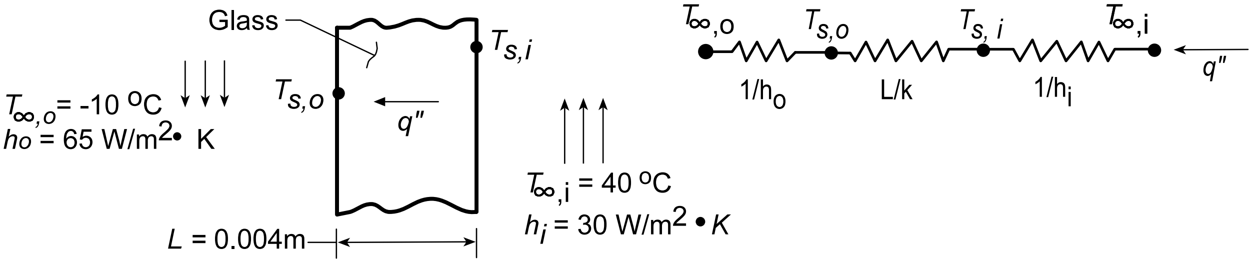
PROBLEM 3.2

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ\mathrm{C} - \left(-10^\circ\mathrm{C}\right)}{\frac{1}{65\,\mathrm{W/m^2\cdot K}} + \frac{0.004\,\mathrm{m}}{1.4\,\mathrm{W/m\cdot K}} + \frac{1}{30\,\mathrm{W/m^2\cdot K}}}$$

$$q'' = \frac{50^\circ\mathrm{C}}{(0.0154 + 0.0029 + 0.0333)\,\mathrm{m^2\cdot K/W}} = 968\,\mathrm{W/m^2}\,.$$

Hence, with $q'' = h_i\left(T_{\infty,i} - T_{\infty,o}\right)$, the inner surface temperature is

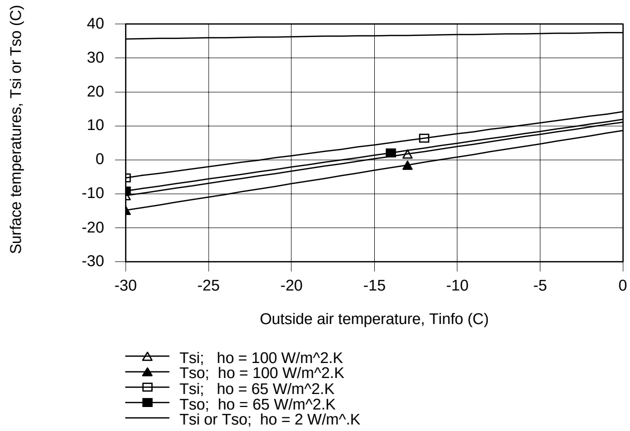
$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ\mathrm{C} - \frac{968\,\mathrm{W/m^2}}{30\,\mathrm{W/m^2\cdot K}} = 7.7^\circ\mathrm{C}$$ <

Similarly for the outer surface temperature with $q'' = h_o\left(T_{s,o} - T_{\infty,o}\right)$ find

$$T_{s,o} = T_{\infty,o} - \frac{q''}{h_o} = -10^\circ\mathrm{C} - \frac{968\,\mathrm{W/m^2}}{65\,\mathrm{W/m^2\cdot K}} = 4.9^\circ\mathrm{C}$$ <

(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of h_o = 2, 65, and 100 W/m²·K. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with h_o = 2 W/m²·K, $T_{s,i} - T_{s,o}$, is too small to show on the plot.

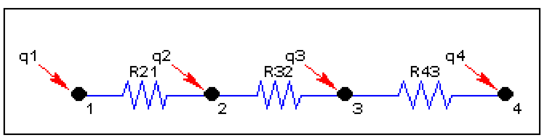
Continued …..



COMMENTS: (1) The largest resistance is that associated with convection at the inner surface. The values of $T_{s,i}$ and $T_{s,o}$ could be increased by increasing the value of h_i.

(2) The *IHT Thermal Resistance Network Model* was used to create a model of the window and generate the above plot. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:




```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 =  Tinfo           // Outside air temperature, C
//q1 =                // Heat rate, W
T2 =  Tso             // Outer surface temperature, C
q2 =  0               // Heat rate, W; node 2, no external heat source
T3 =  Tsi             // Inner surface temperature, C
q3 =  0               // Heat rate, W; node 2, no external heat source
T4 = Tinfi            // Inside air temperature, C
//q4 =                // Heat rate, W
```

// Thermal Resistances:

```
R21 =  1 / ( ho * As )       // Convection thermal resistance, K/W; outer surface
R32 =  L / ( k * As )        // Conduction thermal resistance, K/W; glass
R43 =  1 / ( hi * As )       // Convection thermal resistance, K/W; inner surface
```

// Other Assigned Variables:

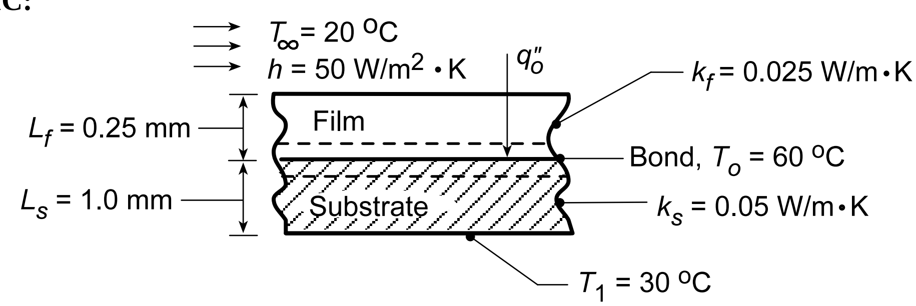
```
Tinfo = -10           // Outside air temperature, C
ho = 65               // Convection coefficient, W/m^2.K; outer surface
L = 0.004             // Thickness, m; glass
k = 1.4               // Thermal conductivity, W/m.K; glass
Tinfi = 40            // Inside air temperature, C
hi = 30               // Convection coefficient, W/m^2.K; inner surface
As = 1                // Cross-sectional area, m^2; unit area
```

PROBLEM 3.4

KNOWN: Curing of a transparent film by radiant heating with substrate and film surface subjected to known thermal conditions.

