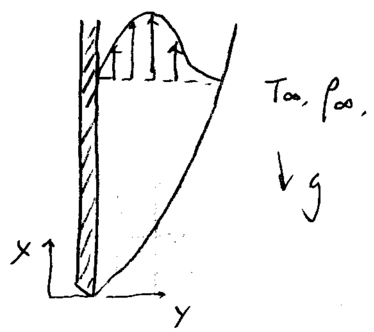
B. Comparison with forced convection.

$$\bar{N}_u = f(Re_L, Pr)$$

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho u L}{\mu}$$

— What is the form of Re for natural convection



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}}_{\text{inertia}} = \underbrace{g\beta(T - T_\infty)}_{\text{buoyancy}} + \underbrace{\nu\frac{\partial^2 u}{\partial y^2}}_{\text{friction}}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

where β : volumetric thermal expansion coefficient

$$\beta \equiv -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T}$$

$$= -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T}$$

① Velocity scale

— Buoyancy balanced by friction

$$u \sim \frac{\alpha}{L} Ra^{1/2}$$

$$Ra = \frac{g\beta(T_w - T_\infty)L^3}{\alpha\nu} \quad \text{— Rayleigh number}$$

— Buoyancy balanced by inertia

$$u \sim \frac{\alpha}{L}(Ra\,Pr)^{1/2}$$

$$Gr = \frac{Ra}{Pr} = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} \quad \text{— Grashof number}$$

$$Re = \frac{uL}{\nu} = (Gr)^{1/2}$$

C. In general $\bar{N}u_L = f(Re_L, Gr_L, Pr)$ when $\frac{Gr_L}{Re_L^2} \approx 1$

Forced convection $\frac{Gr_L}{Re_L^2} \ll 1$. $\bar{N}u_L = f(Re_L, Pr)$

Free convection $\frac{Gr_L}{Re_L^2} \gg 1$ $\bar{N}u_L = f(Gr_L, Pr)$

D. Transition of B.L. in free convection is characterized by a critical $Ra_x \approx 10^9$

E. Laminar flow free convection possesses similarity solution that provides

$$Nu_x = \frac{hx}{k} = \left(\frac{Gr_x}{4}\right)^{1/4} g(Pr)$$

$$\bar{N}u_L = \frac{\bar{h}L}{k} = \frac{4}{3}\left(\frac{Gr_x}{4}\right)^{1/4} g(Pr) = \frac{4}{3} Nu_L$$

$$g(Pr) = \frac{0.75\, Pr^{1/2}}{(0.609 + 1.221\, Pr^{1/2} + 1.238\, Pr)^{1/4}}$$

