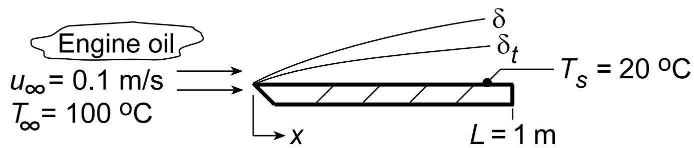
PROBLEM 7.2

KNOWN: Temperature and velocity of engine oil. Temperature and length of flat plate.

FIND: (a) Velocity and thermal boundary layer thickness at trailing edge, (b) Heat flux and surface shear stress at trailing edge, (c) Total drag force and heat transfer per unit plate width, and (d) Plot the boundary layer thickness and local values of the shear stress, convection coefficient, and heat flux as a function of x for $0 \le x \le 1$ m.

SCHEMATIC:



ASSUMPTIONS: (1) Critical Reynolds number is 5×10^5, (2) Flow over top and bottom surfaces.

PROPERTIES: *Table A.5*, Engine Oil (T_f = 333 K): $\rho = 864\ \mathrm{kg/m^3}$, $\nu = 86.1 \times 10^{-6}\ \mathrm{m^2/s}$, k = 0.140 W/m·K, Pr = 1081.

ANALYSIS: (a) Calculate the Reynolds number to determine nature of the flow,

$$\mathrm{Re_L} = \frac{u_\infty L}{\nu} = \frac{0.1\,\mathrm{m/s} \times 1\,\mathrm{m}}{86.1 \times 10^{-6}\,\mathrm{m^2/s}} = 1161$$

Hence the flow is laminar at x = L, from Eqs. 7.19 and 7.24, and

$$\delta = 5L\,\mathrm{Re}_L^{-1/2} = 5(1\,\mathrm{m})(1161)^{-1/2} = 0.147\,\mathrm{m} \qquad <$$

$$\delta_t = \delta\,\mathrm{Pr}^{-1/3} = 0.147\,\mathrm{m}(1081)^{-1/3} = 0.0143\,\mathrm{m} \qquad <$$

(b) The local convection coefficient, Eq. 7.23, and heat flux at x = L are

$$h_L = \frac{k}{L} 0.332\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} = \frac{0.140\,\mathrm{W/m \cdot K}}{1\mathrm{m}} 0.332(1161)^{1/2}(1081)^{1/3} = 16.25\,\mathrm{W/m^2 \cdot K}$$

$$q''_x = h_L(T_s - T_\infty) = 16.25\,\mathrm{W/m^2 \cdot K}(20-100)^\circ\mathrm{C} = -1300\,\mathrm{W/m^2} \qquad <$$

Also, the local shear stress is, from Eq. 7.20,

$$\tau_{s,L} = \frac{\rho u_\infty^2}{2} 0.664\,\mathrm{Re}_L^{-1/2} = \frac{864\,\mathrm{kg/m^3}}{2}(0.1\,\mathrm{m/s})^2\,0.664(1161)^{-1/2}$$

$$\tau_{s,L} = 0.0842\,\mathrm{kg/m \cdot s^2} = 0.0842\,\mathrm{N/m^2} \qquad <$$

(c) With the drag force per unit width given by $D' = 2L\bar{\tau}_{s,L}$ where the factor of 2 is included to account for both sides of the plate, it follows that

$$D' = 2L\left(\rho u_\infty^2/2\right)1.328\,\mathrm{Re}_L^{-1/2} = (1\mathrm{m})864\,\mathrm{kg/m^3}(0.1\,\mathrm{m/s})^2/2\ 1.328(1161)^{-1/2} = 0.337\,\mathrm{N/m} \qquad <$$

