ME 323 Midterm Exam. 11/07/02, 58pts total   Name____________________

True/False. Circle the correct answer. (1pt each, 7pts total)

1. k, h and σ are functions of temperature. T F   k, h are / σ is not
2. In general, k is a function of temperature and position. T F   k is a fn of position for anisotropic material.
3. The electrical resistor analogy for heat transfer is exact for steady 1-D heat problems with constant properties involving conduction, convection, and radiation. T F
   Thermal resistance method is suggested by eqn that determines H.T rate. $R = \frac{\Delta T}{q}$
4. The heat transfer coefficient h is a constant for heat transfer problems involving convection. T F   h is a fn of physical properties, flow condition, and surface geometry
5. Convection is the most important heat transfer mode affecting the Earth's surface temperature. T F   Radiation.
6. Conduction is the dominant heat transfer mechanism at low temperatures. T F
   Both conduction & convection can be dominant
7. Radiation is the dominant heat transfer mechanism at high temperature. T F

$$q''_{rad} = \varepsilon\sigma(T_s^4 - T_\infty^4)$$

Assortment of problems of increasing value and effort….

8. (3pts) What are the respective units of k, h, and σ?

k : $W/m \cdot K$

h : $W/m^2 K$

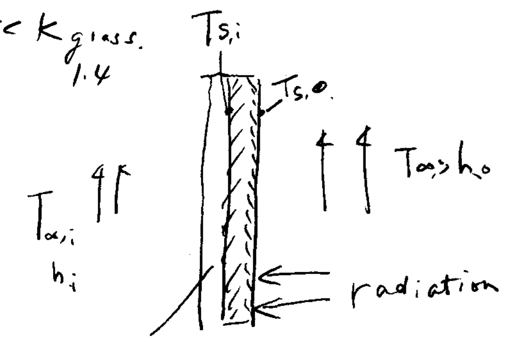
σ : $W/m^2 \cdot K^4$

9. (3pts) Describe the process of convection heat transfer at fluid-solid interface.

Convection is heat transfer between a solid surface and a moving fluid with different temperature. Between the wall and fluid layer right next to the wall there is pure conduction, but in the boundary layer that develops along the wall, the motion of bulk fluid enhances the heat transfer by carrying the thermal energy downstream. In the meanwhile the heat penetrates the fluid so as to increase the thickness of BL.

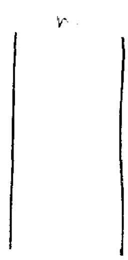
10. (6pts) A curtain over a window provides a thermal solution. With your knowledge of the three heat transfer mechanisms describe how a curtain accomplishes its objectives.

$K_{cotton} << K_{glass}$
$0.06 \quad 1.4$



Generally curtain prevents bulk air in the room from contacting the glass to form convection. Because the [air: poor conductivity] thermal conductivity is low for curtain, the heat loss through it is relatively slow. In addition the curtain reflects radiation heating from sun, and again because its low k, the heat added to the room air is relatively slow.

11. (6pts) The heat flux on a surface of width w is given by $q''(y) = (k\,T_o/L)(1+(y/L)^2)$. What is the total heat transfer per unit width w for $0 < y < L$.



$$q = \int_0^L q''(y)\,dy$$

$$= \int_0^L k\frac{T_o}{L}\left(1+\left(\frac{y}{L}\right)^2\right)dy$$

$$= k\frac{T_o}{L}\int_0^L \left[1+\left(\frac{y}{L}\right)^2\right]dy$$

$$= k\,T_o \int_0^L \left[1+\left(\frac{y}{L}\right)^2\right]d\frac{y}{L}$$

$$= k\,T_o\left(\frac{y}{L}+\frac{1}{3}\left(\frac{y}{L}\right)^3\right)\Big|_0^L$$

