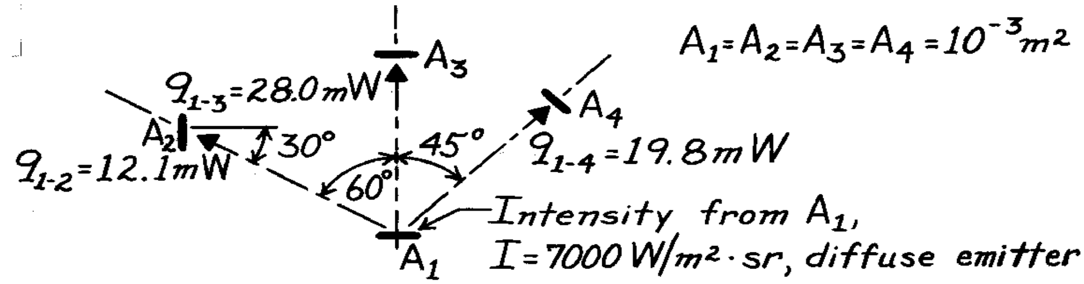
PROBLEM 12.1

KNOWN: Rate at which radiation is intercepted by each of three surfaces (see (Example 12.1).

FIND: Irradiation, G[W/m^2], at each of the three surfaces.

SCHEMATIC:



ANALYSIS: The irradiation at a surface is the rate at which radiation is incident on a surface per unit area of the surface. The irradiation at surface j due to emission from surface 1 is

$$G_j = \frac{q_{1-j}}{A_j}.$$

With $A_1 = A_2 = A_3 = A_4 = 10^{-3}$ m^2 and the incident radiation rates $q_{1\text{-}j}$ from the results of Example 12.1, find

$$G_2 = \frac{12.1\times10^{-3}\,\text{W}}{10^{-3}\,\text{m}^2} = 12.1\,\text{W/m}^2 \qquad <$$

$$G_3 = \frac{28.0\times10^{-3}\,\text{W}}{10^{-3}\,\text{m}^2} = 28.0\,\text{W/m}^2 \qquad <$$

$$G_4 = \frac{19.8\times10^{-3}\,\text{W}}{10^{-3}\,\text{m}^2} = 19.8\,\text{W/m}^2. \qquad <$$

COMMENTS: The irradiation could also be computed from Eq. 12.15, which, for the present situation, takes the form

$$G_j = I_1 \cos\theta_j\, \omega_{1-j}$$

where $I_1 = I = 7000$ W/m^2·sr and $\omega_{1\text{-}j}$ is the solid angle subtended by surface 1 with respect to j. For example,

$$G_2 = I_1 \cos\theta_2\, \omega_{1-2}$$

$$G_2 = 7000\,\text{W/m}^2\cdot\text{sr}\times$$

$$\cos 30^\circ \frac{10^{-3}\,\text{m}^2\times\cos 60^\circ}{(0.5\text{m})^2}$$

$$G_2 = 12.1\,\text{W/m}^2.$$

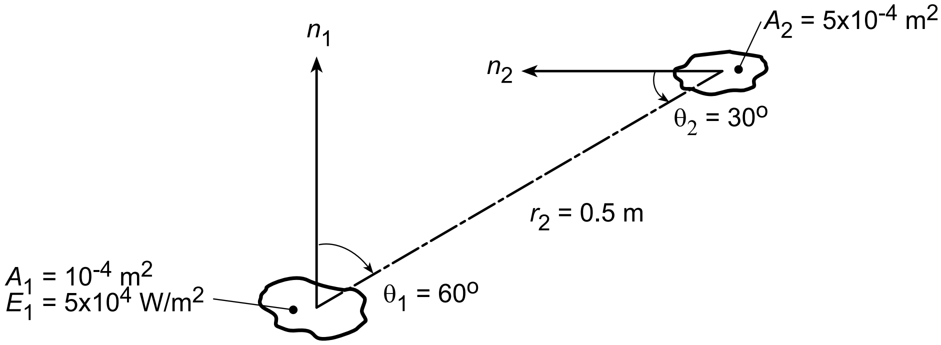
Note that, since A_1 is a diffuse radiator, the intensity I is independent of direction.