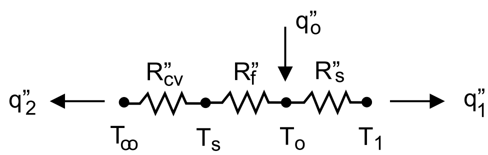
FIND: (a) Thermal circuit for this situation, (b) Radiant heat flux, q''_o (W/m^2), to maintain bond at curing temperature, T_o, (c) Compute and plot q''_o as a function of the film thickness for $0 \le L_f \le 1$ mm, and (d) If the film is not transparent, determine q''_o required to achieve bonding; plot results as a function of L_f.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) All the radiant heat flux q''_o is absorbed at the bond, (4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit for this situation is shown at the right. Note that terms are written on a per unit area basis.



(b) Using this circuit and performing an energy balance on the film-substrate interface,

$$q''_o = q''_1 + q''_2 \qquad\qquad q''_o = \frac{T_o - T_\infty}{R''_{cv} + R''_f} + \frac{T_o - T_1}{R''_s}$$

where the thermal resistances are

$$R''_{cv} = 1/h = 1\big/50\ \mathrm{W/m^2 \cdot K} = 0.020\ \mathrm{m^2 \cdot K/W}$$

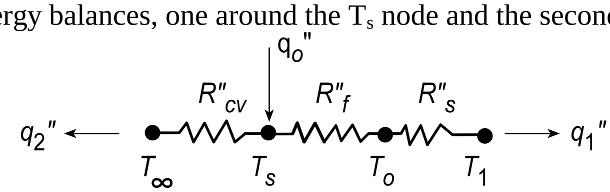
$$R''_f = L_f/k_f = 0.00025\ \mathrm{m}/0.025\ \mathrm{W/m \cdot K} = 0.010\ \mathrm{m^2 \cdot K/W}$$

$$R''_s = L_s/k_s = 0.001\ \mathrm{m}/0.05\ \mathrm{W/m \cdot K} = 0.020\ \mathrm{m^2 \cdot K/W}$$

$$q''_o = \frac{(60-20)^\circ\mathrm{C}}{[0.020+0.010]\ \mathrm{m^2 \cdot K/W}} + \frac{(60-30)^\circ\mathrm{C}}{0.020\ \mathrm{m^2 \cdot K/W}} = (133+1500)\ \mathrm{W/m^2} = 2833\ \mathrm{W/m^2} \qquad <$$

(c) For the transparent film, the radiant flux required to achieve bonding as a function of film thickness L_f is shown in the plot below.

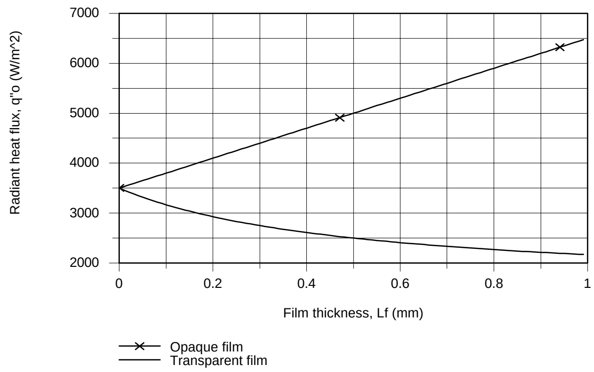
(d) If the film is opaque (not transparent), the thermal circuit is shown below. In order to find q''_o, it is necessary to write two energy balances, one around the T_s node and the second about the T_o node.



.
The results of the analyses are plotted below.

Continued...

PROBLEM 3.4 (Cont.)

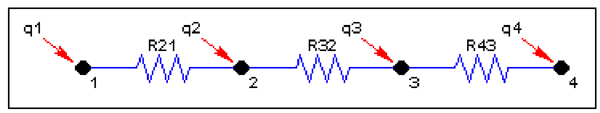


COMMENTS: (1) When the film is transparent, the radiant flux is absorbed on the bond. The flux required decreases with increasing film thickness. Physically, how do you explain this? Why is the relationship not linear?

(2) When the film is opaque, the radiant flux is absorbed on the surface, and the flux required increases with increasing thickness of the film. Physically, how do you explain this? Why is the relationship linear?

(3) The IHT Thermal Resistance Network Model was used to create a model of the film-substrate system and generate the above plot. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:



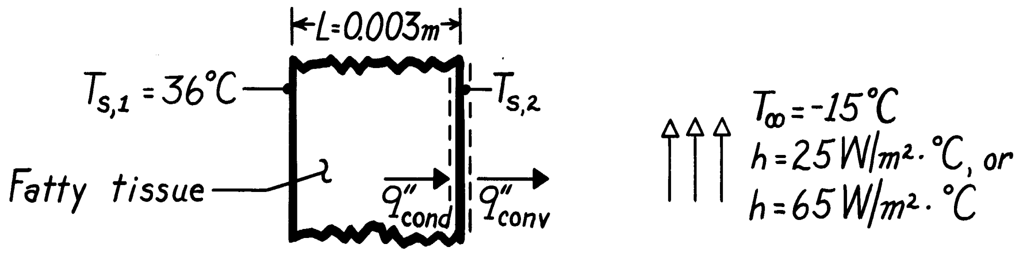

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 = Tinf          // Ambient air temperature, C
//q1 =             // Heat rate, W; film side
T2 = Ts            // Film surface temperature, C
q2 = 0             // Radiant flux, W/m^2; zero for part (a)
T3 = To            // Bond temperature, C
q3 = qo            // Radiant flux, W/m^2; part (a)
T4 = Tsub          // Substrate temperature, C
//q4 =             // Heat rate, W; substrate side

