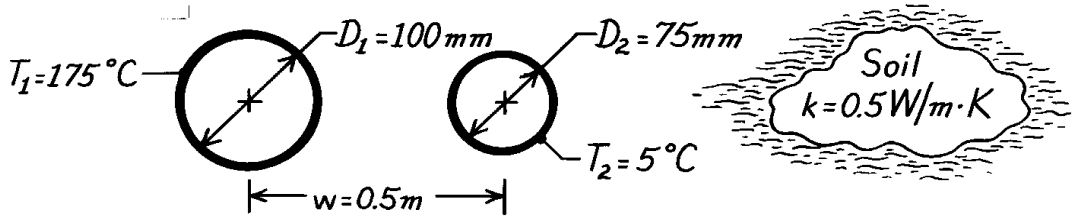
PROBLEM 4.18

KNOWN: Surface temperatures of two parallel pipe lines buried in soil.

FIND: Heat transfer per unit length between the pipe lines.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) Pipe lines are buried very deeply, approximating burial in an infinite medium, (5) Pipe length >> D_1 or D_2 and w > D_1 or D_2.

ANALYSIS: The heat transfer rate per unit length from the hot pipe to the cool pipe is

$$q' = \frac{q}{L} = \frac{S}{L} k \left(T_1 - T_2\right).$$

The shape factor S for this configuration is given in Table 4.1 as

$$S = \frac{2\pi L}{\cosh^{-1}\left[\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1 D_2}\right]}.$$

Substituting numerical values,

$$\frac{S}{L} = 2\pi \,/\cosh^{-1}\left[\frac{4\times(0.5\text{m})^2 - (0.1\text{m})^2 - (0.075\text{m})^2}{2\times 0.1\text{m}\times 0.075\text{m}}\right] = 2\pi \,/\cosh^{-1}(65.63)$$

$$\frac{S}{L} = 2\pi \,/4.88 = 1.29.$$

Hence, the heat rate per unit length is

$$q' = 1.29\times 0.5\text{W/m}\cdot\text{K}\left(175-5\right)^{\circ}\text{C} = 110\text{ W/m}. \qquad <$$

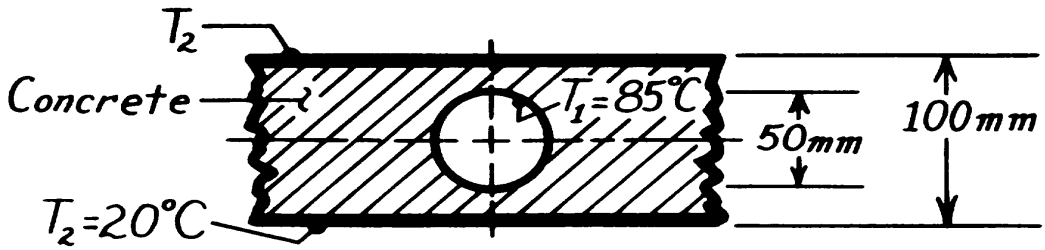
COMMENTS: The heat gain to the cooler pipe line will be larger than 110 W/m if the soil temperature is greater than 5°C. How would you estimate the heat gain if the soil were at 25°C?

PROBLEM 4.19

KNOWN: Tube embedded in the center plane of a concrete slab.

FIND: (a) The shape factor and heat transfer rate per unit length using the appropriate tabulated relation, (b) Shape factor using flux plot method.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Concrete slab infinitely long in horizontal plane, L >> z.

PROPERTIES: *Table A-3*, Concrete, stone mix (300K): k = 1.4 W/m·K.

ANALYSIS: (a) If we relax the restriction that z >> D/2, the embedded tube-slab system corresponds to the fifth case of Table 4.1. Hence,

$$S = \frac{2p\,L}{\ell n\left(8z/p\ D\right)}$$

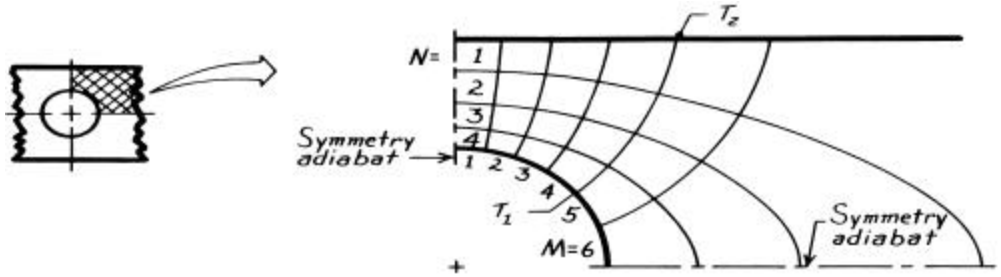
where L is the length of the system normal to the page, z is the half-thickness of the slab and D is the diameter of the tube. Substituting numerical values, find

$$S = 2p\ L/\ell n\left(8\times 50\text{mm}/p\ 50\text{mm}\right) = 6.72L.$$

Hence, the heat rate per unit length is

$$q' = \frac{q}{L} = \frac{S}{L}k\left(T_1 - T_2\right) = 6.72\times 1.4\,\frac{W}{m\cdot K}\left(85-20\right)^{\circ}C = 612\ W.$$