PROBLEM 12.2

KNOWN: A diffuse surface of area $A_1 = 10^{-4} m^2$ emits diffusely with total emissive power $E = 5 \times 10^4$ W/m^2 .

FIND: (a) Rate this emission is intercepted by small surface of area $A_2 = 5 \times 10^{-4}\ m^2$ at a prescribed location and orientation, (b) Irradiation G_2 on A_2, and (c) Compute and plot G_2 as a function of the separation distance r_2 for the range $0.25 \le r_2 \le 1.0$ m for zenith angles $\theta_2 = 0$, 30 and 60°.

SCHEMATIC:



ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) A_1 may be approximated as a differential surface area and that $A_2/r_2^2 << 1$.

ANALYSIS: (a) The rate at which emission from A_1 is intercepted by A_2 follows from Eq. 12.5 written on a total rather than spectral basis.

$$q_{1\to2} = I_{e,1}(\theta,\phi) A_1 \cos\theta_1 d\omega_{2-1}. \tag{1}$$

Since the surface A_1 is diffuse, it follows from Eq. 12.13 that

$$I_{e,1}(\theta,\phi) = I_{e,1} = E_1/\pi \ . \tag{2}$$

The solid angle subtended by A_2 with respect to A_1 is

$$d\omega_{2-1} \approx A_2 \cdot \cos\theta_2 / r_2^2 \ . \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1) with numerical values gives

$$q_{1\to2} = \frac{E_1}{\pi} \cdot A_1 \cos\theta_1 \cdot \frac{A_2 \cos\theta_2}{r_2^2} = \frac{5\times10^4\ W/m^2}{\pi\ sr} \times \left(10^{-4} m^2 \times \cos 60^\circ\right) \times \left[\frac{5\times10^{-4} m^2 \times \cos 30^\circ}{(0.5m)^2}\right] sr \tag{4}$$

$$q_{1\to2} = 15,915\ W/m^2 sr \times \left(5\times10^{-5} m^2\right) \times 1.732\times10^{-3} sr = 1.378\times10^{-3} W.$$ <

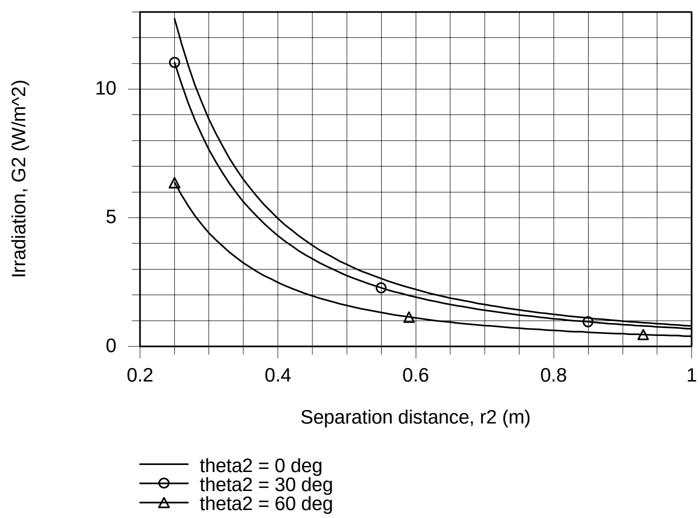
(b) From section 12, 2.3, the irradiation is the rate at which radiation is incident upon the surface per unit surface area,

$$G_2 = \frac{q_{1\to2}}{A_2} = \frac{1.378\times10^{-3} W}{5\times10^{-4} m^2} = 2.76\ W/m^2 \tag{5}$$ <

(c) Using the IHT workspace with the foregoing equations, the G_2 was computed as a function of the separation distance for selected zenith angles. The results are plotted below.

Continued...

PROBLEM 12.2 (Cont.)