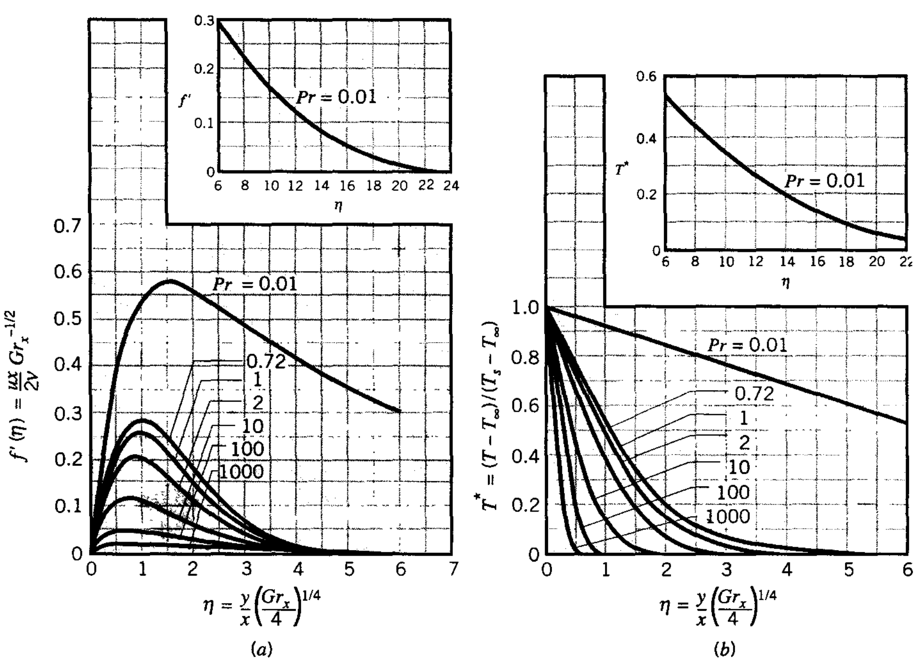


FIGURE 9.4 Laminar, free convection boundary layer conditions on an isothermal, vertical surface. (*a*) Velocity profiles. (*b*) Temperature profiles [3].

II. External natural convection

A. Isothermal surfaces

In general $\bar{Nu}_L = \dfrac{\bar{h} L}{k} = C Ra_L^n$

w/ $n = \frac{1}{4}$ for laminar flow

$n = \frac{1}{3}$ for turbulent flow. $10^9 \le Ra_L \le 10^{13}$

Again, properties evaluated at $T_f = \dfrac{T_\infty + T_s}{2}$

1). Vertical flat plate.

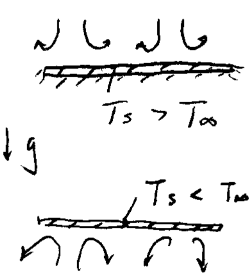
$$\bar{Nu}_L = \left\{ 0.825 + \frac{0.387 \, Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{\frac{9}{16}}\right]^{8/27}} \right\}^2 \quad \text{for all } Ra_L \quad (9.26)$$

for laminar flow, a slightly better correlation

$$\bar{Nu} = 0.68 + \frac{0.670 \, Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{\frac{9}{16}}\right]^{4/9}}, \quad Ra_L \lesssim 10^9 \quad (9.27)$$

2). Horizontal flat plate

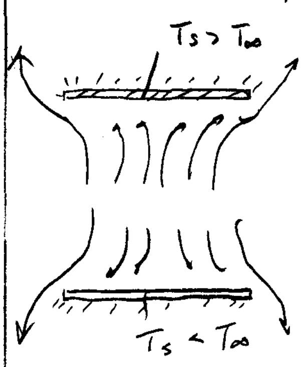
i. Hot surface facing up (cold surface facing down)



$\bar{Nu}_L = 0.54 \, Ra_L^{1/4} \quad (10^4 \lesssim Ra_L \le 10^7)$

$\bar{Nu}_L = 0.15 \, Ra_L^{1/3} \quad (10^7 \lesssim Ra_L \lesssim 10^{11})$

ii. Hot surface facing down (cold surface facing up)



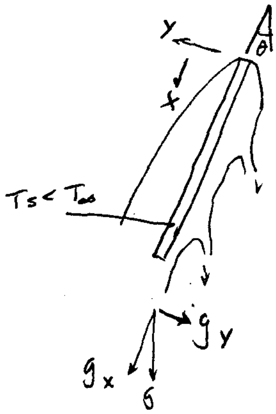
$\bar{Nu}_L = 0.27 \, Ra^{1/4} \quad (10^5 \lesssim Ra_L \lesssim 10^{10})$

* ineffective

$L \equiv \dfrac{A_s}{P}$ $\quad A_s$: surface area

P : perimeter.

3). Inclined plate.



Top: reduction in H.T. compared to vertical plate.

Bottom: increase in H.T. due to buoyancy component g_y

Top: increase in H.T.

Bottom: reduction in H.T.

— $0° \le \theta \le 60°$: for top & bottom of cooled and heated plate respectively. g replaced by $g\cos\theta$ in (9.26) & (9.27).

— $0° \le \theta \le 60°$: for bottom & top of cooled and heated plate respectively. use literature.