// Thermal Resistances:
R21 =  1 / ( h * As )       // Convection resistance, K/W
R32 =  Lf / (kf * As)       // Conduction resistance, K/W; film
R43 =  Ls / (ks * As)       // Conduction resistance, K/W; substrate

// Other Assigned Variables:
Tinf = 20          // Ambient air temperature, C
h = 50             // Convection coefficient, W/m^2.K
Lf = 0.00025       // Thickness, m; film
kf = 0.025         // Thermal conductivity, W/m.K; film
To = 60            // Cure temperature, C
Ls = 0.001         // Thickness, m; substrate
ks = 0.05          // Thermal conductivity, W/m.K; substrate
Tsub = 30          // Substrate temperature, C
As = 1             // Cross-sectional area, m^2; unit area
```

PROBLEM 3.7

KNOWN: A layer of fatty tissue with fixed inside temperature can experience different outside convection conditions.

FIND: (a) Ratio of heat loss for different convection conditions, (b) Outer surface temperature for different convection conditions, and (c) Temperature of still air which achieves same cooling as moving air (*wind chill* effect).

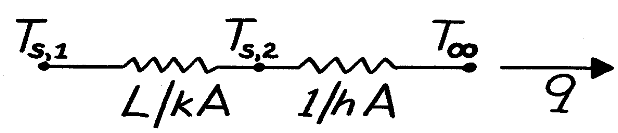
SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) Homogeneous medium with constant properties, (4) No internal heat generation (metabolic effects are negligible), (5) Negligible radiation effects.

PROPERTIES: *Table A-3*, Tissue, fat layer: k = 0.2 W/m·K.

ANALYSIS: The thermal circuit for this situation is



Hence, the heat rate is

$$q = \frac{T_{s,1} - T_\infty}{R_{tot}} = \frac{T_{s,1} - T_\infty}{L/kA + 1/hA}.$$

Therefore,

$$\frac{q''_{calm}}{q''_{windy}} = \frac{\left[\frac{L}{k} + \frac{1}{h}\right]_{windy}}{\left[\frac{L}{k} + \frac{1}{h}\right]_{calm}}.$$

Applying a surface energy balance to the outer surface, it also follows that

$$q''_{cond} = q''_{conv}.$$

Continued …..

Hence,

$$\frac{k}{L}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_{\infty}\right)$$

$$T_{s,2}=\frac{T_{\infty}+\dfrac{k}{hL}T_{s,1}}{1+\dfrac{k}{hL}}.$$

To determine the wind chill effect, we must determine the heat loss for the windy day and use it to evaluate the hypothetical ambient air temperature, T'_{∞}, which would provide the same heat loss on a calm day, Hence,

$$q''=\frac{T_{s,1}-T_{\infty}}{\left[\dfrac{L}{k}+\dfrac{1}{h}\right]_{\text{windy}}}=\frac{T_{s,1}-T'_{\infty}}{\left[\dfrac{L}{k}+\dfrac{1}{h}\right]_{\text{calm}}}$$

From these relations, we can now find the results sought:

(a) $$\frac{q''_{\text{calm}}}{q''_{\text{windy}}}=\frac{\dfrac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\dfrac{1}{65\text{ W/m}^2\cdot\text{K}}}{\dfrac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\dfrac{1}{25\text{ W/m}^2\cdot\text{K}}}=\frac{0.015+0.0154}{0.015+0.04}$$

$$\frac{q''_{\text{calm}}}{q''_{\text{windy}}}=0.553$$ <

(b) $$\left.T_{s,2}\right]_{\text{calm}}=\frac{-15^{\circ}\text{C}+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}36^{\circ}\text{C}}{1+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}}=22.1^{\circ}\text{C}$$ <

$$\left.T_{s,2}\right]_{\text{windy}}=\frac{-15^{\circ}\text{C}+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}36^{\circ}\text{C}}{1+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}}=10.8^{\circ}\text{C}$$ <

(c) $$T'_{\infty}=36^{\circ}\text{C}-\left(36+15\right)^{\circ}\text{C}\frac{\left(0.003/0.2+1/25\right)}{\left(0.003/0.2+1/65\right)}=-56.3^{\circ}\text{C}$$ <

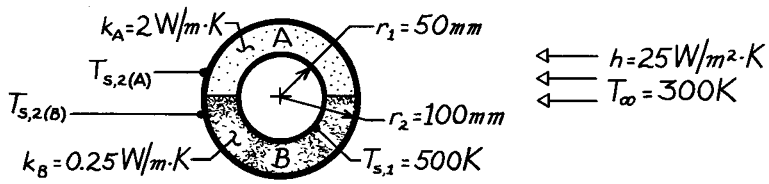
COMMENTS: The wind chill effect is equivalent to a decrease of $T_{s,2}$ by 11.3°C and increase in the heat loss by a factor of $(0.553)^{-1}$ = 1.81.

PROBLEM 3.52

KNOWN: Inner surface temperature of insulation blanket comprised of two semi-cylindrical shells of different materials. Ambient air conditions.

FIND: (a) Equivalent thermal circuit, (b) Total heat loss and material outer surface temperatures.

SCHEMATIC:



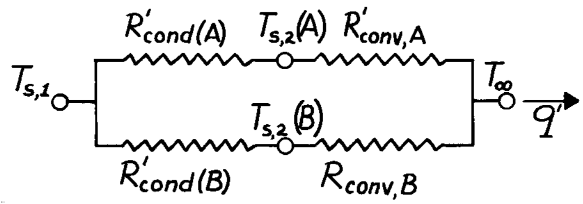
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction, (3) Infinite contact resistance between materials, (4) Constant properties.

ANALYSIS: (a) The thermal circuit is,

$$R'_{conv,A} = R'_{conv,B} = 1/\pi\ r_2 h$$