For all zenith angles, G_2 decreases with increasing separation distance r_2. From Eq. (3), note that $d\omega_{2-1}$ and, hence G_2, vary inversely as the square of the separation distance. For any fixed separation distance, G_2 is a maximum when $\theta_2 = 0°$ and decreases with increasing θ_2, proportional to $\cos\theta_2$.

COMMENTS: (1) For a diffuse surface, the intensity, I_e, is independent of direction and related to the emissive power as $I_e = E/\pi$. Note that π has the units of $[sr]$ in this relation.

(2) Note that Eq. 12.5 is an important relation for determining the radiant power leaving a surface in a prescribed manner. It has been used here on a total rather than spectral basis.

(3) Returning to part (b) and referring to Figure 12.10, the irradiation on A2 may be expressed as

$$G_2 = I_{i,2} \cos\theta_2 \frac{A_1 \cos\theta_1}{r_2^2}$$

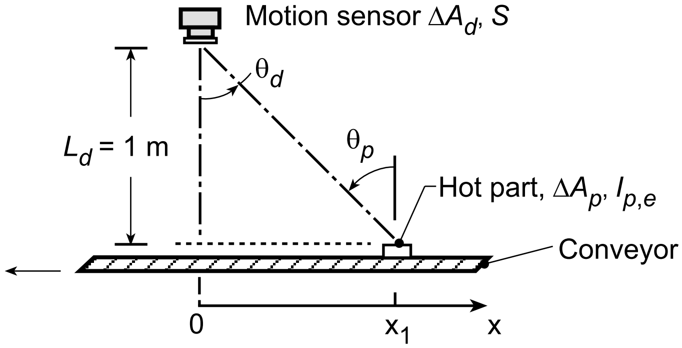
Show that the result is $G_2 = 2.76\ W/m^2$. Explain how this expression follows from Eq. (12.15).

PROBLEM 12.13

KNOWN: Hot part, ΔA_p, located a distance x_1 from an origin directly beneath a motion sensor at a distance $L_d = 1$ m.

FIND: (a) Location x_1 at which sensor signal S_1 will be 75% that corresponding to x = 0, directly beneath the sensor, S_o, and (b) Compute and plot the signal ratio, S/S_o, as a function of the part position x_1 for the range $0.2 \le S/S_o \le 1$ for L_d = 0.8, 1.0 and 1.2 m; compare the x-location for each value of L_d at which $S/S_o = 0.75$.

SCHEMATIC:



ASSUMPTIONS: (1) Hot part is diffuse emitter, (2) $L_d^2 >> \Delta A_p, \Delta A_o$.

ANALYSIS: (a) The sensor signal, S, is proportional to the radiant power leaving ΔA_p and intercepted by ΔA_d,

$$S \sim q_{p\to d} = I_{p,e}\Delta A_p \cos\theta_p \Delta\omega_{d-p} \qquad (1)$$

when

$$\cos\theta_p = \cos\theta_d = \frac{L_d}{R} = L_d \Big/ (L_d^2 + x_1^2)^{1/2} \qquad (2)$$

$$\Delta\omega_{d-p} = \frac{\Delta A_d \cdot \cos\theta_d}{R^2} = \Delta A_d \cdot L_d \Big/ (L_d^2 + x_1^2)^{3/2} \qquad (3)$$

Hence,

$$q_{p\to d} = I_{p,e}\Delta A_p \Delta A_d \frac{L_d^2}{(L_d^2 + x_1^2)^2} \qquad (4)$$

It follows that, with S_o occurring when x= 0 and L_d = 1 m,

$$\frac{S}{S_o} = \frac{L_d^2 \big/ (L_d^2 + x_1^2)^2}{L_d^2 \big/ (L_d^2 + 0^2)^2} = \left[\frac{L_d^2}{L_d^2 + x_1^2}\right]^2 \qquad (5)$$