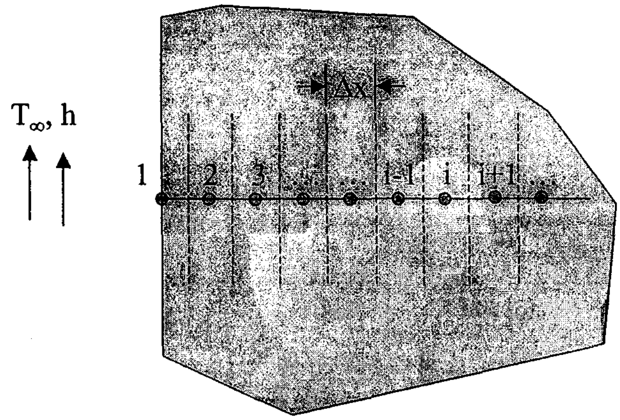
$$= k\,T_o\left(1+\frac{1}{3}\right)$$

$$= \frac{4}{3}k\,T_o$$

12. (6 pts) A 1-D transient conduction problem in a wall, where the conductivity k is a function of position x is governed by the equation

$$\rho c_p \frac{dT}{dt} = \frac{d}{dx}\left(k \frac{dT}{dx}\right)$$

The above equation needs be discretized using the finite difference method for numerical solution. Discretize the conduction term in the above equation at node i.



$$k = k(x)$$

$$\rho c_p \frac{dT}{dt} = \frac{d}{dx}\left(k(x)\frac{dT}{dx}\right)$$

$$\rho c_p \frac{dT}{dt} = \frac{dT}{dx}\frac{dk}{dx} + k\frac{d^2T}{dx^2}$$

$$\left.\frac{dT}{dt}\right|_i \approx \frac{T_i^{p+1} - T_i^p}{\Delta t}$$

$$\left.\frac{dT}{dx}\right|_i \approx \frac{T_{i+1}^p - T_{i-1}^p}{2\Delta x}$$

$$\left.\frac{dk}{dx}\right|_i \approx \frac{K_{i+1} - K_{i-1}}{2\Delta x}$$

$$\frac{\left.\frac{dT}{dx}\right|_{i+\frac{1}{2}} - \left.\frac{dT}{dx}\right|_{i-\frac{1}{2}}}{\Delta x}$$

$$\left.\frac{d^2T}{dx^2}\right|_i \approx \frac{T_{i+1}^p - 2T_i^p + T_{i-1}^p}{\Delta x^2}$$

$$\frac{\frac{T_{i+1}-T_i}{\Delta x} - \frac{T_i - T_{i-1}}{\Delta x}}{\Delta x}$$

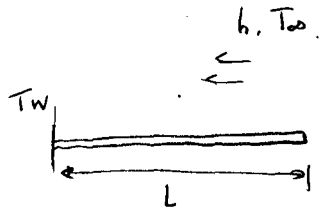
$$\rho c_p \frac{T_i^{p+1} - T_i^p}{\Delta t} = \frac{T_{i+1}^p - T_{i-1}^p}{2\Delta x}\,\frac{K_{i+1} - K_{i-1}}{2\Delta x} + K_i \frac{T_{i+1}^p - 2T_i^p + T_{i-1}^p}{\Delta x^2}$$

$$T_i^{p+1} = T_i^p + \frac{1}{\rho c_p}\frac{\Delta t}{4\Delta x^2}(k_{i+1} - k_{i-1})(T_{i+1}^p - T_{i-1}^p) + \frac{k_i}{\rho c_p}\frac{\Delta t}{\Delta x^2}(T_{i+1}^p - 2T_i^p + T_{i-1}^p)$$

$$= T_i^p + \frac{1}{4\rho c_p}\frac{\Delta t}{\Delta x^2}(k_{i+1} - k_{i-1})\,T_{i+1}^p - \frac{1}{4\rho c_p}\frac{\Delta t}{4\Delta x^2}(k_{i+1} - k_{i-1})\,T_{i-1}^p + \frac{k_i}{\rho c_p}\frac{\Delta t}{\Delta x^2}T_{i+1}^p - \frac{2k_i}{\rho c_p}\frac{\Delta t}{\Delta x^2}T_i^p + \frac{k_i}{\rho c_p}\frac{\Delta t}{\Delta x^2}T_{i-1}^p$$

$$= \frac{1}{4\rho c_p}\frac{\Delta t}{\Delta x^2}(k_{i+1} + 4k_i - k_{i-1})\,T_{i+1}^p + \left(1 - \frac{2K_i}{\rho c_p}\frac{\Delta t}{\Delta x^2}\right)T_i^p - \frac{1}{4\rho c_p}\frac{\Delta t}{\Delta x^2}(K_{i+1} - 4k_i + K_{i-1})\,T_{i-1}^p$$

13. (7pts) 'Enough' hair can serve as insulation, but an isolated hair acts like a fin performing the opposite function. Identify the steps, boundary conditions, assumptions, etc. you would use to compute the heat loss from a single hair. Use the 'fin solutions' table attached and sound argument to justify any necessary assumptions.