$$R'_{cond(A)} = \frac{\ln(r_2/r_1)}{\pi\ k_A}$$

$$R'_{cond(\mathbf{B})} = \frac{\ln(r_2/r_i)}{\pi\ k_B}$$



<

The conduction resistances follow from Section 3.3.1 and Eq. 3.28. Each resistance is larger by a factor of 2 than the result of Eq. 3.28 due to the reduced area.

(b) Evaluating the thermal resistances and the heat rate $\left(q' = q'_A + q'_B\right)$,

$$R'_{conv} = \left(\pi \times 0.1\text{m} \times 25\ \text{W/m}^2 \cdot \text{K}\right)^{-1} = 0.1273\ \text{m} \cdot \text{K/W}$$

$$R'_{cond(A)} = \frac{\ln(0.1\text{m}/0.05\text{m})}{\pi \times 2\ \text{W/m} \cdot \text{K}} = 0.1103\ \text{m} \cdot \text{K/W} \quad R'_{cond(B)} = 8\ R'_{cond(A)} = 0.8825\ \text{m} \cdot \text{K/W}$$

$$q' = \frac{T_{s,1} - T_\infty}{R'_{cond(A)} + R'_{conv}} + \frac{T_{s,1} - T_\infty}{R'_{cond(B)} + R'_{conv}}$$

$$q' = \frac{(500 - 300)\text{K}}{(0.1103 + 0.1273)\text{m} \cdot \text{K/W}} + \frac{(500 - 300)\text{K}}{(0.8825 + 0.1273)\text{m} \cdot \text{K/W}} = (842 + 198)\text{W/m} = 1040\ \text{W/m}.$$ <

Hence, the temperatures are

$$T_{s,2(A)} = T_{s,1} - q'_A R'_{cond(A)} = 500\text{K} - 842\frac{\text{W}}{\text{m}} \times 0.1103\frac{\text{m} \cdot \text{K}}{\text{W}} = 407\text{K}$$ <

$$T_{s,2(B)} = T_{s,1} - q'_B R'_{cond(B)} = 500\text{K} - 198\frac{\text{W}}{\text{m}} \times 0.8825\frac{\text{m} \cdot \text{K}}{\text{W}} = 325\text{K}.$$ <

COMMENTS: The total heat loss can also be computed from $q' = \left(T_{s,1} - T_\infty\right)/R_{equiv}$,

where $R_{equiv} = \left[\left(R'_{cond(A)} + R'_{conv,A}\right)^{-1} + \left(R'_{cond(B)} + R'_{conv,B}\right)^{-1}\right]^{-1} = 0.1923\ \text{m} \cdot \text{K/W}.$

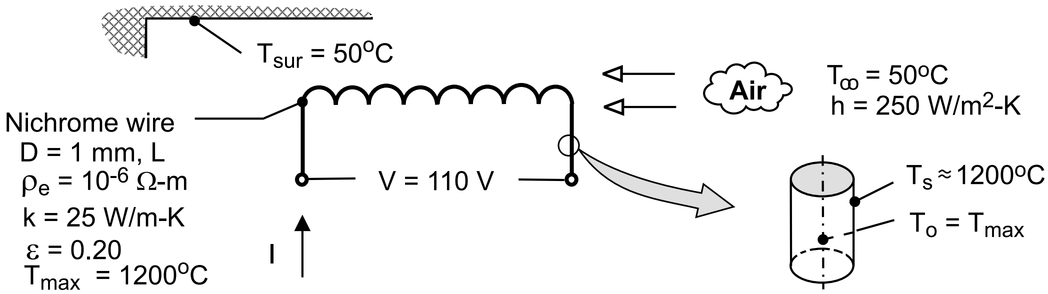
Hence $q' = (500 - 300)\text{K}/0.1923\ \text{m} \cdot \text{K/W} = 1040\ \text{W/m}.$

PROBLEM 3.86

KNOWN: Diameter, resistivity, thermal conductivity, emissivity, voltage, and maximum temperature of heater wire. Convection coefficient and air exit temperature. Temperature of surroundings.

FIND: Maximum operating current, heater length and power rating.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform wire temperature, (3) Constant properties, (4) Radiation exchange with large surroundings.

ANALYSIS: Assuming a uniform wire temperature, $T_{max} = T(r = 0) \equiv T_o \approx T_s$, the maximum volumetric heat generation may be obtained from Eq. (3.55), but with the total heat transfer coefficient, $h_t = h + h_r$, used in lieu of the convection coefficient h. With

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.20 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,(1473 + 323)\,\mathrm{K}\left(1473^2 + 323\right)^2 \mathrm{K}^2 = 46.3\,\mathrm{W/m^2 \cdot K}$$

$$h_t = (250 + 46.3)\,\mathrm{W/m^2 \cdot K} = 296.3\,\mathrm{W/m^2 \cdot K}$$

$$\dot{q}_{max} = \frac{2h_t}{r_o}\left(T_s - T_\infty\right) = \frac{2\left(296.3\,\mathrm{W/m^2 \cdot K}\right)}{0.0005\mathrm{m}}(1150°\mathrm{C}) = 1.36 \times 10^9\,\mathrm{W/m^3}$$

Hence, with $\dot{q} = \dfrac{I^2 R_e}{\forall} = \dfrac{I^2\left(\rho_e L / A_c\right)}{LA_c} = \dfrac{I^2 \rho_e}{A_c^2} = \dfrac{I^2 \rho_e}{\left(\pi D^2/4\right)^2}$

$$I_{max} = \left(\frac{\dot{q}_{max}}{\rho_e}\right)^{1/2} \frac{\pi D^2}{4} = \left(\frac{1.36 \times 10^9\,\mathrm{W/m^3}}{10^{-6}\,\Omega \cdot \mathrm{m}}\right)^{1/2} \frac{\pi\,(0.001\mathrm{m})^2}{4} = 29.0\,\mathrm{A}$$ <

Also, with $\Delta E = I\,R_e = I\,(\rho_e L/A_c)$,

$$L = \frac{\Delta E \cdot A_c}{I_{max}\,\rho_e} = \frac{110\,\mathrm{V}\left[\pi\,(0.001\mathrm{m})^2/4\right]}{29.0\,\mathrm{A}\left(10^{-6}\,\Omega \cdot \mathrm{m}\right)} = 2.98\mathrm{m}$$ <

and the power rating is

$$P_{elec} = \Delta E \cdot I_{max} = 110\,\mathrm{V}\,(29\,\mathrm{A}) = 3190\,\mathrm{W} = 3.19\,\mathrm{kW}$$ <

COMMENTS: To assess the validity of assuming a uniform wire temperature, Eq. (3.53) may be used to compute the centerline temperature corresponding to $\dot{q}_{max}$ and a surface temperature of 1200°C. It follows that $T_o = \dfrac{\dot{q}\,r_o^2}{4k} + T_s = \dfrac{1.36 \times 10^9\,\mathrm{W/m^3}\,(0.0005\mathrm{m})^2}{4\,(25\,\mathrm{W/m \cdot K})} + 1200°\mathrm{C} = 1203°\mathrm{C}$. With only a