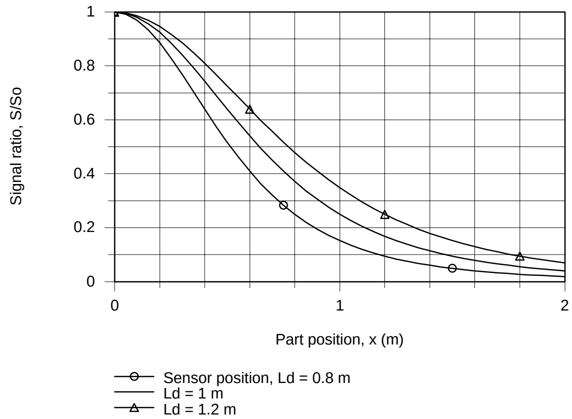
so that when $S/S_o = 0.75$, find,

$x_1 = 0.393$ m <

(b) Using Eq. (5) in the IHT workspace, the signal ratio, S/S_o, has been computed and plotted as a function of the part position x for selected L_d values.

Continued...

PROBLEM 12.13 (Cont.)



When the part is directly under the sensor, x = 0, $S/S_o = 1$ for all values of L_d. With increasing x, S/S_o decreases most rapidly with the smallest L_d. From the IHT model we found the part position x corresponding to $S/S_o = 0.75$ as follows.

S/S_o	L_d (m)	x_1 (m)
0.75	0.8	0.315
0.75	1.0	0.393
0.75	1.2	0.472

If the sensor system is set so that when S/S_o reaches 0.75 a process is initiated, the technician can use the above plot and table to determine at what position the part will begin to experience the treatment process.

PROBLEM 13.1

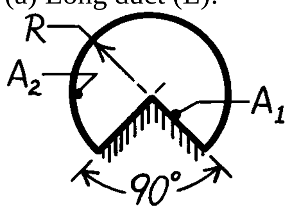
KNOWN: Various geometric shapes involving two areas A_1 and A_2.

FIND: Shape factors, F_{12} and F_{21}, for each configuration.

ASSUMPTIONS: Surfaces are diffuse.

ANALYSIS: The analysis is not to make use of tables or charts. The approach involves use of the reciprocity relation, Eq. 13.3, and summation rule, Eq. 13.4. Recognize that reciprocity applies to two surfaces; summation applies to an enclosure. Certain shape factors will be identified by inspection. Note L is the length normal to page.

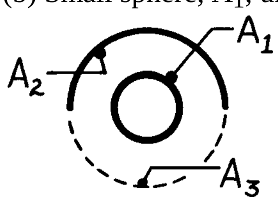
(a) Long duct (L):



By inspection, $F_{12} = 1.0$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2\,RL}{(3/4)\cdot 2\pi RL} \times 1.0 = \frac{4}{3\pi} = 0.424$ <

(b) Small sphere, A_1, under concentric hemisphere, A_2, where $A_2 = 2A$

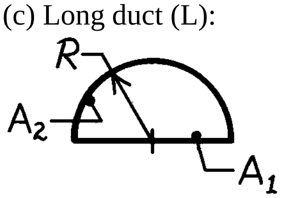


Summation rule $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} \times 0.5 = 0.25.$ <

(c) Long duct (L):



<

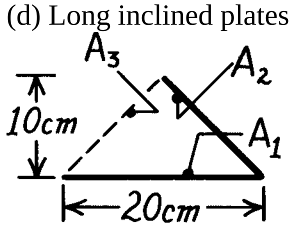
By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{\pi RL} \times 1.0 = \frac{2}{\pi} = 0.637$ <

Summation rule, $F_{22} = 1 - F_{21} = 1 - 0.64 = 0.363.$ <

By inspection,

$F_{12} = 1.0$

(d) Long inclined plates (L):

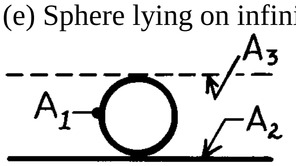


Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20L}{10(2)^{1/2} L} \times 0.5 = 0.707.$ <

(e) Sphere lying on infinite plane



Summation rule, $F_{11} + F_{12} + F_{13} = 1$

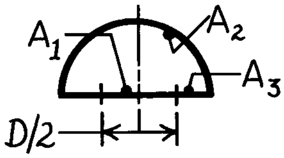
But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} \to 0$ since $A_2 \to \infty.$ <

Continued …..

