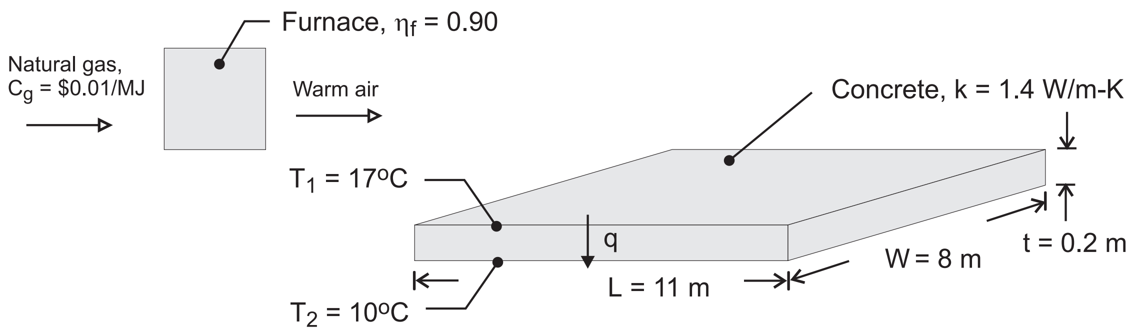
PROBLEM 1.3

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,\mathrm{W/m\cdot K}\left(11\,\mathrm{m}\times 8\,\mathrm{m}\right)\frac{7°\mathrm{C}}{0.20\,\mathrm{m}} = 4312\ \mathrm{W} \qquad <$$

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,\mathrm{W}\times \$0.01/\mathrm{MJ}}{0.9\times 10^6\,\mathrm{J/MJ}}\left(24\,\mathrm{h/d}\times 3600\,\mathrm{s/h}\right) = \$4.14/\mathrm{d} \qquad <$$

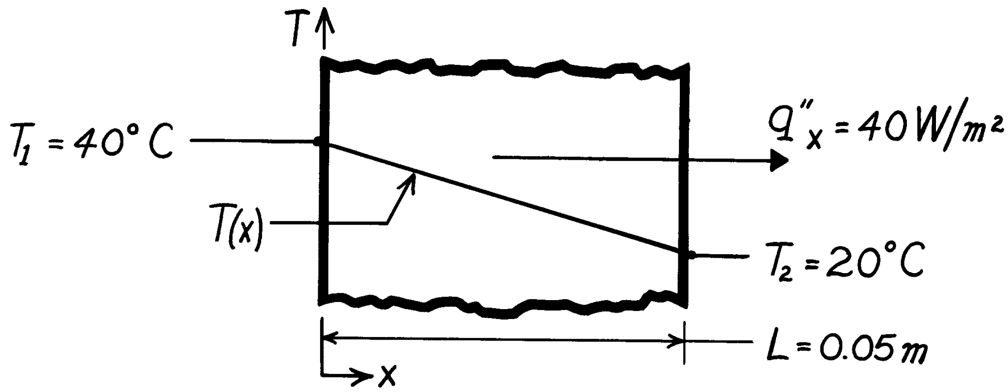
COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 1.4

KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q''_x \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40\text{-}20)^{\circ} C}$$

$$k = 0.10 \text{ W/m} \cdot \text{K}.$$ <

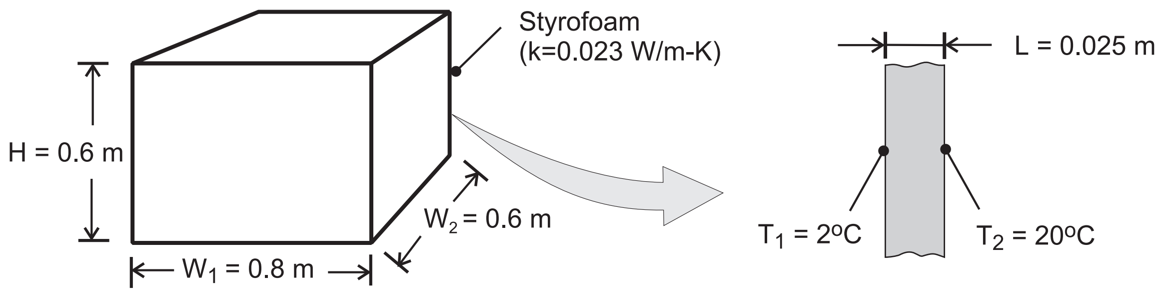
COMMENTS: Note that the °C or K temperature units may be used interchangeably when evaluating a temperature difference.