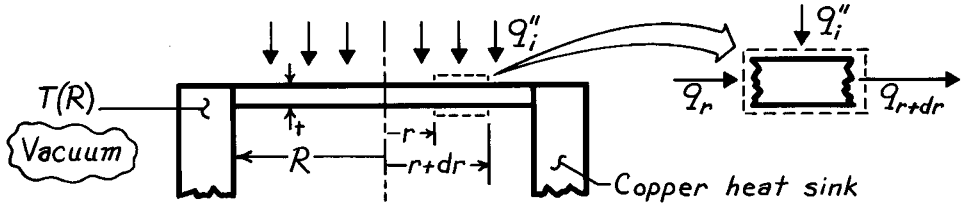
3°C temperature difference between the centerline and surface of the wire, the assumption is *excellent*.

PROBLEM 3.98

KNOWN: Radius, thickness, and incident flux for a radiation heat gauge.

FIND: Expression relating incident flux to temperature difference between center and edge of gauge.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r (negligible temperature drop across foil thickness), (3) Constant properties, (4) Uniform incident flux, (5) Negligible heat loss from foil due to radiation exchange with enclosure wall, (6) Negligible contact resistance between foil and heat sink.

ANALYSIS: Applying energy conservation to a circular ring extending from r to r + dr,

$$q_r + q_i''(2\pi\ r dr) = q_{r+dr}, \qquad q_r = -k(2\pi\ rt)\frac{dT}{dr}, \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr.$$

Rearranging, find that

$$q_i''(2\pi\ r dr) = \frac{d}{dr}\left[(-k2\pi\ rt)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_i''}{kt}r.$$

Integrating,

$$r\frac{dT}{dr} = -\frac{q_i'' r^2}{2kt} + C_1 \qquad \text{and} \qquad T(r) = -\frac{q_i'' r^2}{4kt} + C_1 \ln r + C_2.$$

With $dT/dr|_{r=0} = 0$, $C_1 = 0$ and with $T(r = R) = T(R)$,

$$T(R) = -\frac{q_i'' R^2}{4kt} + C_2 \qquad \text{or} \qquad C_2 = T(R) + \frac{q_i'' R^2}{4kt}.$$

Hence, the temperature distribution is

$$T(r) = \frac{q_i''}{4kt}\left(R^2 - r^2\right) + T(R).$$

Applying this result at r = 0, it follows that

$$q_i'' = \frac{4kt}{R^2}\left[T(0) - T(R)\right] = \frac{4kt}{R^2}\Delta T. \qquad <$$

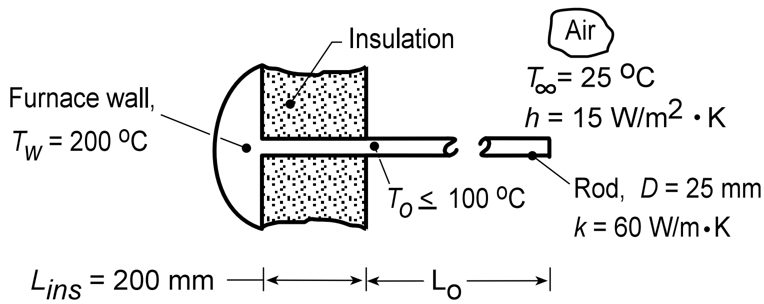
COMMENTS: This technique allows for determination of a radiation flux from measurement of a temperature difference. It becomes inaccurate if emission from the foil becomes significant.

PROBLEM 3.111

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \le 100°C$? If not, what design parameters would you change?

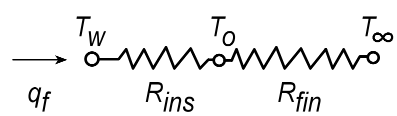
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.76,

$$R_{fin} = \theta_b/q_f = \frac{1}{\left(hPkA_c\right)^{1/2}\tanh\left(mL_o\right)} \qquad (2)$$

$$m = \left(hP/kA_c\right)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}\left(T_w - T_\infty\right) \qquad (6)$$ <

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25^\circ C + \frac{6.298}{6.790 + 6.298}\left(200 - 25\right)^\circ C = 109^\circ C$$ <

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909\times 10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi\left(0.025\,m\right)^2/4 = 4.909\times 10^{-4}\,m^2$$

$$R_{fin} = 1\Big/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh\left(6.324\times 0.200\right) = 6.298\,K/W$$

$$\left(hPkA_c\right) = \left(15\,W/m^2\cdot K\times\pi\left(0.025\,m\right)\times 60\,W/m\cdot K\times 4.909\times 10^{-4}\,m^2\right) = 0.0347\,W^2/K^2$$

Continued...

PROBLEM 3.111 (Cont.)

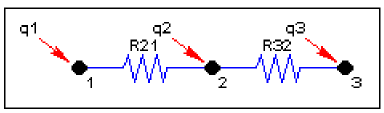
$$m = \left(hP/kA_c\right)^{1/2} = \left(15\,W/m^2\cdot K \times \pi\,(0.025\,m)/60\,W/m\cdot K \times 4.909\times10^{-4}\,m^2\right)^{1/2} = 6.324\,m^{-1}$$

Consider the following design changes aimed at reducing $T_o \le 100°C$. (1) Increasing length of the fin portions: with $L_o = 200$ mm, the fin already behaves as an infinitely long fin. Hence, increasing L_o will not result in reducing T_o. (2) Decreasing the thermal conductivity: backsolving the above equation set with $T_0 = 100°C$, find the required thermal conductivity is $k = 14$ W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for $T_o = 100°C$, the required insulation thickness would be $L_{ins} = 211$ mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by “tack welding” (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit.

COMMENTS: (1) Would replacing the rod by a thick-walled tube provide a practical solution?

(2) The *IHT Thermal Resistance Network Model* and the *Thermal Resistance Tool* for a *fin* with an *adiabatic tip* were used to create a model of the rod. The Workspace is shown below.

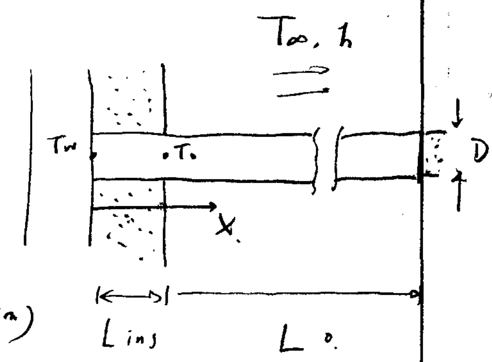
// Thermal Resistance Network Model:
// The Network:




```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal
points at which there is no external source of heat. */
T1 = Tw             // Furnace wall temperature, C
//q1 =              // Heat rate, W
T2 = To             // To, beginning of rod exposed length
q2 =  0             // Heat rate, W; node 2; no external heat source
T3 =  Tinf          // Ambient air temperature, C
//q3 =              // Heat rate, W

// Thermal Resistances:
// Rod - conduction resistance
R21 =  Lins / (k * Ac)        // Conduction resistance, K/W
Ac = pi * D^2 / 4             // Cross sectional area of rod, m^2

// Thermal Resistance Tools - Fin with Adiabatic Tip:
R32 =  Rfin                   //  Resistance of fin, K/W
/* Thermal resistance of a fin of uniform cross sectional area Ac, perimeter P, length L, and thermal
conductivity k with an adiabatic tip condition experiencing convection with a fluid at Tinf and coefficient h, */
Rfin = 1/ ( tanh (m*Lo) * (h * P * k * Ac ) ^ (1/2) )          // Case B, Table 3.4
m = sqrt(h*P / (k*Ac))
P = pi * D                    // Perimeter, m

// Other Assigned Variables:
Tw = 200            // Furnace wall temperature, C
k = 60              // Rod thermal conductivity, W/m.K
Lins = 0.200        // Insulated length, m
D = 0.025           // Rod diameter, m
h = 15              // Convection coefficient, W/m^2.K
Tinf = 25           // Ambient air temperature,C
Lo = 0.200          // Exposed length, m
```

3.111

Find T_o and $T(x)$



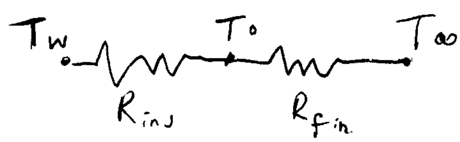
Assumption: ① Steady state

② 1-D conduction (x-direction)

③ Negligible contact resistance

④. Negligible radiation effects.

Analysis. Part 1 Using thermal circuit.



$$R_{ins} = \frac{L_{ins}}{k A_c}$$

For R_{fin}, using case B of tip condition $\left(\frac{\partial T}{\partial x} \Big|_L = 0 \right)$.

$$q_f = M \tanh mL_o$$

$$= \sqrt{hPkA_c}\, \theta_b \tanh mL_o$$

$$= \sqrt{hPkA_c} \tanh mL_o (T_o - T_\infty)$$

$$R_{fin} = \frac{T_o - T_\infty}{q_f} = \frac{1}{\sqrt{hPkA_c} \tanh mL_o}$$

$$\frac{T_w - T_o}{R_{ins}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}}$$

$$\Rightarrow T_o = T_w - \frac{R_{ins}}{R_{ins} + R_{fin}} (T_w - T_\infty)$$

$$= \frac{R_{fin}}{R_{ins} + R_{fin}} T_w + \frac{R_{ins}}{R_{ins} + R_{fin}} T_\infty$$

$$T_0 = \frac{R_{fin} T_w + R_{ins} T_\infty}{R_{ins} + R_{fin}}$$

$$= \frac{T_w + \frac{R_{ins}}{R_{fin}} T_\infty}{1 + \frac{R_{ins}}{R_{fin}}}$$

$$\frac{R_{ins}}{R_{fin}} = \frac{\frac{L_{ins}}{k A_c}}{\frac{1}{\sqrt{hPkA_c}\tanh mL_0}} = \frac{L_{ins}\sqrt{hPkA_c}\tanh mL_0}{k A_c}$$

$$= L_{ins}\, m \tanh mL_0$$

$$\therefore T_0 = \frac{T_w + L_{ins}\, m \tanh mL_0\, T_\infty}{1 + L_{ins}\, m \tanh mL_0} \quad (1)$$