PROBLEM 13.1 (Cont.)

(f) Hemisphere over a disc of diameter D/2; find also F_{22} and F_{23}.



By inspection, $F_{12} = 1.0$ <

Summation rule for surface A_3 is written as

$$F_{31} + F_{32} + F_{33} = 1. \quad \text{Hence, } F_{32} = 1.0.$$

By reciprocity, $$F_{23} = \frac{A_3}{A_2} F_{32}$$

$$F_{23} = \left\{\left[\frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4}\right] / \frac{\pi D^2}{2}\right\} 1.0 = 0.375.$$

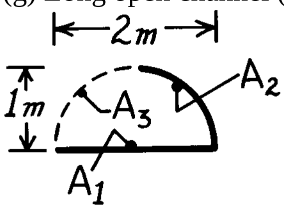
By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \left\{\frac{\pi}{4}\left[\frac{D}{2}\right]^2 / \frac{\pi D^2}{2}\right\} \times 1.0 = 0.125.$$ <

Summation rule for A_2, $$F_{21} + F_{22} + F_{23} = 1 \text{ or}$$

$$F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5.$$ <

Note that by inspection you can deduce $F_{22} = 0.5$

(g) Long open channel (L):



Summation rule for A_1

$$F_{11} + F_{12} + F_{13} = 0$$

but $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$. <

By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi 1)/4 \times L} = \frac{4}{\pi} \times 0.50 = 0.637.$$

COMMENTS: (1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

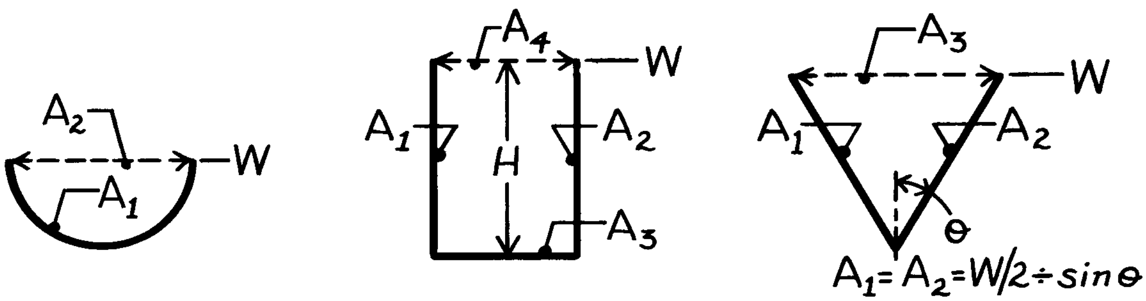
(2) Recognize that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.

PROBLEM 13.2

KNOWN: Geometry of semi-circular, rectangular and V grooves.

FIND: (a) View factors of grooves with respect to surroundings, (b) View factor for sides of V groove, (c) View factor for sides of rectangular groove.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces, (2) Negligible end effects, "long grooves".

ANALYSIS: (a) Consider a unit length of each groove and represent the surroundings by a hypothetical surface (dashed line).

Semi-Circular Groove:

$$F_{21} = 1; \qquad F_{12} = \frac{A_2}{A_1} F_{21} = \frac{W}{(\pi W/2)} \times 1$$

$$F_{12} = 2/\pi. \qquad <$$

Rectangular Groove:

$$F_{4(1,2,3)} = 1; \qquad F_{(1,2,3)4} = \frac{A_4}{A_1 + A_2 + A_3} F_{4(1,2,3)} = \frac{W}{H + W + H} \times 1$$

$$F_{(1,2,3)4} = W/(W + 2H). \qquad <$$

V Groove:

$$F_{3(1,2)} = 1; \qquad F_{(1,2)3} = \frac{A_3}{A_1 + A_2} F_{3(1,2)} = \frac{W}{\frac{W/2}{\sin\theta} + \frac{W/2}{\sin\theta}}$$

$$F_{(1,2)3} = \sin\theta.$$

(b) From Eqs. 13.3 and 13.4, $\quad F_{12} = 1 - F_{13} = 1 - \frac{A_3}{A_1} F_{31}.$