Assumptions: ① steady-state

② 1-D H.T (r direction)

③ uniform cross-sectional area

④ no heat loss at the end.

(infinite long fin)

$\theta(L) = 0.$

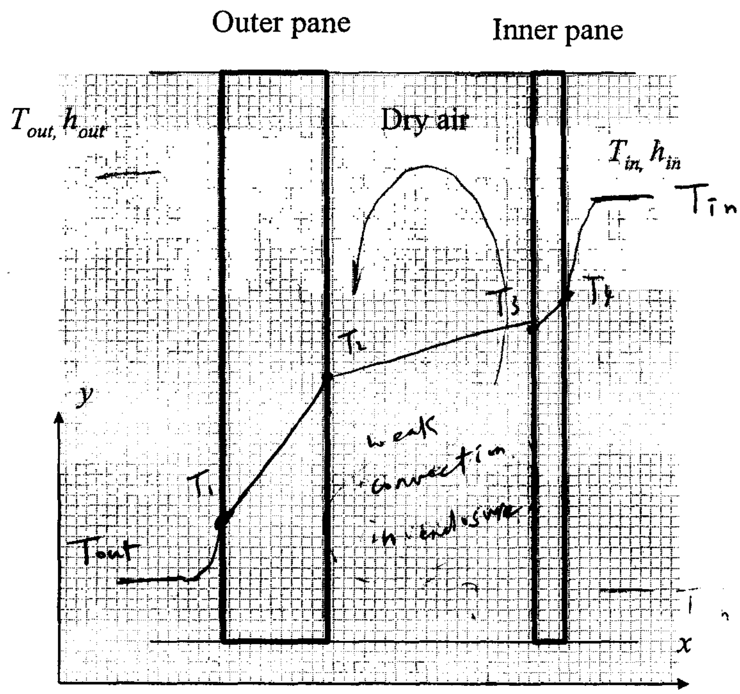
$q_f = M = \sqrt{hPkA_c}\,\theta_b$

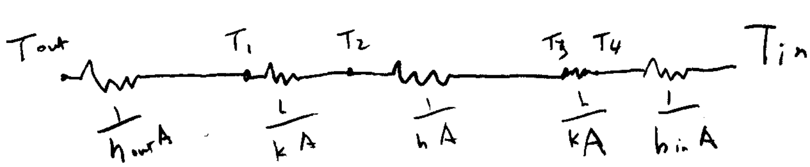
$$m = \frac{hP}{kA_c} = \frac{h}{k}\frac{2\pi r}{\pi r^2} = \frac{h}{k}\frac{2}{r}$$

$(r \simeq 100\ \mu m)$

14. (8pts) A 'safety' double pane window consists of a significantly thicker outer pane than the inner pane. Assuming the external heat transfer coefficient is much larger than the interior heat transfer coefficient, using the grid overlaid on the sketch draw a fairly quantitative temperature profile you might expect on the schematic (to scale) provided below assuming the inside temperature is 100 F and the outside temperature is 0 F.

not required! ⇓





($h_{out} >> h_{in}$ $\quad L_{out} >> L_{in}$.

