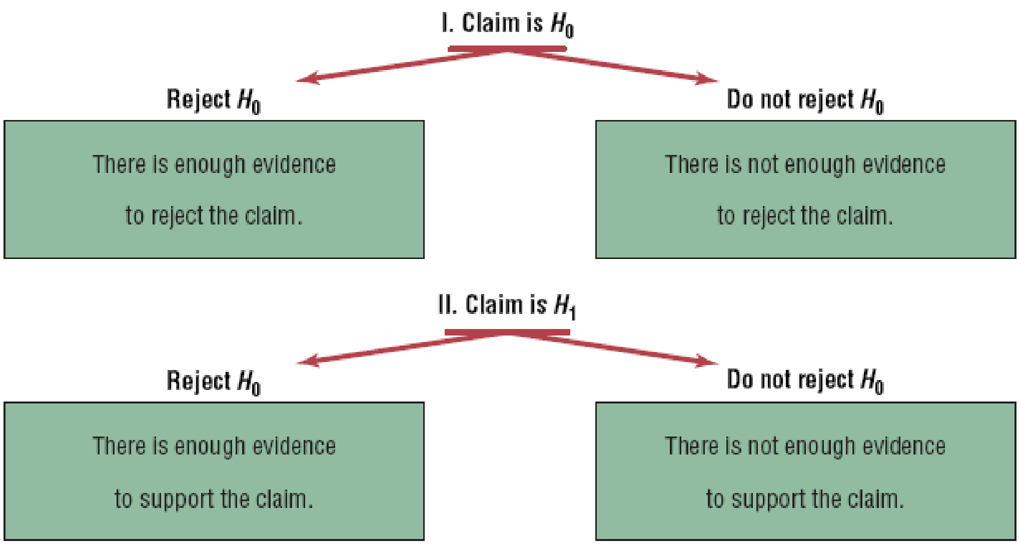
The rejection rule:

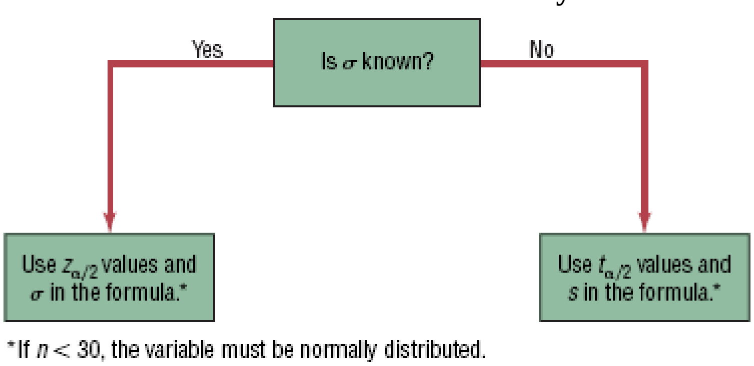
- p-value method, reject H_0 when the p-value $\leq \alpha$.
- Critical value method, reject H_0 when the test statistic is in the critical tail(s).
- Confidence Interval method, reject H_0 when the hypothesized value found in H_0 is outside the bounds of the confidence interval.



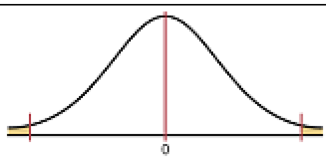
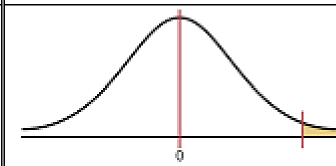
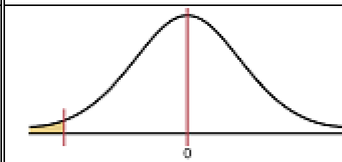
Common Symobls

n = Sample Size
N = Population Size
$\bar{x}$ = Sample Mean
μ = Population Mean
S = Sample Standard Deviation
σ = Population Standard Deviation
σ^2 = σ^2 = Population Variance
S^2 = Sample Variance
p = Population Proportion
$\hat{p}$ = Sample Proportion
r = Sample Correlation Coefficient
ρ = Population Correlation Coefficient

When n < 30 the variable must be normally distributed.

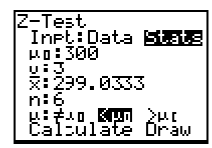


Look for these key words to help set up your hypotheses:

Two-tailed Test	Right-tailed Test	Left-tailed Test
Null Hypothesis		
=	≤	≥
Is	Is less than or equal to	Is greater than or equal to
Is equal to	Is at most	Is at least
Is exactly the same as	Is not more than	Is not less than
Has not changed from	Within	
Is the same as		
		
Alternative Hypothesis		
≠	>	<
Is not	More than	Less than
Is not equal to	Greater than	Below
Is different from	Above	Lower than
Has changed from	Higher than	Shorter than
Is not the same as	Longer than	Smaller than
	Bigger than	Decreased
	Increased	Reduced

One Sample Tests:

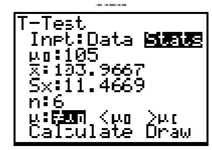
Means



$H_0: \mu = \mu_0$

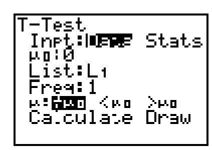
$H_1: \mu \neq \mu_0$

Test statistic when σ is given in the problem: $z = \dfrac{\bar{x} - \mu_0}{\left(\sigma / \sqrt{n}\right)}$



Test statistic when σ is unknown: $t = \dfrac{\bar{x} - \mu_0}{\left(s / \sqrt{n}\right)}$ with df= n – 1

- Calculator Short Cut: Press the [STAT] key, go to the [TESTS] menu, arrow down to either the [Z-Test] or [t-test] option and press the [ENTER] key. Arrow over to the [Stats] menu and press the [ENTER] key. Then type in value for the hypothesized mean (μ_0),standard deviation, sample mean, sample size, arrow over to the ≠, <, > sign that is in the alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic and p-value.
- *Or* (If you have raw data in a list) Select the [Data] menu and press the [ENTER] key. Then type in the value for the hypothesized mean (μ_0), type in your list name (TI-84 L_1 is above the 1 key) (TI-89 list names are in the main folder under Var-Link), leave Freq:1 alone, arrow over to the ≠, <, > sign that is in the alternative hypothesis statement then press the [ENTER]key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic and the p-value.