4). Long vertical cylinders.

if $\delta \ll D \Leftarrow \dfrac{D}{L} \ll \dfrac{35}{Gr^{1/4}}$

Curvature effect ignored. use (9.26), (9.27)

5). Long horizontal cylinders — isothermal surface

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1 + \left(\frac{0.559}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}} \right\}^2$$

— by Churchill & Chu. $Ra_D \le 10^{12}$

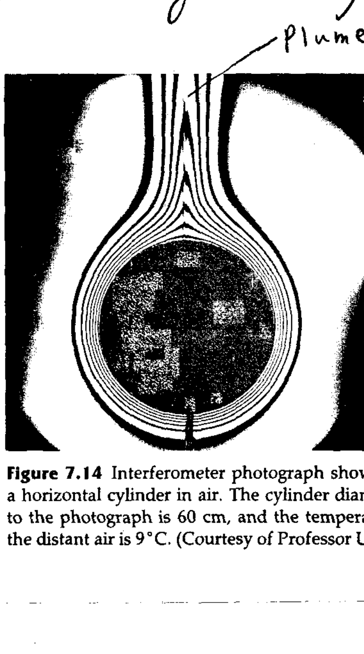
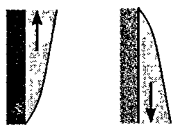
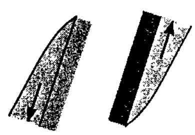
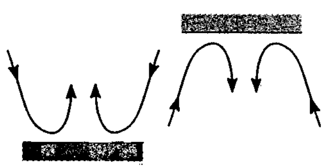
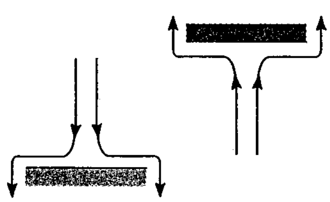
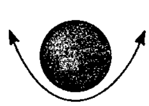
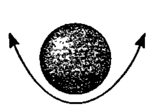


Figure 7.14 Interferometer photograph showing the laminar boundary layer around a horizontal cylinder in air. The cylinder diameter is 6 cm, the length perpendicular to the photograph is 60 cm, and the temperature difference between the cylinder and the distant air is 9°C. (Courtesy of Professor U. Grigull, Technical University of Munich.)

TABLE 9.2 Summary of free convection empirical correlations for immersed geometries

Geometry	Recommended Correlation	Restrictions
1. Vertical plates[a] 	Equation 9.26	None
2. Inclined plates Cold surface up or hot surface down 	Equation 9.26 $g \rightarrow g \cos\theta$	$0 \lesssim \theta \lesssim 60°$
3. Horizontal plates (a) Hot surface up or cold surface down 	Equation 9.30	$10^4 \lesssim Ra_L \lesssim 10^7$
	Equation 9.31	$10^7 \lesssim Ra_L \lesssim 10^{11}$
(b) Cold surface up or hot surface down 	Equation 9.32	$10^5 \lesssim Ra_L \lesssim 10^{10}$
4. Horizontal cylinder 	Equation 9.34	$Ra_D \lesssim 10^{12}$
5. Sphere 	Equation 9.35	$Ra_D \lesssim 10^{11}$ $Pr \gtrsim 0.7$

[a] The correlation may be applied to a vertical cylinder if $(D/L) \gtrsim (35/Gr_L^{1/4})$

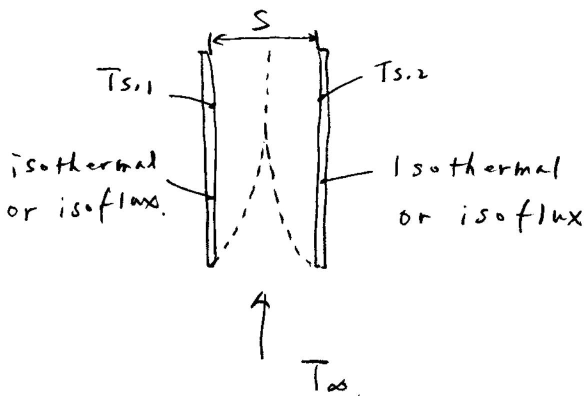
6). Sphere

$$\overline{Nu}_D = 2 + \frac{0.589\, Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}} \qquad Pr \gtrsim 0.7 \quad Ra_D \lesssim 10^{11}$$

* if $Ra_D \to 0$, $\overline{Nu}_D \to 2$. conduction.