$T_{in} = 100\ F.$ $\quad T_{out} = 0.$

$$q = \frac{T_{in} - T_{out}}{R_{tot}}$$

$$R_{tot} = \frac{1}{h_{out}A} + \frac{L_{out}}{kA} + \frac{1}{hA} + \frac{L_{in}}{kA} + \frac{1}{h_{in}A}$$

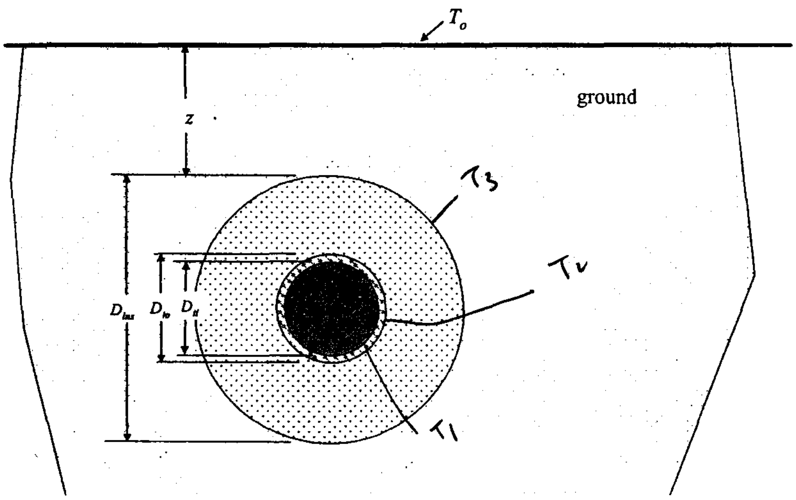
$$\frac{T_1 - T_\infty}{\frac{1}{h_{out}A}} = q = \frac{T_{in} - T_4}{\frac{1}{h_{in}A}}$$

$\therefore T_1 - T_\infty < T_{in} - T_4$

$$\frac{T_2 - T_1}{\frac{L_{out}}{kA}} = \frac{T_4 - T_3}{\frac{L_{in}}{kA}} \Rightarrow T_2 - T_1 >> T_4 - T_3$$

$T_2 \simeq T_3$)

15. (12 pts) A long insulated pipe of length L is buried in the ground a distance z from the surface. The pipe carries a heated fluid of temperature T_f flowing at a rate yielding heat transfer coefficient h_f. The temperature of the outside air/ground interface is given by T_o. A schematic of this situation (not necessarily to scale) is sketched in the figure below with some important dimensions given.



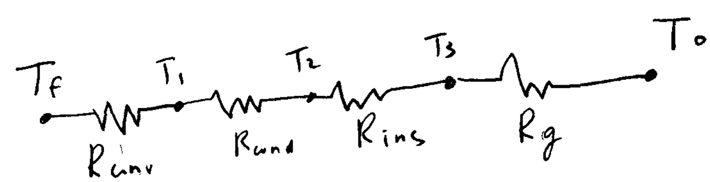
A. Draw the resistance model for this thermal problem. DO NOT SOLVE.

B. Write, label, identify the important thermal resistances (make table if necessary) Use values from the figure where appropriate in your labels.

C. For a metal pipe conveying a liquid with h = 10, what is the controlling thermal resistance if k_{ins} = 0.1, D_{ins} = 0.10, D_{to} = 0.033, D_{ti} = 0.027, k_t = 20, k_{ground} = 3, and z = 0.25? (SI units used)

ignore contact resistance ?

A.



B.

R_{conv} $\quad \dfrac{1}{h_f A} = \dfrac{1}{h_f \pi D_{ti} L} = \dfrac{1}{10 \cdot \pi \cdot 0.027 \cdot L} = \dfrac{1.179}{L}$

R_{cond} $\quad \dfrac{\ln D_{to}/D_{ti}}{2\pi k_t L} = \dfrac{\ln \frac{0.033}{0.027}}{2 \cdot \pi \cdot 20 \cdot L} = \dfrac{0.0016}{L}$

R_{ins} $\quad \dfrac{\ln D_{ins}/D_{to}}{2\pi k_{ins} L} = \dfrac{\ln \frac{0.1}{0.033}}{2 \cdot \pi \cdot 0.1 \cdot L} = \dfrac{1.76}{L}$ ✓ controlling

$R_g = \dfrac{1}{k_g \dfrac{2\pi L}{\cosh^{-1}(2z/D_{ins})}} = \dfrac{1}{3 \dfrac{2\pi L}{(\ln(\frac{4\times 0.3}{0.1}))}} = \dfrac{0.13}{L}$

$(L >> D)$

$z = 0.25 + 0.05$
$= 0.3 > 0.15$