Part 2. q is constant.

① heat eqn. for plane wall.

$$\frac{d^2T}{dx^2} = 0 \Rightarrow T = C_1 x + C_2$$

B.C's: $T = T_w$ @ $x = 0$ $\Rightarrow C_2 = T_w$.

$T = T_o$ @ $x = L_{ins}$ $\Rightarrow C_1 = \frac{T_o - T_w}{L_{ins}}$

$$\therefore T = \frac{T_o - T_w}{L_{ins}} x + T_w$$

$$\Rightarrow \left.\frac{dT}{dx}\right|_{x=L_{ins}} = \frac{T_o - T_w}{L_{ins}}$$

$$q = -kA_c \frac{dT}{dx} = -kA_c \frac{T_o - T_w}{L_{ins}}$$

② From case B. $\frac{\theta}{\theta_b} = \frac{\cosh m(L_o - x)}{\cosh mL} \Rightarrow \frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$

$$q_f = M \tanh mL_o$$

constant q

$$\Rightarrow q = q_f \Rightarrow kA_c \frac{T_w - T_o}{L_{ins}} = \sqrt{hPkA_c}\,(T_o - T_\infty) \tanh mL_o$$

$$T_w - T_o = \frac{L_{ins}\sqrt{hPkA_c} \tanh mL_o}{kA_c} (T_o - T_\infty)$$

$$T_w + \frac{L_{ins}\sqrt{hPkA_c} \tanh mL_o}{kA_c} T_\infty = \left(1 + \frac{L_{ins}\sqrt{hPkA_c} \tanh mL_o}{kA_c}\right) T_o$$

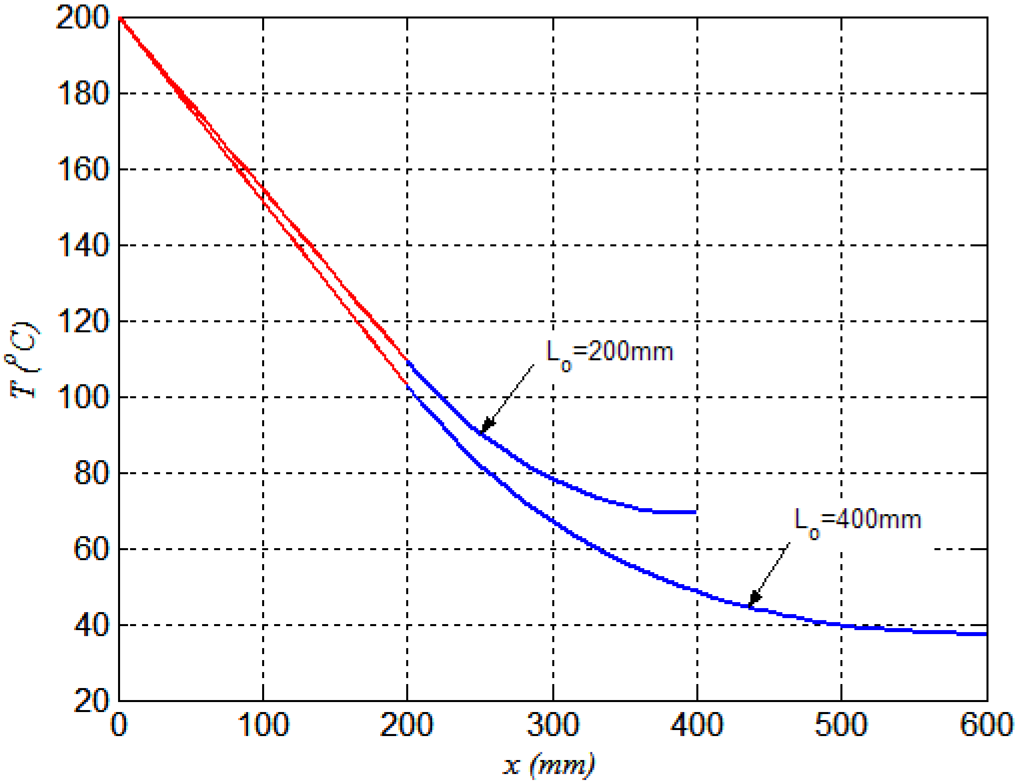
$$T_w + L_{ins}\, m \tanh mL_o\, T_\infty = (1 + L_{ins}\, m \tanh mL_o)\, T_o$$

$$T_o = \frac{T_w + L_{ins}\, m \tanh mL_o\, T_\infty}{1 + L_{ins}\, m \tanh mL_o} \quad (2)$$

(2) is the same as (1)

For Lo = 200 mm, To =109 deg(C)
Lo = 400 mm, To =103 deg(C)

plot of $T(x)$ for $L_o = 200, 400$ mm. Is shown in Fig. 1 on next page



200
180
160
140
120
100
80
60
40
20
0
100
200
300
400
500
600
T (°C)
x (mm)
L_o=200mm
L_o=400mm

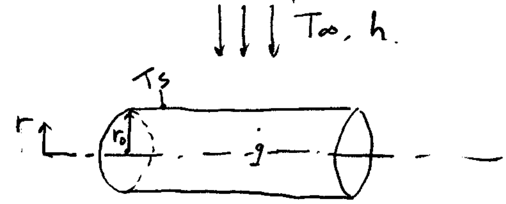
Fig 1

Supplemental problems:

A. Known: Conduction and convection coefficient, temperature of air, Surface temperature.

Find: temperature distribution

Schematic:



Assumptions: (1) Steady state Conditions
(2). 1-D Conduction.
(3) Negligible radiation effects
(4) Constant properties.

Analysis:

1-D conduction means the heat is transfered only in r direction.

Starting with full equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \quad (1)$$

Based on assumptions (1), (2), and (4), equation (1) can be simplified to

$$\frac{k}{r}\frac{d}{dr}\left(r \frac{dT}{dr}\right) + \dot{q} = 0 \quad (2)$$

Boundary conditions: $\frac{\partial T}{\partial r} = 0$ @ $r = 0$ (B1)

$T = T_s$ @ $r = r_0$ (B2)

Rearranging eqn. ② gives

$$d\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} r\,dr \qquad (3)$$

Integrating eqn ③ once,

$$r\frac{dT}{dr} = -\frac{\dot{q}}{2k}r^2 + C_1 \qquad (4)$$

Separating variables gives

$$dT = \left(-\frac{\dot{q}}{2k}r + \frac{C_1}{r}\right)dr \qquad (5)$$

Integrating eqn. ⑤ gives

$$T = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2 \qquad (6)$$

Applying boundary conditions (B1) and (B2) gives