(b) To find the shape factor using the flux plot method, first identify the symmetrical section bounded by the symmetry adiabats formed by the horizontal and vertical center lines. Selecting four temperature increments (N = 4), the flux plot can be constructed.



From Eq. 4.26, the shape factor of the symmetrical section is

$$S_o = ML/N = 6L/4 = 1.5L.$$

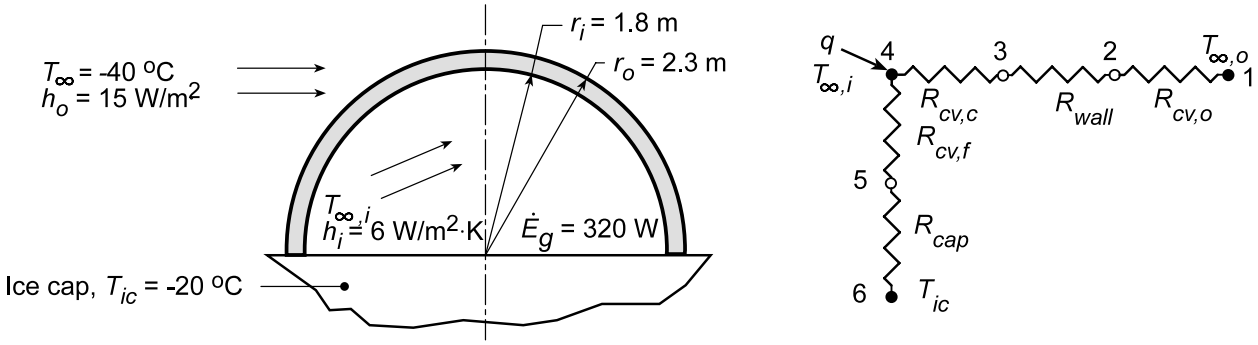
For the tube-slab system, it follows that $S = 4S_o = 6.0L$, which compares favorably with the result obtained from the shape factor relation.

PROBLEM 4.28

KNOWN: Igloo constructed in hemispheric shape sits on ice cap; igloo wall thickness and inside/outside convection coefficients (h_i, h_o) are prescribed.

FIND: (a) Inside air temperature $T_{\infty,i}$ when outside air temperature is $T_{\infty,o}$ = -40°C assuming occupants provide 320 W within igloo, (b) Perform parameter sensitivity analysis to determine which variables have significant effect on T_i.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficient is the same on floor and ceiling of igloo, (3) Floor and ceiling are at uniform temperature, (4) Floor-ice cap resembles disk on semi-infinite medium, (5) One-dimensional conduction through igloo walls.

PROPERTIES: Ice and compacted snow (given): k = 0.15 W/m·K.

ANALYSIS: (a) The thermal circuit representing the heat loss from the igloo to the outside air and through the floor to the ice cap is shown above. The heat loss is

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{cv,c} + R_{wall} + R_{cv,o}} + \frac{T_{\infty,i} - T_{ic}}{R_{cv,f} + R_{cap}}.$$

Convection, ceiling: $$R_{cv,c} = \frac{2}{h_i\left(4\pi r_i^2\right)} = \frac{2}{6\,W/m^2\cdot K \times 4\pi\,(1.8\,m)^2} = 0.00819\,K/W$$

Convection, outside: $$R_{cv,o} = \frac{2}{h_o\left(4\pi r_o^2\right)} = \frac{2}{15\,W/m^2\cdot K \times 4\pi\,(2.3\,m)^2} = 0.00201\,K/W$$

Convection, floor: $$R_{cv,f} = \frac{1}{h_i\left(\pi r_i^2\right)} = \frac{1}{6\,W/m^2\cdot K \times \pi\,(1.8\,m)^2} = 0.01637\,K/W$$

Conduction, wall: $$R_{wall} = 2\left[\frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)\right] = 2\left[\frac{1}{4\pi \times 0.15\,W/m\cdot K}\left(\frac{1}{1.8} - \frac{1}{2.3}\right)m\right] = 0.1281\,K/W$$