From Symmetry, $\quad F_{31} = 1/2.$

Hence, $F_{12} = 1 - \frac{W}{(W/2)/\sin\theta} \times \frac{1}{2}$ or $F_{12} = 1 - \sin\theta.$ <

(c) From Fig. 13.4, with X/L = H/W =2 and Y/L $\to \infty$,

$$F_{12} \approx 0.62. \qquad <$$

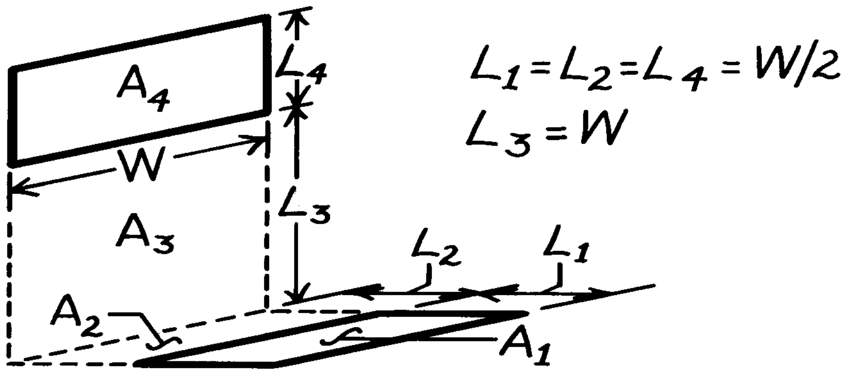
COMMENTS: (1) Note that for the V groove, $F_{13} = F_{23} = F_{(1,2)3} = \sin\theta$, (2) In part (c), Fig. 13.4 could also be used with Y/L = 2 and X/L = ∞. However, obtaining the limit of F_{ij} as X/L $\to \infty$ from the figure is somewhat uncertain.

PROBLEM 13.10

KNOWN: Arrangement of perpendicular surfaces without a common edge.

FIND: (a) A relation for the view factor F_{14} and (b) The value of F_{14} for prescribed dimensions.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces.

ANALYSIS: (a) To determine F_{14}, it is convenient to define the hypothetical surfaces A_2 and A_3. From Eq. 13.6,

$$(A_1 + A_2) F_{(1,2)(3,4)} = A_1 F_{1(3,4)} + A_2 F_{2(3,4)}$$

where $F_{(1,2)(3,4)}$ and $F_{2(3,4)}$ may be obtained from Fig. 13.6. Substituting for $A_1 F_{1(3,4)}$ from Eq. 13.5 and combining expressions, find

$$A_1 F_{1(3,4)} = A_1 F_{13} + A_1 F_{14}$$

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2) F_{(1,2)(3,4)} - A_1 F_{13} - A_2 F_{2(3,4)}\right].$$

Substituting for $A_1 F_{13}$ from Eq. 13.6, which may be expressed as

$$(A_1 + A_2) F_{(1,2)3} = A_1 F_{13} + A_2 F_{23}.$$

The desired relation is then

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2) F_{(1,2)(3,4)} + A_2 F_{23} - (A_1 + A_2) F_{(1,2)3} - A_2 F_{2(3,4)}\right].$$ <

(b) For the prescribed dimensions and using Fig. 13.6, find these view factors:

Surfaces (1,2)(3,4) $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.45, \quad F_{(1,2)(3,4)} = 0.22$

Surfaces 23 $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{23} = 0.28$

Surfaces (1,2)3 $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{(1,2)3} = 0.20$

