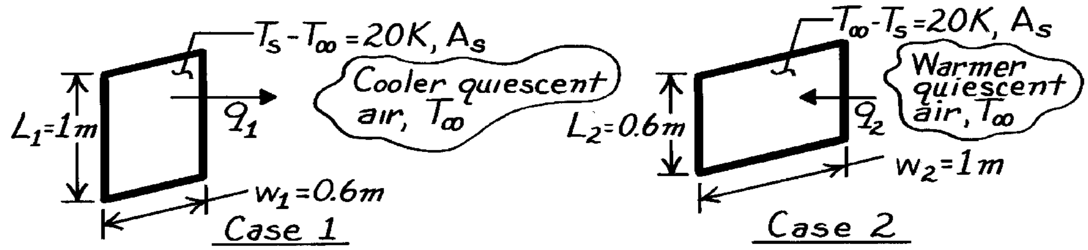
PROBLEM 9.5

KNOWN: Heat transfer rate by convection from a vertical surface, 1m high by 0.6m wide, to quiescent air that is 20K cooler.

FIND: Ratio of the heat transfer rate for the above case to that for a vertical surface that is 0.6m high by 1m wide with quiescent air that is 20K warmer.

SCHEMATIC:



ASSUMPTIONS: (1) Thermophysical properties independent of temperature; evaluate at 300K; (2) Negligible radiation exchange with surroundings, (3) Quiescent ambient air.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, $\alpha = 22.5 \times 10^{-6}\ m^2/s$.

ANALYSIS: The rate equation for convection between the plates and quiescent air is

$$q = \overline{h}_L\ A_s \Delta T \tag{1}$$

where ΔT is either $(T_s - T_\infty)$ or $(T_\infty - T_s)$; for both cases, $A_s = Lw$. The desired heat transfer ratio is then

$$\frac{q_1}{q_2} = \frac{\overline{h}_{L1}}{\overline{h}_{L2}}. \tag{2}$$

To determine the dependence of $\overline{h}_L$ on geometry, first calculate the Rayleigh number,

$$Ra_L = g\ b\ \Delta T L^3 / na \tag{3}$$

and substituting property values at 300K, find,

Case 1: $Ra_{L1} = 9.8\ m/s^2\ (1/300K)\ 20K\ (1m)^3/15.89 \times 10^{-6}\ m^2/s \times 22.5 \times 10^{-6}\ m^2/s = 1.82 \times 10^9$

Case 2: $Ra_{L2} = Ra_{L1}\ (L_2/L_1)^3 = 1.82 \times 10^4\ (0.6m/1.0m)^3 = 3.94 \times 10^8$.

Hence, Case 1 is turbulent and Case 2 is laminar. Using the correlation of Eq. 9.24,

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = C\left(Ra_L\right)^n \qquad \overline{h}_L = \frac{k}{L} C\ Ra_L^n \tag{4}$$

where for *Case 1*: $C_1 = 0.10$, $n_1 = 1/3$ and for *Case 2*: $C_2 = 0.59$, $n_2 = 1/4$. Substituting Eq. (4) into the ratio of Eq. (2) with numerical values, find

$$\frac{q_1}{q_2} = \frac{\left(C_1/L_1\right) Ra_{L1}^{n_1}}{\left(C_2/L_2\right) Ra_{L2}^{n_2}} = \frac{(0.10/1m)\left(1.82 \times 10^9\right)^{1/3}}{(0.59/0.6m)\left(3.94 \times 10^8\right)^{1/4}} = 0.881 \qquad <$$

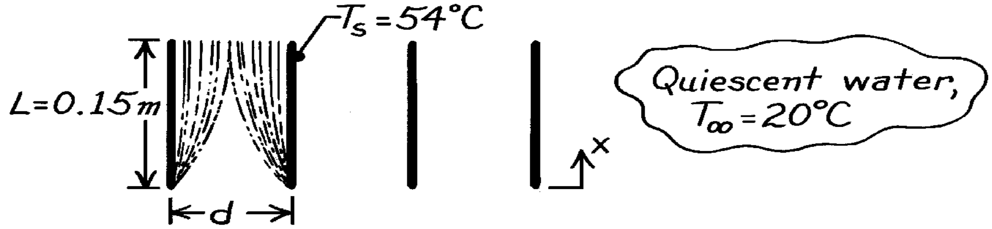
COMMENTS: Is this result to be expected? How do you explain this effect of plate orientation on the heat rates?