PROBLEM 1.8

KNOWN: Dimensions and thermal conductivity of food/beverage container. Inner and outer surface temperatures.

FIND: Heat flux through container wall and total heat load.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer through bottom wall, (3) Uniform surface temperatures and one-dimensional conduction through remaining walls.

ANALYSIS: From Fourier's law, Eq. 1.2, the heat flux is

$$q'' = k\frac{T_2 - T_1}{L} = \frac{0.023\ \text{W/m}\cdot\text{K}\ (20-2)^\circ\ \text{C}}{0.025\ \text{m}} = 16.6\ \text{W/m}^2 \qquad <$$

Since the flux is uniform over each of the five walls through which heat is transferred, the heat load is

$$q = q'' \times A_{total} = q''\left[H\left(2W_1 + 2W_2\right) + W_1 \times W_2\right]$$

$$q = 16.6\ \text{W/m}^2\left[0.6\text{m}\left(1.6\text{m} + 1.2\text{m}\right) + \left(0.8\text{m} \times 0.6\text{m}\right)\right] = 35.9\ \text{W} \qquad <$$

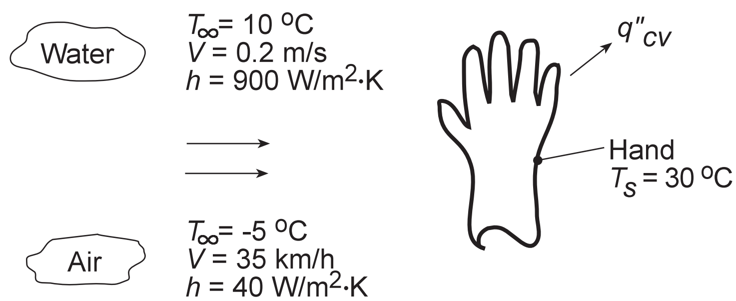
COMMENTS: The corners and edges of the container create local departures from one-dimensional conduction, which increase the heat load. However, for H, W_1, W_2 >> L, the effect is negligible.

PROBLEM 1.13

KNOWN: Hand experiencing convection heat transfer with moving air and water.

FIND: Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m^2 under normal room conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

ANALYSIS: The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h\left(T_s - T_\infty\right)$$

For the air stream:

$$q''_{air} = 40\,W/m^2 \cdot K\left[30 - (-5)\right]K = 1{,}400\,W/m^2 \qquad <$$

For the water stream:

$$q''_{water} = 900\,W/m^2 \cdot K\left(30 - 10\right)K = 18{,}000\,W/m^2 \qquad <$$

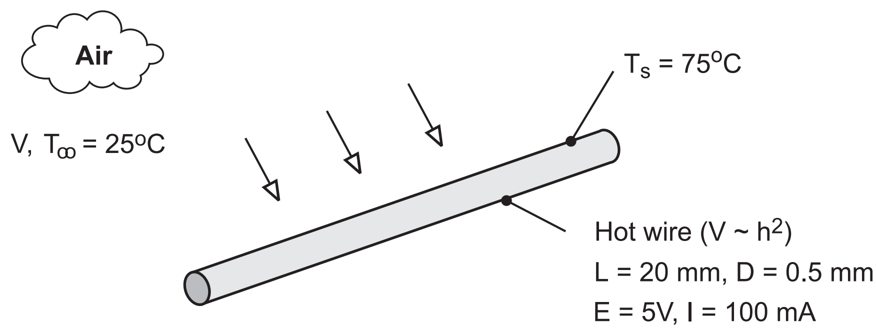
COMMENTS: The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only 30 W/m^2 which is a factor of 400 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.

PROBLEM 1.17

KNOWN: Length, diameter and calibration of a hot wire anemometer. Temperature of air stream. Current, voltage drop and surface temperature of wire for a particular application.