For laminar flow, the average value $\bar{h}_L$ over the distance 0 to L is twice the local value, h_L,

$$\bar{h}_L = 2h_L = 32.5\,\mathrm{W/m^2 \cdot K}$$

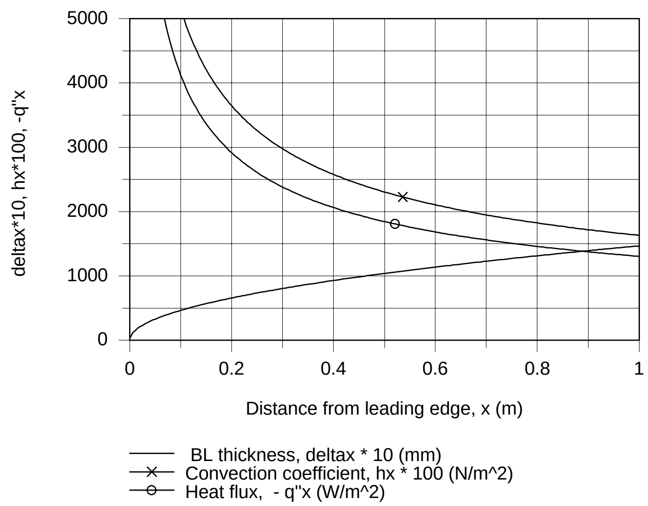
The total heat transfer rate per unit width of the plate is

$$q' = 2L\bar{h}_L(T_s - T_\infty) = 2(1\mathrm{m})32.5\,\mathrm{W/m^2 \cdot K}(20-100)^\circ\mathrm{C} = -5200\,\mathrm{W/m} \qquad <$$

Continued...

PROBLEM 7.2 (Cont.)

(c) Using IHT with the foregoing equations, the boundary layer thickness, and local values of the convection coefficient and heat flux were calculated and plotted as a function of x.



COMMENTS: (1) Note that since Pr >> 1, $\delta \gg \delta_t$. That is, for the high Prandtl liquids, the velocity boundary layer will be much thicker than the thermal boundary layer.

(2) A copy of the *IHT Workspace* used to generate the above plot is shown below.

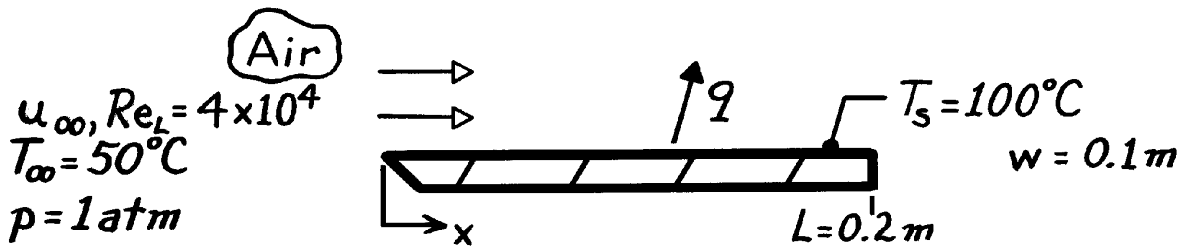
```
// Boundary layer thickness, delta
delta = 5 * x * Rex ^-0.5
delta_mm = delta * 1000
delta_plot = delta_mm * 10          // Scaling parameter for convenience in  plotting

// Convection coefficient and heat flux, q''x
q''x = hx * (Ts - Tinf)
Nux = 0.332 * Rex^0.5 * Pr^(1/3)
Nux = hx * x / k
hx_plot = 100 * hx                  // Scaling parameter for convenience in plotting
q''x_plot = ( -1 ) * q''x           // Scaling parameter for convenience in plotting

// Reynolds number
Rex = uinf * x / nu

// Properties Tool: Engine oil
// Engine Oil property functions : From Table A.5
// Units: T(K)
rho = rho_T("Engine Oil",Tf)            // Density, kg/m^3
cp = cp_T("Engine Oil",Tf)              // Specific heat, J/kg·K
nu = nu_T("Engine Oil",Tf)              // Kinematic viscosity, m^2/s
k = k_T("Engine Oil",Tf)                // Thermal conductivity, W/m·K
Pr = Pr_T("Engine Oil",Tf)              // Prandtl number


// Assigned variables
Tf = (Ts + Tinf) / 2                    // Film temperature, K
Tinf = 100 + 273                        // Freestream temperature, K
Ts = 20 + 273                           // Surface temperature, K
uinf = 0.1                              // Freestream velocity, m/s
x = 1                                   // Plate length, m
```

PROBLEM 7.15

KNOWN: Temperature, pressure and Reynolds number for air flow over a flat plate of uniform surface temperature.