1) - 6) summarized in table 9.2.

7). Parallel plate channels. (Fins etc.)



$T_{s,1} = T_{s,2}$ symmetric

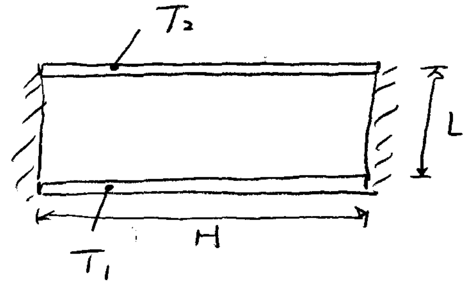
$T_{s,1} \neq T_{s,2}$ asymmetric.

S_{opt} : yields maximum $\bar{h} A_s$.

S_{max} : maximizes single plate performance, no overlap of adjoining B.L

III. Natural convection in enclosures.

A). Horizontal enclosures.



$$q'' = h\,(T_1 - T_2)$$

1. Heated from above : $T_2 > T_1$

   Thermally stable. pure conduction.

   $$h = \frac{k}{L} \Rightarrow Nu_L = 1$$

2. Heated from below. / Cooled from above

   $T_1 > T_2$

- $Ra_L \lesssim 1708$. thermally stable.

  Conduction (liquid) only, $Nu_L = 1$

- $1708 < Ra_L \lesssim 5\times10^4$ buoyancy & viscous flow.

  Thermally unstable.

- $Ra_L > 5\times10^5$,

  turbulent motion

  $$\overline{Nu}_L = 0.069\, Ra_L^{1/3}\, Pr^{0.074}$$

  $$3\times10^5 \lesssim Ra_L \leq 7\times10^9$$

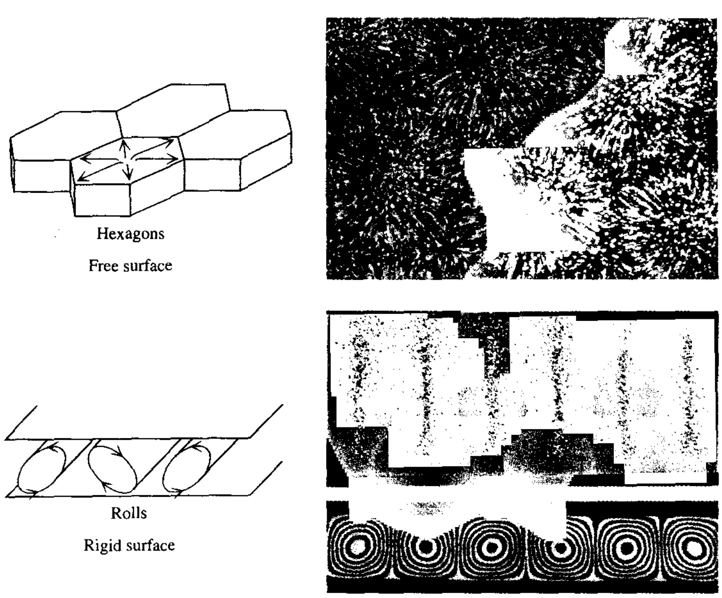
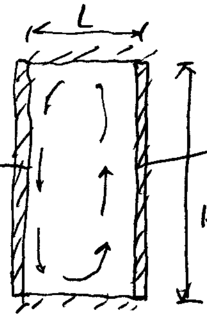


Figure 7.22 Two-dimensional rolls and three-dimensional hexagonal cells in a fluid layer heated from below. (Oertel [52], with permission from G. Braun, Karlsruhe.)

B) Vertical enclosure.