FIND: Air velocity

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from the wire by natural convection or radiation.

ANALYSIS: If all of the electric energy is transferred by convection to the air, the following equality must be satisfied

$$P_{elec} = EI = hA\left(T_s - T_\infty\right)$$

where $A = \pi DL = \pi\left(0.0005\text{m} \times 0.02\text{m}\right) = 3.14 \times 10^{-5}\text{m}^2.$

Hence,

$$h = \frac{EI}{A\left(T_s - T_\infty\right)} = \frac{5\text{V} \times 0.1\text{A}}{3.14 \times 10^{-5}\text{m}^2\left(50\ ^\circ\text{C}\right)} = 318\ \text{W/m}^2 \cdot \text{K}$$

$$V = 6.25 \times 10^{-5} h^2 = 6.25 \times 10^{-5}\left(318\ \text{W/m}^2 \cdot \text{K}\right)^2 = 6.3\ \text{m/s}$$ <

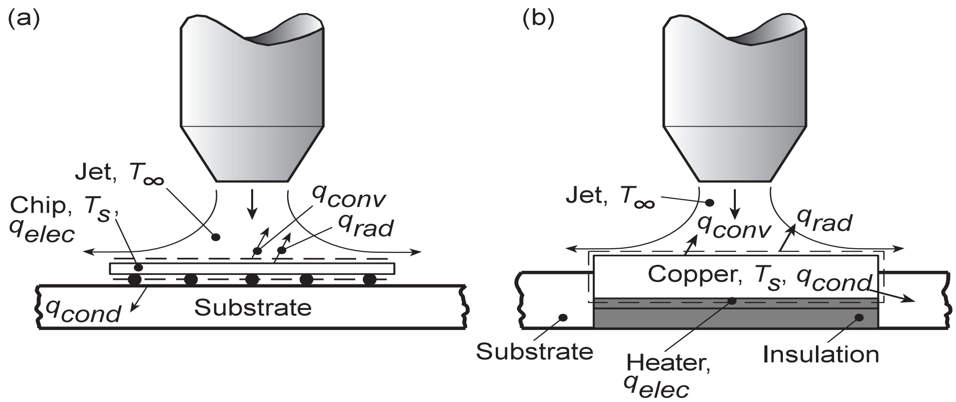
COMMENTS: The convection coefficient is sufficiently large to render buoyancy (natural convection) and radiation effects negligible.

PROBLEM 1.20

KNOWN: Air jet impingement is an effective means of cooling logic chips.

FIND: Procedure for measuring convection coefficients associated with a 10 mm × 10 mm chip.

SCHEMATIC:



ASSUMPTIONS: Steady-state conditions.

ANALYSIS: One approach would be to use the actual chip-substrate system, Case (a), to perform the measurements. In this case, the electric power dissipated in the chip would be transferred from the chip by radiation and conduction (to the substrate), as well as by convection to the jet. An energy balance for the chip yields $q_{elec} = q_{conv} + q_{cond} + q_{rad}$. Hence, with $q_{conv} = hA\left(T_s - T_\infty\right)$, where A = 100 mm^2 is the surface area of the chip,

$$h = \frac{q_{elec} - q_{cond} - q_{rad}}{A\left(T_s - T_\infty\right)} \qquad (1)$$

While the electric power (q_{elec}) and the jet (T_∞) and surface (T_s) temperatures may be measured, losses from the chip by conduction and radiation would have to be estimated. Unless the losses are negligible (an unlikely condition), the accuracy of the procedure could be compromised by uncertainties associated with determining the conduction and radiation losses.

A second approach, Case (b), could involve fabrication of a heater assembly for which the conduction and radiation losses are controlled and minimized. A 10 mm × 10 mm copper block (k ~ 400 W/m·K) could be inserted in a poorly conducting substrate (k < 0.1 W/m·K) and a patch heater could be applied to the back of the block and insulated from below. If conduction to both the substrate and insulation could thereby be rendered negligible, heat would be transferred almost exclusively through the block. If radiation were rendered negligible by applying a low emissivity coating (ε < 0.1) to the surface of the copper block, virtually all of the heat would be transferred by convection to the jet. Hence, q_{cond} and q_{rad} may be neglected in equation (1), and the expression may be used to accurately determine h from the known (A) and measured (q_{elec}, T_s, T_∞) quantities.