Conduction, ice cap: $$R_{cap} = \frac{1}{kS} = \frac{1}{4kr_i} = \frac{1}{4\times 0.15\,W/m\cdot K \times 1.8\,m} = 0.9259\,K/W$$

where S was determined from the shape factor of Table 4.1. Hence,

$$q = 320\,W = \frac{T_{\infty,i} - (-40)^\circ C}{(0.00818 + 0.1281 + 0.0020)\,K/W} + \frac{T_{\infty,i} - (-20)^\circ C}{(0.01637 + 0.9259)\,K/W}$$

$$320\ W = 7.232(T_{\infty,i} + 40) + 1.06(T_{\infty,i} + 20) \qquad T_{\infty,i} = 1.1°C. \quad <$$

Continued...

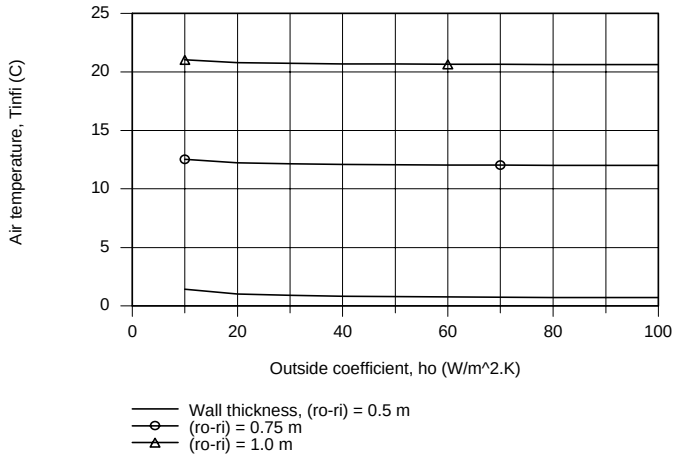
PROBLEM 4.28 (Cont.)

(b) Begin the parameter sensitivity analysis to determine important variables which have a significant influence on the inside air temperature by examining the thermal resistances associated with the processes present in the system and represented by the network.

Process	Symbols		Value (K/W)
Convection, outside	$R_{cv,o}$	R21	0.0020
Conduction, wall	R_{wall}	R32	0.1281
Convection, ceiling	$R_{cv,c}$	R43	0.0082
Convection, floor	$R_{cv,f}$	R54	0.0164
Conduction, ice cap	R_{cap}	R65	0.9259

It follows that the convection resistances are negligible relative to the conduction resistance across the igloo wall. As such, only changes to the wall thickness will have an appreciable effect on the inside air temperature relative to the outside ambient air conditions. We don't want to make the igloo walls thinner and thereby allow the air temperature to dip below freezing for the prescribed environmental conditions.

Using the *IHT Thermal Resistance Network Model*, we used the circuit builder to construct the network and perform the energy balances to obtain the inside air temperature as a function of the outside convection coefficient for selected increased thicknesses of the wall.



COMMENTS: (1) From the plot, we can see that the influence of the outside air velocity which controls the outside convection coefficient h_o is negligible.

(2) The thickness of the igloo wall is the dominant thermal resistance controlling the inside air temperature.

Given

$$\rho C_p \frac{\partial T}{\partial t} = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + \dot{q} \quad (1)$$

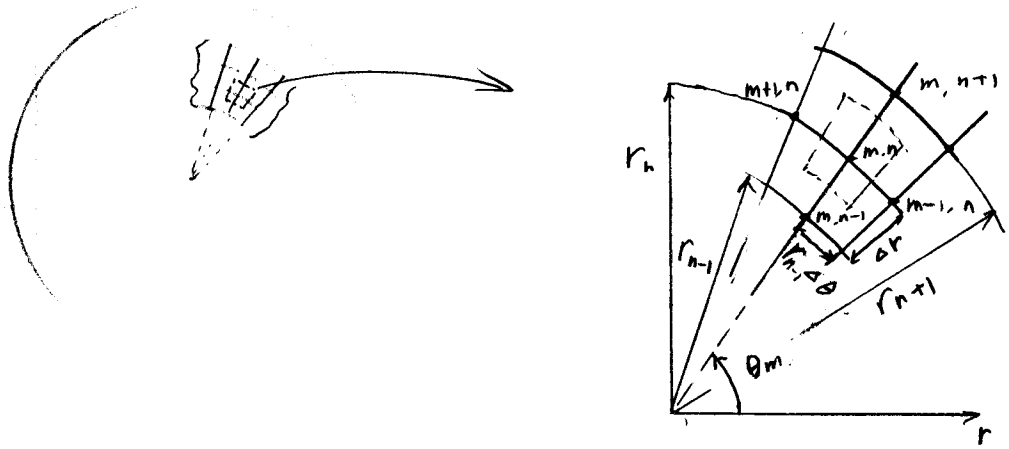
Assume — steady state, $\frac{\partial}{\partial t} = 0$

