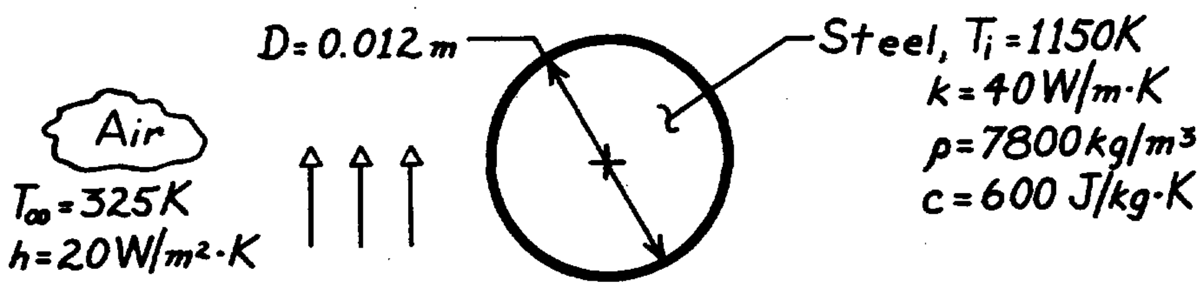
PROBLEM 5.5

KNOWN: Diameter and initial temperature of steel balls cooling in air.

FIND: Time required to cool to a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation effects, (2) Constant properties.

ANALYSIS: Applying Eq. 5.10 to a sphere ($L_c = r_o/3$),

$$Bi = \frac{hL_c}{k} = \frac{h\left(r_o/3\right)}{k} = \frac{20\ W/m^2 \cdot K\ \left(0.002m\right)}{40\ W/m \cdot K} = 0.001.$$

Hence, the temperature of the steel remains approximately uniform during the cooling process, and the lumped capacitance method may be used. From Eqs. 5.4 and 5.5,

$$t = \frac{r V c_p}{hA_s} \ln\frac{T_i - T_\infty}{T - T_\infty} = \frac{r\left(p D^3/6\right)c_p}{hp\ D^2} \ln\frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{7800 kg/m^3 \left(0.012m\right) 600 J/kg \cdot K}{6 \times 20\ W/m^2 \cdot K} \ln\frac{1150 - 325}{400 - 325}$$

$$t = 1122\ s\ = 0.312h$$ <

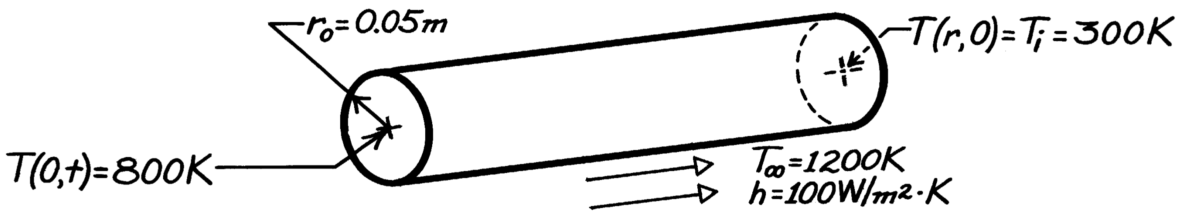
COMMENTS: Due to the large value of T_i, radiation effects are likely to be significant during the early portion of the transient. The effect is to shorten the cooling time.

PROBLEM 5.9

KNOWN: Diameter and radial temperature of AISI 1010 carbon steel shaft. Convection coefficient and temperature of furnace gases.

FIND: Time required for shaft centerline to reach a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

PROPERTIES: AISI 1010 carbon steel, *Table A.1* $\left(\bar{T} = 550\text{ K}\right)$: $r = 7832\text{ kg/m}^3$, k = 51.2 W/m·K, c = 541 J/kg·K, $\alpha = 1.21\times10^{-5}\text{ m}^2\text{/s}$.