Surfaces 2(3,4) $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.5, \quad F_{2(3,4)} = 0.31$

Using the relation above, find

$$F_{14} = \frac{1}{(WL_1)}\left[(WL_1 + WL_2)0.22 + (WL_2)0.28 - (WL_1 + WL_2)0.20 - (WL_2)0.31\right]$$

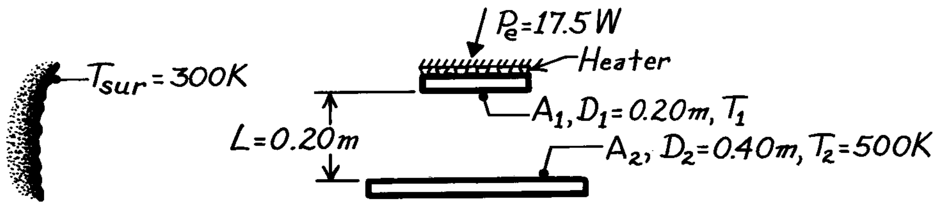
$$F_{14} = \left[2(0.22) + 1(0.28) - 2(0.20) - 1(0.31)\right] = 0.01.$$ <

PROBLEM 13.21

KNOWN: Coaxial, parallel black plates with surroundings. Lower plate (A_2) maintained at prescribed temperature T_2 while electrical power is supplied to upper plate (A_1).

FIND: Temperature of the upper plate T_1.

SCHEMATIC:



ASSUMPTIONS: (1) Plates are black surfaces of uniform temperature, and (2) Backside of heater on A_1 insulated.

ANALYSIS: The net radiation heat rate leaving A_i is

$$P_e = \sum_{j=1}^{N} q_{ij} = A_1 F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1 F_{13}\sigma\left(T_1^4 - T_3^4\right)$$

$$P_e = A_1\sigma\left[F_{12}\left(T_1^4 - T_2^4\right) + F_{13}\left(T_1^4 - T_{sur}^4\right)\right] \qquad (1)$$

From Fig. 13.5 for coaxial disks (see Table 13.2),

$$R_1 = r_1/L = 0.10\ \text{m}/0.20\ \text{m} = 0.5 \qquad R_2 = r_2/L = 0.20\ \text{m}/0.20\ \text{m} = 1.0$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+1^2}{(0.5)^2} = 9.0$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(r_2/r_1\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{9 - \left[9^2 - 4\left(0.2/0.1\right)^2\right]^{1/2}\right\} = 0.469.$$

From the summation rule for the enclosure A_1, A_2 and A_3 where the last area represents the surroundings with $T_3 = T_{sur}$,

$$F_{12} + F_{13} = 1 \qquad F_{13} = 1 - F_{12} = 1 - 0.469 = 0.531.$$

Substituting numerical values into Eq. (1), with $A_1 = \pi D_1^2/4 = 3.142\times10^{-2}\,\text{m}^2$,

$$17.5\ \text{W} = 3.142\times10^{-2}\,\text{m}^2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[0.469\left(T_1^4 - 500^4\right)\text{K}^4 + 0.531\left(T_1^4 - 300^4\right)\text{K}^4\right]$$

$$9.823\times10^9 = 0.469\left(T_1^4 - 500^4\right) + 0.531\left(T_1^4 - 300^4\right)$$

find by trial-and-error that $T_1 = 456\ \text{K}.$ $<$

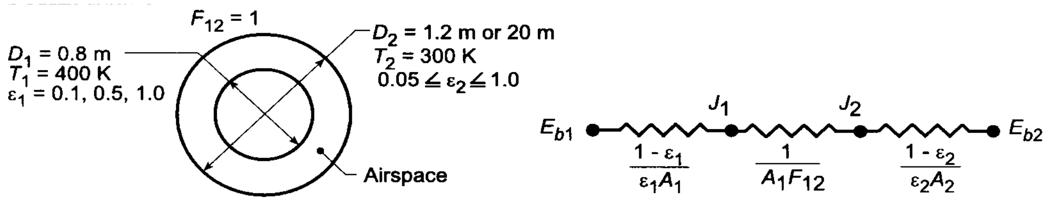
COMMENTS: Note that if the upper plate were adiabatic, T_1 = 427 K.