FIND: (a) Rate of heat transfer from the plate, (b) Rate of heat transfer if air velocity is doubled and pressure is increased to 10 atm.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation, (4) $Re_{x_c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): k = 0.0299 W/m·K, Pr = 0.70.

ANALYSIS: (a) The heat rate is

$$q = \overline{h}_L (w\times L) \ (T_s - T_\infty).$$

Since the flow is laminar over the entire plate for $Re_L = 4\times10^4$, it follows that

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664(40{,}000)^{1/2}(0.70)^{1/3} = 117.9.$$

Hence $$\overline{h}_L = 117.9\frac{k}{L} = 117.9\frac{0.0299\ W/m\cdot K}{0.2m} = 17.6\ W/m^2\cdot K$$

and $$q = 17.6\frac{W}{m^2\cdot K}(0.1m\times0.2m)\ (100-50)^\circ C = 17.6\ W.$$ <

(b) With $p_2 = 10\ p_1$, it follows that $\rho_2 = 10\ \rho_1$ and $\nu_2 = \nu_1/10$. Hence

$$Re_{L,2} = \left(\frac{u_\infty L}{n}\right)_2 = 2\times10\left(\frac{u_\infty L}{n}\right)_1 = 20\ Re_{L,1} = 8\times10^5$$

and mixed boundary layer conditions exist on the plate. Hence

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = \left(0.037\ Re_L^{4/5} - 871\right)\ Pr^{1/3} = \left[0.037\times\left(8\times10^5\right)^{4/5} - 871\right]\ (0.70)^{1/3}$$

$$\overline{Nu}_L = 961.$$

Hence, $$\overline{h}_L = 961\frac{0.0299\ W/m\cdot K}{0.2m} = 143.6\ W/m^2\cdot K$$

$$q = 143.6\frac{W}{m^2\cdot K}(0.1m\times0.2m)\ (100-50)^\circ C = 143.6\ W.$$ <

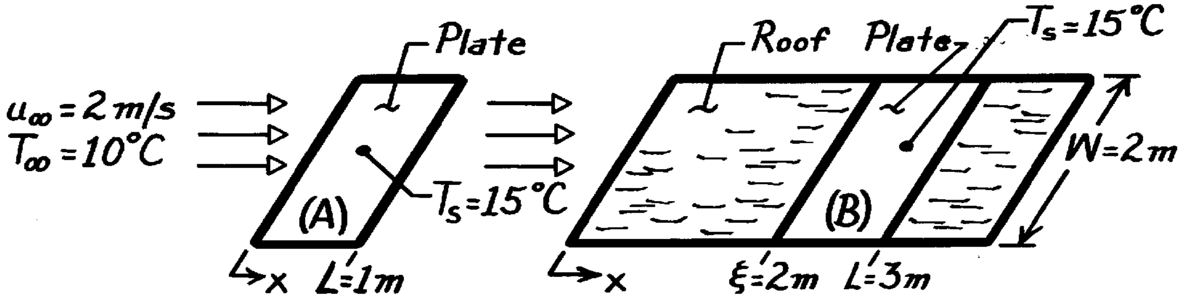
COMMENTS: Note that, in calculating $Re_{L,2}$, ideal gas behavior has been assumed. It has also been assumed that k, μ and Pr are independent of pressure over the range considered.

PROBLEM 7.37

KNOWN: Cover plate dimensions and temperature for flat plate solar collector. Air flow conditions.

FIND: (a) Heat loss with simultaneous velocity and thermal boundary layer development, (b) Heat loss with unheated starting length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Boundary layer is not disturbed by roof-plate interface, (4) $Re_{x,c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 285.5K, 1 atm): $\nu = 14.6 \times 10^{-6}\ m^2/s$, k = 0.0251 W/m·K, Pr = 0.71.