ANALYSIS: The Biot number is

$$\text{Bi} = \frac{hr_o/2}{k} = \frac{100\text{ W/m}^2\cdot\text{K}\left(0.05\text{ m/2}\right)}{51.2\text{ W/m}\cdot\text{K}} = 0.0488.$$

Hence, the lumped capacitance method can be applied. From Equation 5.6,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hAs}{r\,Vc}\right)t\right] = \exp\left[-\frac{4h}{r\,cD}t\right]$$

$$\ln\left(\frac{800-1200}{300-1200}\right) = -0.811 = -\frac{4\times100\text{ W/m}^2\cdot\text{K}}{7832\text{ kg/m}^3\left(541\text{ J/kg}\cdot\text{K}\right)0.1\text{ m}}t$$

$$t = 859\text{ s.}$$ <

COMMENTS: To check the validity of the foregoing result, use the one-term approximation to the series solution. From Equation 5.49c,

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{-400}{-900} = 0.444 = C_1\exp\left(-V_1^2\text{Fo}\right)$$

For Bi = hr_o/k = 0.0976, Table 5.1 yields ς_1 = 0.436 and C_1 = 1.024. Hence

$$\frac{-\left(0.436\right)^2\left(1.2\times10^{-5}\text{ m}^2/\text{s}\right)}{\left(0.05\text{ m}\right)^2}t = \ln\left(0.434\right) = -0.835$$

$$t = 915\text{ s.}$$

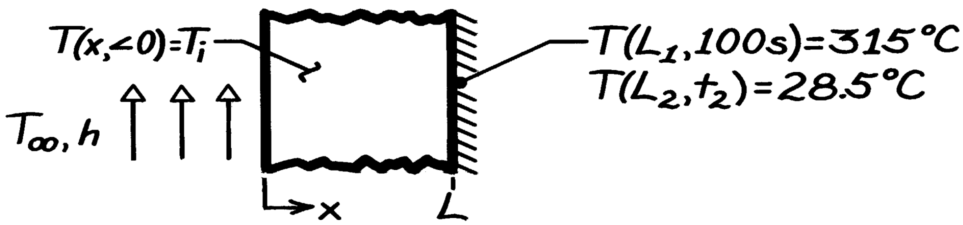
The results agree to within 6%. The lumped capacitance method underestimates the actual time, since the response at the centerline lags that at any other location in the shaft.

PROBLEM 5.31

KNOWN: One-dimensional wall, initially at a uniform temperature, T_i, is suddenly exposed to a convection process (T_∞, h). For wall #1, the time (t_1 = 100s) required to reach a specified temperature at x = L is prescribed, $T(L_1, t_1)$ = 315°C.

FIND: For wall #2 of different thickness and thermal conditions, the time, t_2, required for $T(L_2, t_2)$ = 28°C.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: The properties, thickness and thermal conditions for the two walls are:

Wall	L(m)	$\alpha(m^2/s)$	k(W/m·K)	T_i(°C)	T_∞(°C)	$h(W/m^2{\cdot}K)$
1	0.10	15×10^{-6}	50	300	400	200
2	0.40	25×10^{-6}	100	30	20	100

The dimensionless functional dependence for the one-dimensional, transient temperature distribution, Eq. 5.38, is

$$q^* = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = f\left(x^*, \text{Bi}, \text{Fo}\right)$$

where

$$x^* = x/L \qquad \text{Bi} = hL/k \qquad \text{Fo} = a\,t/L^2.$$

If the parameters x*, Bi, and Fo are the same for both walls, then $q_1^* = q_2^*$. Evaluate these parameters:

Wall	x*	Bi	Fo	θ*
1	1	0.40	0.150	0.85
2	1	0.40	$1.563\times10^{-4}\ t_2$	0.85

where

$$q_1^* = \frac{315-400}{300-400} = 0.85 \qquad q_2^* = \frac{28.5-20}{30-20} = 0.85.$$

It follows that

$$\text{Fo}_2 = \text{Fo}_1 \qquad 1.563\times10^{-4} t_2 = 0.150$$

$$t_2 = 960\text{s}.$$

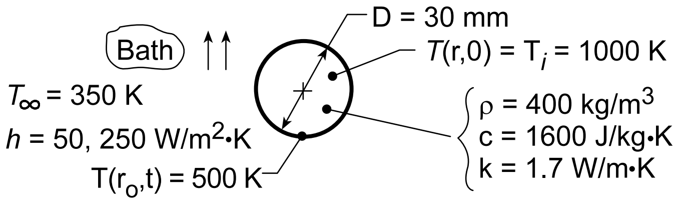
<

PROBLEM 5.49

KNOWN: A long cylinder, initially at a uniform temperature, is suddenly quenched in a large oil bath.