PROBLEM 9.7

KNOWN: Thin, vertical plates of length 0.15m at 54°C being cooled in a water bath at 20°C.

FIND: Minimum spacing between plates such that no interference will occur between free-convection boundary layers.

SCHEMATIC:



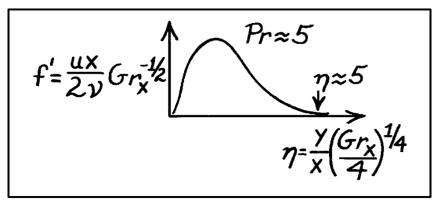
ASSUMPTIONS: (a) Water in bath is quiescent, (b) Plates are at uniform temperature.

PROPERTIES: *Table A-6*, Water ($T_f = (T_s + T_\infty)/2 = (54 + 20)°C/2 = 310K$): $\rho = 1/v_f = 993.05$ kg/m^3, $\mu = 695 \times 10^{-6}$ N·s/m^2, $\nu = \mu/\rho = 6.998 \times 10^{-7}$ m^2/s, Pr = 4.62, $\beta = 361.9 \times 10^{-6}$ K^{-1}.

ANALYSIS: The minimum separation distance will be twice the thickness of the boundary layer at the trailing edge where x = 0.15m. Assuming laminar, free convection boundary layer conditions, the similarity parameter, η, given by Eq. 9.13, is

$$h = \frac{y}{x}\left(Gr_x / 4\right)^{1/4}$$

where y is measured normal to the plate (see Fig. 9.3). According to Fig. 9.4, the boundary layer thickness occurs at a value $\eta \approx 5$.
It follows then that,



$$y_{bl} = h\ x\ \left(Gr_x / 4\right)^{-1/4}$$

where $\quad Gr_x = \dfrac{g\,b\ \left(T_s - T_\infty\right) x^3}{n^2}$

$$Gr_x = 9.8\ \text{m/s}^2 \times 361.9 \times 10^{-6}\ \text{K}^{-1}(54-20)\text{K} \times (0.15\text{m})^3 / \left(6.998 \times 10^{-7}\ \text{m}^2/\text{s}\right)^2 = 8.310 \times 10^8.$$ <

Hence, $y_{bl} = 5 \times 0.15\text{m}\left(8.310 \times 10^8 / 4\right)^{-1/4} = 6.247 \times 10^{-3}\text{m} = 6.3\ \text{mm}$

and the minimum separation is

$$d = 2\ y_{bl} = 2 \times 6.3\ \text{mm} = 12.6\ \text{mm}.$$ <

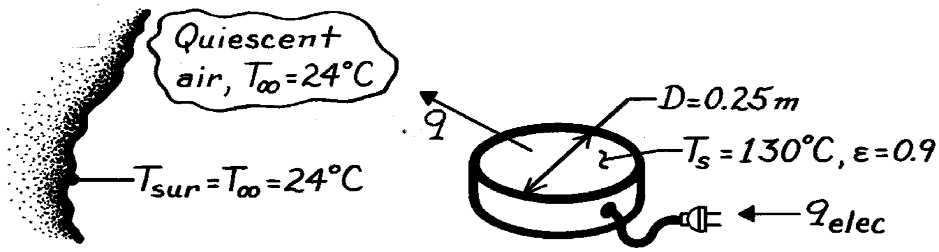
COMMENTS: According to Eq. 9.23, the critical Grashof number for the onset of turbulent conditions in the boundary layer is $Gr_{x,c}\,Pr \approx 10^9$. For the conditions above, $Gr_x\,Pr = 8.31 \times 10^8 \times 4.62 = 3.8 \times 10^9$. We conclude that the boundary layer is indeed turbulent at x = 0.15m and our calculation is only an estimate which is likely to be low. Therefore, the plate separation should be greater than 12.6 mm.