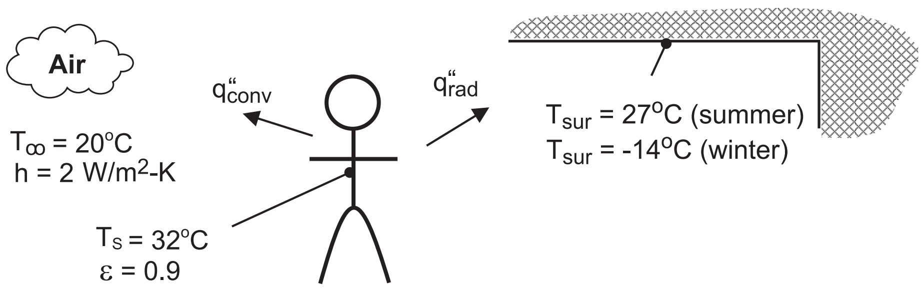
COMMENTS: Since convection coefficients associated with gas flows are generally small, concurrent heat transfer by radiation and/or conduction must often be considered. However, jet impingement is one of the more effective means of transferring heat by convection and convection coefficients well in excess of 100 W/m^2·K may be achieved.

PROBLEM 1.24

KNOWN: Air and wall temperatures of a room. Surface temperature, convection coefficient and emissivity of a person in the room.

FIND: Basis for difference in comfort level between summer and winter.

SCHEMATIC:



ASSUMPTIONS: (1) Person may be approximated as a small object in a large enclosure.

ANALYSIS: Thermal comfort is linked to heat loss from the human body, and a *chilled* feeling is associated with excessive heat loss. Because the temperature of the room air is fixed, the different summer and winter comfort levels can not be attributed to convection heat transfer from the body. In both cases, the heat flux is

Summer and Winter: $$q''_{conv} = h\left(T_s - T_\infty\right) = 2\ W/m^2 \cdot K \times 12\ ^\circ C = 24\ W/m^2$$

However, the heat flux due to radiation will differ, with values of

Summer: $$q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(305^4 - 300^4\right) K^4 = 28.3\ W/m^2$$

Winter: $$q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(305^4 - 287^4\right) K^4 = 95.4\ W/m^2$$

There is a significant difference between winter and summer radiation fluxes, and the chilled condition is attributable to the effect of the colder walls on radiation.

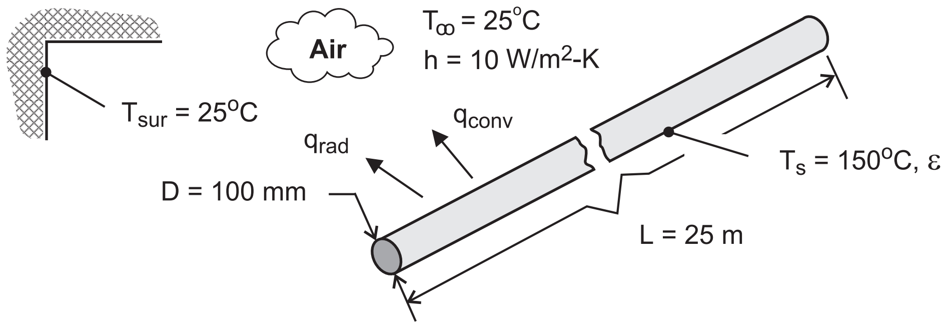
COMMENTS: For a representative surface area of A = 1.5 m^2, the heat losses are q_{conv} = 36 W, $q_{rad(summer)}$ = 42.5 W and $q_{rad(winter)}$ = 143.1 W. The winter time radiation loss is significant and if maintained over a 24 h period would amount to 2,950 kcal.

PROBLEM 1.28

KNOWN: Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

FIND: (a) Rate of heat loss, (b) Annual cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

ANALYSIS: (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi\left(0.1\text{m} \times 25\text{m}\right) = 7.85\text{m}^2$.

