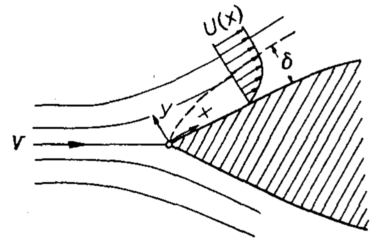

Fig. 6.1. Boundary–layer flow along a wall

boundary–layer thickness is proportional to the square root of the kinematic viscosity:

$$\delta \sim \sqrt{\nu} .$$

In the simplifications of the Navier–Stokes equations which follow, it will be assumed that this boundary–layer thickness is very small compared to a still unspecified linear dimension of the body l:

$$\delta \ll l .$$

Thus the solutions of the boundary–layer equations have an asymptotic character for very high Reynolds numbers.

If we use the free stream velocity V and a characteristic dimension of the body l as reference values, the relation $\delta \sim \sqrt{\nu}$ leads to the dimensionally correct representation

$$\frac{\delta}{l} \sim \frac{1}{\sqrt{\mathrm{Re}}} \quad \text{with} \quad \mathrm{Re} = \frac{Vl}{\nu} . \tag{6.1}$$

That is, the boundary–layer thickness tends to zero with increasing Reynolds number.

We now want to establish what simplifications of the Navier–Stokes equations arise if (only) the asymptotic solutions at high Reynolds number are to be determined. Instead of going along the path of Chap. 5, by first solving the complete Navier–Stokes equations and then determining the asymptotic solution for $\mathrm{Re} \to \infty$, the asymptotic solution will now be ascertained directly from correspondingly simplified differential equations. Consider first the two–dimensional problem in Fig. 6.1, assuming the wall is flat. Let the x axis lie along the wall, and the y axis be perpendicular to it. We now want to write the continuity equation and the Navier–Stokes equations in dimensionless form. All lengths will be referred to the characteristic length l already introduced and all velocities to the free stream velocity V. The pressure is made dimensionless with ϱV^2 and the time with l/V. Furthermore, the Reynolds number, which is very large by assumption, is

$$\mathrm{Re} = \frac{\varrho V l}{\mu} = \frac{V l}{\nu} . \tag{6.2}$$

From H. Schlichting, Boundary Layer Theory

Thus the equations read, in dimensionless notation:
momentum equation in the x direction:

$$\underset{1}{\frac{\partial u^*}{\partial t^*}} + \underset{1}{u^*}\,\underset{1}{\frac{\partial u^*}{\partial x^*}} + \underset{\delta^*}{v^*}\,\underset{\frac{1}{\delta^*}}{\frac{\partial u^*}{\partial y^*}} = -\frac{\partial p^*}{\partial x^*} + \underset{\delta^{*2}}{\frac{1}{\mathrm{Re}}}\left(\underset{1}{\frac{\partial^2 u^*}{\partial x^{*2}}} + \underset{\frac{1}{\delta^{*2}}}{\frac{\partial^2 u^*}{\partial y^{*2}}}\right), \tag{6.3}$$

momentum equation in the y direction:

$$\underset{\delta^*}{\frac{\partial v^*}{\partial t^*}} + \underset{1}{u^*}\,\underset{\delta^*}{\frac{\partial v^*}{\partial x^*}} + \underset{\delta^*}{v^*}\,\underset{1}{\frac{\partial v^*}{\partial y^*}} = -\frac{\partial p^*}{\partial y^*} + \underset{\delta^{*2}}{\frac{1}{\mathrm{Re}}}\left(\underset{\delta^*}{\frac{\partial^2 v^*}{\partial x^{*2}}} + \underset{\frac{1}{\delta^*}}{\frac{\partial^2 v^*}{\partial y^{*2}}}\right), \tag{6.4}$$

continuity equation:

$$\underset{1}{\frac{\partial u^*}{\partial x^*}} + \underset{1}{\frac{\partial v^*}{\partial y^*}} = 0\,. \tag{6.5}$$