PROBLEM 9.40

KNOWN: Horizontal, circular grill of 0.2m diameter with emissivity 0.9 is maintained at a uniform surface temperature of 130°C when ambient air and surroundings are at 24°C.

FIND: Electrical power required to maintain grill at prescribed surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Room air is quiescent, (2) Surroundings are large compared to grill surface.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 = (24 + 130)°C/2 = 350K$, 1 atm): $\nu = 20.92 \times 10^{-6}\ m^2/s$, $k = 0.030\ W/m\cdot K$, $\alpha = 29.9 \times 10^{-6}\ m^2/s$, $\beta = 1/T_f$.

ANALYSIS: The heat loss from the grill is due to free convection with the ambient air and to radiation exchange with the surroundings.

$$q = A_S\left[\bar{h}\left(T_S - T_\infty\right) + es\left(T_S^4 - T_{sur}^4\right)\right]. \qquad (1)$$

Calculate Ra_L from Eq. 9.25,

$$Ra_L = gb\left(T_S - T_\infty\right)L_c^3/na$$

where for a horizontal disc from Eq. 9.29, $L_c = A_s/P = (\pi D^2/4)/\pi D = D/4$. Substituting numerical values, find

$$Ra_L = \frac{9.8\text{m/s}^2\left(1/350\text{K}\right)\left(130-24\right)\text{K}\left(0.25\text{m}/4\right)^3}{20.92\times10^{-6}\text{m}^2/\text{s}\times29.9\times10^{-6}\text{m}^2/\text{s}} = 1.158\times10^6.$$

Since the grill is an *upper surface heated*, Eq. 9.30 is the appropriate correlation,

$$\overline{Nu}_L = \bar{h}_L L_c/k = 0.54 Ra_L^{1/4} = 0.54\left(1.158\times10^6\right)^{1/4} = 17.72$$

$$\bar{h}_L = \overline{Nu}_L\,k/L_c = 17.72\times0.030\text{W/m}\cdot\text{K}/\left(0.25\text{m}/4\right) = 8.50\text{W/m}^2\cdot\text{K}. \qquad (2)$$

Substituting from Eq. (2) for $\bar{h}$ into Eq. (1), the heat loss or required electrical power, q_{elec}, is

$$q = \frac{p}{4}\left(0.25\text{m}\right)^2\left[8.50\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(130-24\right)\text{K} + 0.9\times5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left(\left(130+273\right)^4 - \left(24+273\right)^4\right)\text{K}^4\right]$$

$$q = 44.2\text{W} + 46.0\text{W} = 90.2\text{W}.$$

<

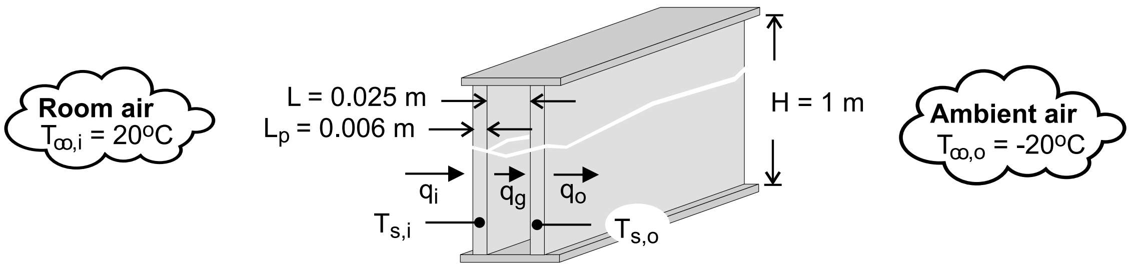
COMMENTS: Note that for this situation, free convection and radiation modes are of equal importance. If the grill were highly polished such that $\varepsilon \approx 0.1$, the required power would be reduced by nearly 50%.