Hence,

$$q = 7.85\text{m}^2\left[10\ \text{W/m}^2 \cdot \text{K}\left(150 - 25\right)\text{K} + 0.8 \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4\left(423^4 - 298^4\right)\text{K}^4\right]$$

$$q = 7.85\text{m}^2\left(1,250 + 1,095\right)\text{w/m}^2 = \left(9813 + 8592\right)\text{W} = 18,405\ \text{W}$$ <

(b) The annual energy loss is

$$E = qt = 18,405\ \text{W} \times 3600\ \text{s/h} \times 24\text{h/d} \times 365\ \text{d/y} = 5.80 \times 10^{11}\ \text{J}$$

With a furnace energy consumption of $E_f = E/\eta_f = 6.45 \times 10^{11}\ \text{J}$, the annual cost of the loss is

$$C = C_g E_f = 0.01\ \$/\text{MJ} \times 6.45 \times 10^5\text{MJ} = \$6450$$ <

COMMENTS: The heat loss and related costs are unacceptable and should be reduced by insulating the steam line.

PROBLEM 1.29

KNOWN: Exact and approximate expressions for the linearized radiation coefficient, h_r and h_{ra}, respectively.

FIND: (a) Comparison of the coefficients with ε = 0.05 and 0.9 and surface temperatures which may exceed that of the surroundings (T_{sur} = 25°C) by 10 to 100°C; also comparison with a free convection coefficient correlation, (b) Plot of the relative error $(h_r - r_{ra})/h_r$ as a function of the furnace temperature associated with a workpiece at T_s = 25°C having ε = 0.05, 0.2 or 0.9.

ASSUMPTIONS: (1) Furnace walls are large compared to the workpiece and (2) Steady-state conditions.

ANALYSIS: (a) The linearized radiation coefficient, Eq. 1.9, follows from the radiation exchange rate equation,

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$$

If $T_s \approx T_{sur}$, the coefficient may be approximated by the simpler expression

$$h_{r,a} = 4\varepsilon\sigma\bar{T}^3 \qquad \bar{T} = \left(T_s + T_{sur}\right)/2$$

For the condition of ε = 0.05, $T_s = T_{sur} + 10 = 35°C = 308$ K and T_{sur} = 25°C = 298 K, find that

$$h_r = 0.05\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(308+298)\left(308^2+298^2\right)K^3 = 0.32\,W/m^2\cdot K \qquad <$$

$$h_{r,a} = 4\times0.05\times5.67\times10^{-8}\,W/m^2\cdot K^4\left((308+298)/2\right)^3 K^3 = 0.32\,W/m^2\cdot K \qquad <$$

The free convection coefficient with T_s = 35°C and $T_\infty = T_{sur}$ = 25°C, find that

$$h = 0.98\Delta T^{1/3} = 0.98\left(T_s - T_\infty\right)^{1/3} = 0.98(308-298)^{1/3} = 2.1\,W/m^2\cdot K \qquad <$$

For the range $T_s - T_{sur}$ = 10 to 100°C with ε = 0.05 and 0.9, the results for the coefficients are tabulated below. For this range of surface and surroundings temperatures, the radiation and free convection coefficients are of comparable magnitude for moderate values of the emissivity, say ε > 0.2. The approximate expression for the linearized radiation coefficient is valid within 2% for these conditions.

(b) The above expressions for the radiation coefficients, h_r and $h_{r,a}$, are used for the workpiece at T_s = 25°C placed inside a furnace with walls which may vary from 100 to 1000°C. The relative error, $(h_r - h_{ra})/h_r$, will be independent of the surface emissivity and is plotted as a function of T_{sur}. For T_{sur} > 150°C, the approximate expression provides estimates which are in error more than 5%. The approximate expression should be used with caution, and only for surface and surrounding temperature differences of 50 to 100°C.

		Coefficients ($W/m^2\cdot K$)		
T_s (°C)	ε	h_r	$h_{r,a}$	h
35	0.05	0.32	0.32	2.1
	0.9	5.7	5.7	
135	0.05	0.51	0.50	4.7
	0.9	9.2	9.0