PROBLEM 13.53

KNOWN: Emissivities, diameters and temperatures of concentric spheres.

FIND: (a) Radiation transfer rate for black surfaces. (b) Radiation transfer rate for diffuse-gray surfaces, (c) Effects of increasing the diameter and assuming blackbody behavior for the outer sphere. (d) Effect of emissivities on net radiation exchange.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody or diffuse-gray surface behavior.

ANALYSIS: (a) Assuming blackbody behavior, it follows from Eq. 13.13

$$q_{12} = A_1 F_{12} \sigma \left(T_1^2 - T_2^4\right) = \pi (0.8\,\text{m})^2 (1) 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \left[(400\ \text{K})^4 - (300\ \text{K})^4\right] = 1995\ \text{W}.$$ <

(b) For diffuse-gray surface behavior, it follows from Eq. 13.26

$$q_{12} = \frac{\sigma A_1 \left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \pi (0.8\,\text{m})^2 \left[400^4 - 300^4\right]\text{K}^4}{\dfrac{1}{0.5} + \dfrac{1-0.05}{0.05}\left(\dfrac{0.4}{0.6}\right)^2} = 191\ \text{W}.$$ <

(c) With D_2 = 20 m, it follows from Eq. 13.26

$$q_{12} = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}\pi (0.8\,\text{m})^2 \left[(400\ \text{K})^4 - (300\ \text{K})^4\right]}{\dfrac{1}{0.5} + \dfrac{1-0.05}{0.05}\left(\dfrac{0.4}{10}\right)^2} = 983\ \text{W}.$$ <

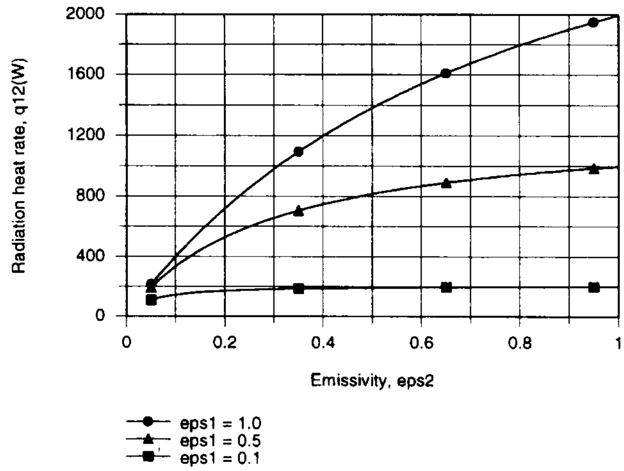
With $\varepsilon_2 = 1$, instead of 0.05, Eq. 13.26 reduces to Eq. 13.27 and

$$q_{12} = \sigma A_1 \varepsilon_1 \left(T_1^4 - T_2^4\right) = 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \pi (0.8\,\text{m})^2\, 0.5 \left[(400\ \text{K})^4 - (300\ \text{K})^4\right] = 998\ \text{W}.$$ <

Continued …..

PROBLEM 13.53 (Cont.)

(d) Using the *IHT Radiation* Tool Pad, the following results were obtained



Net radiation exchange increases with ε_1 and ε_2, and the trends are due to increases in emission from and absorption by surfaces 1 and 2, respectively.

COMMENTS: From part (c) it is evident that the actual surface emissivity of a *large* enclosure has a small effect on radiation exchange with small surfaces in the enclosure. Working with $\varepsilon_2 = 1.0$ instead of $\varepsilon_2 = 0.05$, the value of q_{12} is increased by only (998 – 983)/983 = 1.5%. In contrast, from the results of (d) it is evident that the surface emissivity ε_2 of a *small* enclosure has a large effect on radiation exchange with interior objects, which increases with increasing ε_1.