PROBLEM 9.96

KNOWN: Dimensions of double pane window. Thickness of air gap. Temperatures of room and ambient air.

FIND: (a) Temperatures of glass panes and heat rate through window, (b) Resistance of glass pane relative to smallest convection resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible glass pane thermal resistance, (3) Constant properties.

PROPERTIES: *Table A-3*, Plate glass: k_p = 1.4 W/m·K. *Table A-4*, Air (p = 1 atm). $T_{f,i}$ = 287.6K: $\nu_i = 14.8 \times 10^{-6}$ m^2/s, k_i = 0.0253 W/m·K, $\alpha_i = 20.9 \times 10^{-6}$ m^2/s, Pr_i = 0.710, β_i = 0.00348 K^{-1}. $\overline{T}$ = $(T_{s,i} + T_{s,o})/2$ = 272.8K: $\nu = 13.49 \times 10^{-6}$ m^2/s, k = 0.0241 W/m·K, $\alpha = 18.9 \times 10^{-6}$ m^2/s, Pr = 0.714, β = 0.00367 K^{-1}. $T_{f,o}$ = 258.2K: $\nu_o = 12.2 \times 10^{-6}$ m^2/s, k_o = 0.0230W/m·K, $\alpha = 17.0 \times 10^{-6}$ m^2/s, Pr = 0.718, β_o = 0.00387 K^{-1}.

ANALYSIS: (a) The heat rate may be expressed as

$$q = q_o = \overline{h}_o H^2 \left(T_{s,o} - T_{\infty,o}\right) \tag{1}$$

$$q = q_g = \overline{h}_g H^2 \left(T_{s,i} - T_{s,o}\right) \tag{2}$$

$$q = q_i = \overline{h}_i H^2 \left(T_{\infty,i} - T_{s,i}\right) \tag{3}$$

where $\overline{h}_o$ and $\overline{h}_i$ may be obtained from Eq. (9.26),

$$\overline{Nu}_H = \left\{0.825 + \frac{0.387\,Ra_H^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

with $Ra_H = g\beta_o\left(T_{s,o} - T_{\infty,o}\right)H^3/\alpha_o\nu_o$ and $Ra_H = g\beta_i\left(T_{\infty,i} - T_{s,i}\right)H^3/\alpha_i\nu_i$, respectively. Assuming $10^4 < Ra_L < 10^7$, $\overline{h}_g$ is obtained from

$$\overline{Nu}_L = 0.42\,Ra_L^{1/4}\,Pr^{0.012}\left(H/L\right)^{-0.3}$$

where $Ra_L = g\beta\left(T_{s,i} - T_{s,o}\right)L^3/\alpha\nu$. A simultaneous solution to Eqs. (1) – (3) for the three unknowns yields

Continued …..

$$T_{s,i} = 9.1°C, \quad T_{s,o} = -9.6°C, q = 35.7\ W \qquad <$$

where $\bar{h}_i = 3.29\,W/m^2 \cdot K$, $\bar{h}_o = 3.45\,W/m^2 \cdot K$ and $\bar{h}_g = 1.90\,W/m^2 \cdot K$.

(b) The unit conduction resistance of a glass pane is $R''_{cond} = L_p/k_p = 0.00429\,m^2 \cdot K/W$, and the smallest convection resistance is $R''_{conv,o} = \left(1/\bar{h}_o\right) = 0.290\,m^2 \cdot K/W$. Hence,

$$R''_{cond} << R''_{conv,min} \qquad <$$

and it is reasonable to neglect the thermal resistance of the glass.

COMMENTS: (1) Assuming a heat flux of 35.7 W/m^2 through a glass pane, the corresponding temperature difference across the pane is $\Delta T = q''\left(L_p/k_p\right) = 0.15°C$. Hence, the assumption of an isothermal pane is good. (2) Equations (1) – (3) were solved using the IHT workspace and the temperature-dependent air properties provided by the software. The property values provided in the PROPERTIES section of this solution were obtained from the software.