Aspect ratio : $\frac{H}{L}$

$$q'' = h(T_1 - T_2)$$

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha \nu}$$

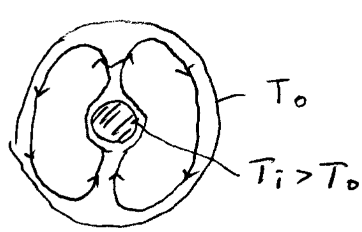
- $Ra_L \lesssim 10^3$, convection weak, mainly conduction
  $$h = \frac{k}{L} \Rightarrow Nu_L = 1$$

- $Ra_L > 10^3$, convection stronger but is concentrated in the B.L.s adjoining the side walls, the core becomes nearly stagnant.
  - * For $1 \lesssim \frac{H}{L} \lesssim 10$, (9.50). (9.51)
  - * Larger $\frac{H}{L}$, (9.52) & (9.53)

- Tilted cavity, stimulated by flat-plate type solar collectors, transition of fluid motion occurs at a critical tilt angle τ^*.

C) Other geometries.

A Concentric cylinders.



B. Concentric Spheres.

IV. Mixed convection : combined forced and natural convection.

Recall that Gr for natural convection is like Re for forced convection.

Natural convection : $\dfrac{Gr_L}{Re_L^2} >> 1$

Forced convection : $\dfrac{Gr_L}{Re_L^2} << 1$

Mixed convection : $\dfrac{Gr_L}{Re_L^2} \approx 1$

Buoyancy induced & Forced motion

- same direction → assisting flow ①
- opposite direction → opposing flow ②
- perpendicular → transverse flow ③

$$Nu^n = Nu_F^n \pm Nu_N^n$$

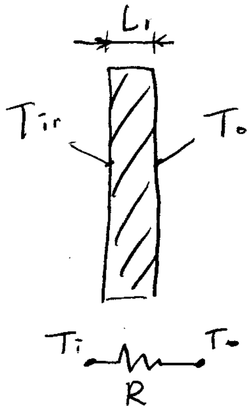
$+$: ① , ③

$-$: ②

$n = 3$, in general

$7/2$, transverse flow over horizontal plate

4, transverse flow over horizontal cylinders (spheres)

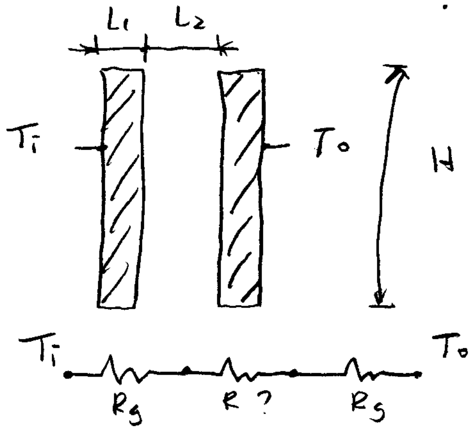


$T_i = 20°C$

$T_o = 0°C$

$A = 1\ m^2$

$k_g = 1.4\ \frac{W}{m \cdot K}$



$L_1 = 6\ mm$

$$R_g = \frac{L_1}{k_g A} = \frac{0.006}{1.4 \cdot 1} = 0.0043\ \frac{K}{W}$$

$$q = \frac{\Delta T}{R_g} = \frac{20}{0.0043} = 4651\ W$$

$L_2 = 10\ mm$

$k_{air} = 0.026\ \frac{W}{m \cdot K}$

$R_g = 0.0043$

$$R_{air} = \frac{L}{k_{air} A} = \frac{0.01}{0.026 \cdot 1} = 0.385$$

$$\Sigma R = 2R_g + R_{air} = 0.394$$

$$q = \frac{\Delta T}{\Sigma R} = 51\ W$$

Q: will there be convection in the gap due to ΔT?

* Aspect ratio: $\frac{H}{L} = \frac{1}{0.01} = 100$
* Properties: $g = 9.8\ m/s^2$ $\quad \beta \simeq \frac{1}{T_f} = \frac{1}{283}$

  $L = 0.01\ m$

  $\nu = 14.38 \times 10^{-6}\ m^2/s$

  $Pr = 0.72$
* $Ra_L = \frac{g \beta \Delta T L^3}{\nu^2} Pr$
* To begin: guess ΔT. $T_1 = 15°C$, $T_2 = 5°C$,

  $\Delta T = 10°C$

$$Ra_L = \frac{9.8 \frac{1}{283} \times 10 \times 0.01^3}{(14.38 \times 10^{-6})^2} \times 0.72 = 1205$$

— weak convection