ANALYSIS: (a) The Reynolds number for the plate of L = 1m is

$$Re_L = \frac{u_\infty L}{n} = \frac{2\ m/s \times 1m}{14.6\times10^{-6}\ m^2/s} = 1.37\times10^5 < Re_{x,c}.$$

For laminar flow

$$\overline{Nu}_L = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664\left(1.37\times10^5\right)^{1/2}(0.71)^{1/3} = 219.2$$

$$q = \frac{k}{L}\overline{Nu}_L\ A_s\left(T_s - T_\infty\right) = \frac{0.0251\ W/m\cdot K}{1m}219.2\left(2m^2\right)5°C = 55\ W.$$ $<$

(b) The Reynolds number for the roof and collector of length L = 3m is

$$Re_L = \frac{2\ m/s\times3m}{14.6\times10^{-6}\ m^2/s} = 4.11\times10^5 < Re_{x,c}.$$

Hence, laminar boundary layer conditions exist throughout and the heat rate is

$$q = \int_x^L q''\,dA = \left(T_s - T_\infty\right)0.332\left(\frac{u_\infty}{n}\right)^{1/2} Pr^{1/3} kW \int_x^L \frac{x^{-1/2}dx}{\left[1-\left(x/x\right)^{3/4}\right]^{1/3}}$$

$$q = \left(5°C\right)0.332\left(\frac{2\ m/s}{14.6\times10^{-6}\ m^2/s}\right)^{1/2}(0.71)^{1/3}\ 0.0251\frac{W}{m\cdot K}2m\int_x^L \frac{x^{-1/2}dx}{\left[1-\left(x/x\right)^{3/4}\right]^{1/3}}$$

Using a numerical technique to evaluate the integral,

$$q = 27.50\int_2^3 \frac{x^{-1/2}dx}{\left[1-\left(2.0/x\right)^{3/4}\right]^{1/3}} = 27.50\times1.417 = 39\ W$$ $<$

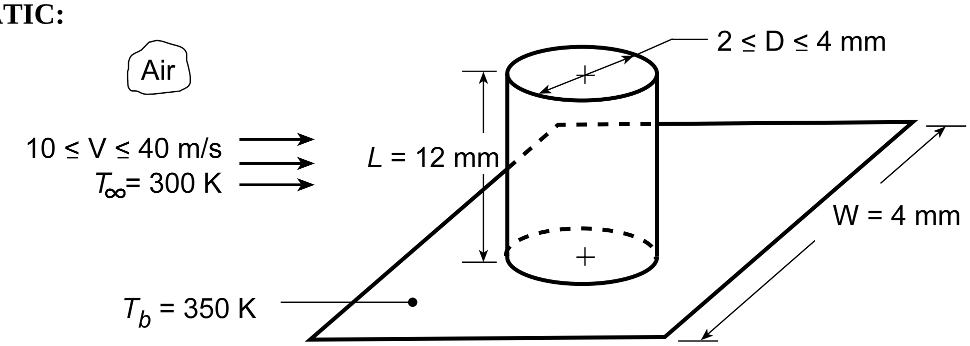
COMMENTS: Values of $\bar{h}$ with and without the unheated starting length are 3.9 and 5.5 $W/m^2\cdot K$. Prior development of the velocity boundary layer decreases $\bar{h}$.

PROBLEM 7.47

KNOWN: Dimensions of chip and pin fin. Chip temperature. Free stream velocity and temperature of air coolant.

FIND: (a) Average pin convection coefficient, (b) Pin heat transfer rate, (c) Total heat rate, (d) Effect of velocity and pin diameter on total heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pin, (3) Constant properties, (4) Convection coefficients on pin surface (tip and side) and chip surface correspond to single cylinder in cross flow, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Copper (350 K): k = 399 W/m·K; *Table A.4*, Air ($T_f \approx 325$ K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.704.