$$C_1 = 0, \qquad C_2 = T_s + \frac{\dot{q}}{4k}r_o^2 \qquad (7)$$

Substituting ⑦ into ⑥ gives

$$\boxed{T = T_s + \frac{\dot{q}}{4k}(r_o^2 - r^2)} \qquad (8)$$

Now, we want to relate T_s to the air temperature T_∞. We do this by applying energy conservation to the surface of the wire, i.e at the surface

$$q_{cond} = q_{conv} \qquad (9)$$

which gives

$$-kA\left.\frac{dT}{dr}\right|_{r=r_o} = hA(T_s - T_\infty) \qquad (10)$$

From ⑧ we can have

$$\left.\frac{dT}{dr}\right|_{r=r_o} = -\frac{\dot{q}}{2k}r_o \qquad (11)$$

Substituting (11) into (10) gives

$$\frac{\dot{q}}{2} r_o = h(T_s - T_\infty) \qquad (12)$$

which gives

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h} \qquad (13)$$

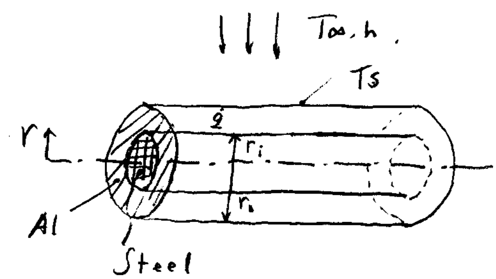
Now substituting (13) into (8) gives

$$\boxed{T = T_\infty + \frac{\dot{q} r_o}{2h} + \frac{\dot{q}}{4k}(r_o^2 - r^2)} \qquad (14)$$

B. Known: Conduction and convection coefficient, temperature associated with air and the surface of the cylinder.

Find: temperature distribution $T(r)$

Schematic.



Assumptions:
1). Steady state conditions
2). 1-D conduction
3). Negligible radiation effects
4). Constant properties

Analysis:

Similar to A, the full eqn is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial t}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \qquad (15)$$

and it can be simplified to

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \dot{q} = 0 \qquad (16)$$

The boundary conditions are

$$\frac{dT}{dr} = 0 \quad @ \quad r = r_i \qquad (B3)$$

$$T = T_s \quad @ \quad r = r_o \qquad (B4)$$

Following the same procedure as in A, rearranging ⑦ followed by integration gives

$$T = -\frac{\dot{q}}{4k} r^2 + C_1 \ln r + C_2 \qquad (17)$$

Applying boundary conditions (B3) and (B4) gives

$$C_1 = \frac{\dot{q} r_i^2}{2k}, \qquad C_2 = T_s + \frac{\dot{q}}{4k} r_o^2 - \frac{\dot{q} r_i^2}{2k} \ln r_o \qquad (18)$$

Substituting C_1, C_2 into (17) gives

$$T = -\frac{\dot{q}}{4k} r^2 + \frac{\dot{q} r_i^2}{2k} \ln r + T_s + \frac{\dot{q}}{4k} r_o^2 - \frac{\dot{q} r_i^2}{2k} \ln r_o$$

$$= \boxed{T_s + \frac{\dot{q}}{4k}(r_o^2 - r^2) + \frac{\dot{q} r_i^2}{2k} \ln r/r_o} \qquad (19)$$

similar to part A, one applies the heat balance to the outer surface of the annular sheath which relates T_s to T_∞:

$$T_s = T_\infty + \frac{\dot{q}}{2h} \frac{r_o^2 - r_i^2}{r_o} \qquad (20)$$

Substituting eqn.20 into eqn.19 gives:

$$T = T_\infty + \frac{\dot{q}}{2h} \frac{r_o^2 - r_i^2}{r_o} + \frac{\dot{q}}{4k}(r_o^2 - r^2) + \frac{\dot{q} r_i^2}{2k} \ln r/r_o \qquad (21)$$

Remark: If the outer radius of the insulation layer is smaller than k/h, adding insulation does keep the transmission line cooler.