FIND: (a) Time required for the surface to reach 500 K, (b) Effect of convection coefficient on surface temperature history.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Fo > 0.2.

ANALYSIS: (a) Check first whether lumped capacitance method is applicable. For h = 50 W/m^2·K,

$$\mathrm{Bi_c} = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{50\,\mathrm{W/m^2 \cdot K}(0.015\,\mathrm{m}/2)}{1.7\,\mathrm{W/m \cdot K}} = 0.221.$$

Since $\mathrm{Bi_c} > 0.1$, method is not suited. Using the approximate series solution for the infinite cylinder,

$$\theta^*(r^*, \mathrm{Fo}) = C_1 \exp\left(-\zeta_1^2 \mathrm{Fo}\right) \times J_0\left(\zeta_1 r^*\right) \tag{1}$$

Solving for Fo and setting $r^* = 1$, find

$$\mathrm{Fo} = -\frac{1}{\zeta_1^2} \ln\left[\frac{\theta^*}{C_1 J_0(\zeta_1)}\right]$$

where $\theta^* = (1, \mathrm{Fo}) = \dfrac{T(r_o, t_o) - T_\infty}{T_i - T_\infty} = \dfrac{(500 - 350)\mathrm{K}}{(1000 - 350)\mathrm{K}} = 0.231.$

From Table 5.1, with Bi = 0.441, find $\zeta_1 = 0.8882$ rad and $C_1 = 1.1019$. From Table B.4, find $J_0(\zeta_1) = 0.8121$. Substituting numerical values into Eq. (2),

$$\mathrm{Fo} = -\frac{1}{(0.8882)^2} \ln\left[0.231/1.1019 \times 0.8121\right] = 1.72.$$

From the definition of the Fourier number, Fo = $\alpha t / r_o^2$, and $\alpha = k/\rho c$,

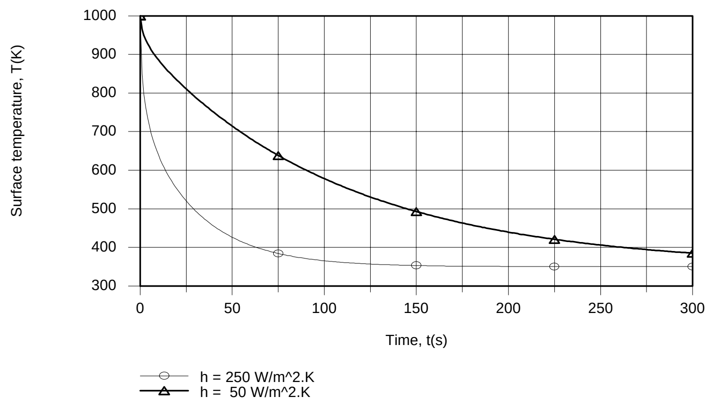
$$t = \mathrm{Fo}\frac{r_o^2}{\alpha} = \mathrm{Fo} \cdot r_o^2 \frac{\rho c}{k}$$

$$t = 1.72(0.015\,\mathrm{m})^2 \times 400\,\mathrm{kg/m^3} \times 1600\,\mathrm{J/kg \cdot K} / 1.7\,\mathrm{W/m \cdot K} = 145\mathrm{s}. \qquad <$$

(b) Using the IHT *Transient Conduction Model* for a *Cylinder*, the following surface temperature histories were obtained.

Continued...

PROBLEM 5.49 (Cont.)



Increasing the convection coefficient by a factor of 5 has a significant effect on the surface temperature, greatly accelerating its approach to the oil temperature. However, even with h = 250 $W/m^2 \cdot K$, Bi = 1.1 and the convection resistance remains significant. Hence, in the interest of accelerated cooling, additional benefit could be achieved by further increasing the value of h.

COMMENTS: For Part (a), note that, since Fo = 1.72 > 0.2, the approximate series solution is appropriate.