ANALYSIS: (a) With V = 10 m/s and D = 0.002 m,

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.002\,m}{18.41 \times 10^{-6}\,m^2/s} = 1087$$

Using the Churchill and Bernstein correlations, Eq. (7.57),

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 16.7$$

$$\bar{h} = \left(\overline{Nu}_D k/D\right) = (16.7 \times 0.0282\,W/m\cdot K/0.002\,m) = 235\,W/m^2\cdot K$$ <

(b) For the fin with tip convection and

$$M = \left(\bar{h}\pi D k \pi D^2/4\right)^{1/2}\theta_b = (\pi/2)\left[235\,W/m^2\cdot K\,(0.002\,m)^3\,399\,W/m\cdot K\right]^{1/2} 50\,K = 2.15\,W$$

$$m = \left(\bar{h}P/kA_c\right)^{1/2} = \left(4 \times 235\,W/m^2\cdot K/399\,W/m\cdot K \times 0.002\,m\right)^{1/2} = 34.3\,m^{-1}$$

$$mL = 34.3\,m^{-1}(0.012\,m) = 0.412$$

$$\left(\bar{h}/mk\right) = \left(235\,W/m^2\cdot K/34.3\,m^{-1} \times 399\,W/m\cdot K\right) = 0.0172.$$

The fin heat rate is

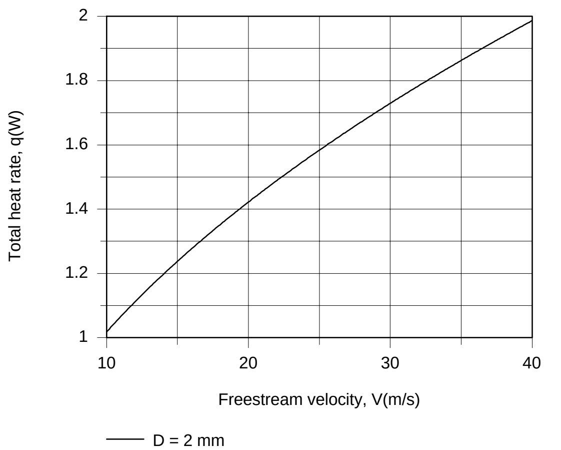
$$q_f = M\frac{\sinh mL + \left(\bar{h}/mk\right)\cosh mL}{\cosh mL + \left(\bar{h}/mk\right)\sinh mL} = 0.868\,W.$$ <

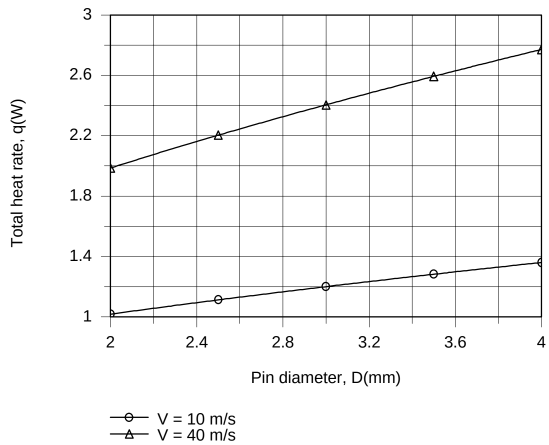
Continued...

(c) The total heat rate is that from the base and through the fin,

$$q = q_b + q_f = \bar{h}\left(W^2 - \pi D^2/4\right)\theta_b + q_f = (0.151 + 0.868)\,W = 1.019\,W\,. \qquad <$$

(d) Using the IHT Extended Surface Model for a Pin Fin with the Correlations Tool Pad for a Cylinder in crossflow and Properties Tool Pad for Air, the following results were generated.





Clearly, there is significant benefit associated with increasing V which increases the convection coefficient and the total heat rate. Although the convection coefficient decreases with increasing D, the increase in the total heat transfer surface area is sufficient to yield an increase in q with increasing D. The maximum heat rate is q = 2.77 W for V = 40 m/s and D = 4 mm.

COMMENTS: Radiation effects should be negligible, although tip and base convection coefficients will differ from those calculated in parts (a) and (d).