— no heat generation, $\dot{q} = 0$

— $T = T(r, \theta)$, $\frac{\partial}{\partial z} = 0$.

$$(1) \Rightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0.$$

$$\Rightarrow \quad \frac{\partial T}{\partial r} + r\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial^2 T}{\partial \theta^2} = 0 \quad (2)$$



The grid can be made by drawing a set of concentric circles of equal distance (Δr) and straight lines that pass the center of the disk. The straight lines divide the disk into angular regions of same angular span ($\Delta\theta$). Thus

$$r_{n+1} - r_n = r_n - r_{n-1} = \Delta r$$

$$\theta_{m+1} - \theta_m = \theta_m - \theta_{m-1} = \Delta\theta$$

Discretization:

$$\left.\frac{\partial T}{\partial r}\right|_{\theta_m, r_n} \approx \frac{T_{m,n+\frac{1}{2}} - T_{m,n-\frac{1}{2}}}{\Delta r} \approx \frac{\frac{T_{m,n+1}+T_{m,n}}{2} - \frac{T_{m,n}+T_{m,n-1}}{2}}{\Delta r}$$

$$= \frac{T_{m,n+1} - T_{m,n-1}}{2\Delta r} \quad (3)$$

$$\left.\frac{\partial^2 T}{\partial r^2}\right|_{\theta_m, r_n} \approx \frac{\left.\frac{\partial T}{\partial r}\right|_{m,n+\frac{1}{2}} - \left.\frac{\partial T}{\partial r}\right|_{m,n-\frac{1}{2}}}{\Delta r} \quad (4)$$

$$\left.\frac{\partial T}{\partial r}\right|_{m,n+\frac{1}{2}} = \frac{T_{m,n+1} - T_{m,n}}{\Delta r} \quad (5)$$

$$\left.\frac{\partial T}{\partial r}\right|_{m,n-\frac{1}{2}} = \frac{T_{m,n} - T_{m,n-1}}{\Delta r} \quad (6)$$

Substituting (5) & (6) into (4) gives

$$\left.\frac{\partial^2 T}{\partial r^2}\right|_{\theta_m, r_n} \approx \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{\Delta r^2} \quad (7)$$

$$\left.\frac{\partial^2 T}{\partial (r\theta)^2}\right|_{\theta_m, r_n} \approx \frac{\left.\frac{\partial T}{\partial (r_n\theta)}\right|_{m+\frac{1}{2},n} - \left.\frac{\partial T}{\partial (r_n\theta)}\right|_{m-\frac{1}{2},n}}{r_n \Delta\theta} \quad (8)$$

$$\left.\frac{\partial T}{\partial (r_n\theta)}\right|_{m+\frac{1}{2},n} = \frac{T_{m+1,n} - T_{m,n}}{r_n \Delta\theta} \quad (9), \qquad \left.\frac{\partial T}{\partial (r_n\theta)}\right|_{m-\frac{1}{2},n} = \frac{T_{m,n} - T_{m-1,n}}{r_n \Delta\theta} \quad (10)$$

Substituting (9) & (10) into (8) gives

$$\left.\frac{\partial^2 T}{\partial (r\theta)^2}\right|_{\theta_m, r_n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{r_n^2 \Delta\theta^2}$$

$$\Rightarrow \left.\frac{\partial^2 T}{\partial \theta^2}\right|_{\theta_m, r_n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{\Delta\theta^2} \quad (11)$$

Substituting (3), (7), (11) into (2)

$$\frac{T_{m,n+1} - T_{m,n-1}}{2\Delta r} + r_n \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{\Delta r^2} + \frac{1}{r_n} \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{\Delta\theta^2} \quad (12)$$

Equation (12) can be rearranged into

$$\left(\frac{1}{2\Delta r}+\frac{r_n}{\Delta r^2}\right)T_{m,n+1}+\left(-\frac{1}{2\Delta r}+\frac{r_n}{\Delta r^2}\right)T_{m,n-1}-2\left(\frac{r_n}{\Delta r^2}+\frac{1}{r_n\Delta\theta^2}\right)T_{m,n}$$
$$+\frac{1}{r_n\Delta\theta^2}T_{m+1,n}+\frac{1}{r_n\Delta\theta^2}T_{m-1,n}=0 \qquad (13).$$

* Sorry, I should have used (i,j) instead of (m,n) for indices.