Proportions

$H_0: p = p_0$

$H_1: p \neq p_0$

Test statistic is $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}}$ where $\hat{p} = \dfrac{x}{n}$

- Calculator Short Cut: Press the [STAT] key, select the [TESTS] menu, arrow down to the option [1-PropZTest] and press the [ENTER] key. Type in the value for the hypothesized proportion (p_0), X, sample size, arrow over to the ≠, <, > sign that is in the alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic and p-value. Note: sometimes you are not given the X value but a percentage instead. The

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calculator will give you an error message if you put in a decimal for X or n. For example if $\hat{p}$ = .22 and n = 124 then .22*124 = 27.28, so use X = 27.

Variance or Standard Deviation

$H_0: \sigma^2 = \sigma_0^2$ or $H_0: \sigma = \sigma_0$
$H_1: \sigma^2 \neq \sigma_0^2$ or $H_1: \sigma \neq \sigma_0$ Both use test statistic $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$ with df= n – 1

Two Sample Tests:

2 Means – Independent Populations

$H_0: \mu_1 = \mu_2$ or $H_0: \mu_1 - \mu_2 = (\mu_1 - \mu_2)_0$ where usually you have $H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 \neq \mu_2$ or $H_1: \mu_1 - \mu_2 \neq (\mu_1 - \mu_2)_0$ where usually you have $H_1: \mu_1 - \mu_2 \neq 0$

Test Statistic when σ_1^2 and σ_2^2 are given: $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

- Press the [STAT] key, select the [TESTS] menu, arrow down to the option [2-SampZTest] and press the [ENTER] key. Arrow over to the [Data] for raw data in lists or [Stats] menu when stats are already given and press the [ENTER] key. Then type in the population standard deviations, the first sample mean and sample size, then the second sample mean and sample size, or the list names for raw data. Arrow over to the ≠, <, > sign that is in the alternative hypothesis statement then press the [ENTER]key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic z and the p-value.

Confidence interval when σ_1^2 and σ_2^2 are given: $(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\left(\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}\right)}$

- Press the [STAT] key, arrow over to the [TESTS] (interval on the TI-89) menu, arrow down to the option [2-SampZInt] and press the [ENTER] key. Arrow over to the [Data] for raw data in lists or [Stats] menu and press the [ENTER] key. Then type in the population standard deviations, the first sample mean and sample size, then the second sample mean and sample size, then enter the confidence level. Arrow down to [Calculate] and press the [ENTER] key. The calculator returns the confidence interval.

Test statistic when σ_1^2 and σ_2^2 are unknown: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

- Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [2-SampTTest] and press the [ENTER] key. Arrow over to the [Data] for raw data in lists or [Stats] menu and press the [Enter] key. Enter the list names or the means, sample standard deviations, sample sizes, and confidence level. Then arrow over to the not equal, <, > sign that is in the alternative hypothesis statement then press the [ENTER] key. Highlight the No option under Pooled for unequal variances. Arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic and the p-value.

with $df = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\left(\left(\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2\right) + \left(\dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2\right)\right)}$

Confidence interval when σ_1^2 and σ_2^2 are unknown: $(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

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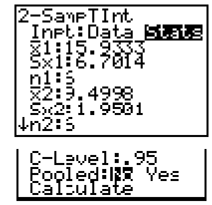
- Press the [STAT] key, arrow over to the [TESTS] (interval on the TI-89) menu, arrow down to the option [2-SampTInt] and press the [ENTER] key. Arrow over to the [Data] for raw data in lists or [Stats] menu and press the [Enter] key. Enter the list names or means, sample standard deviations, sample sizes, and confidence level. Highlight the No option under Pooled for unequal variances. Arrow down to [Calculate] and press the [ENTER] key. The calculator returns the confidence interval.



2 Means – Dependent Populations

Find the difference (d) between each matched pairs.

$H_0: \mu_D = 0$

$H_1: \mu_D \neq 0$

Test statistic: $t = \dfrac{\bar{d} - \mu_0}{\left(s_d / \sqrt{n}\right)}$

- Type the data for group1 in L_1 and group 2 in L_2. Arrow up so that you are highlighted on the label of L_3. Then subtract the two lists $L_1 - L_2$. This puts the differences into L_3 (or just type the differences into a list). In the [STAT] menu, arrow over to the [TESTS] menu, arrow down to the option [T -Test] and press the [ENTER] key. Arrow over to the [Data] menu and press the [ENTER] key. Then type in the hypothesized mean as 0, List: L_3, leave Freq:1 alone, arrow over to the ≠, <, > sign that is the same in the problems alternative hypothesis statement then press the [ENTER]key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the t-test statistic, the p-value, $\bar{x} = \bar{d}$ and $S_x = S_D$.

Confidence Interval: $\bar{d} \pm t_{\alpha/2}\left(\dfrac{s_d}{\sqrt{n}}\right)$ with df= n – 1.

- Type the data for group1 in L_1 and group 2 in L_2. Arrow up so that you are highlighted on the label of L_3. Then subtract the two lists $L_1 - L_2$. This puts the differences into L_3 (or just type the differences into a list). In the [STAT] menu, arrow over to the [TESTS] (interval on the TI-89) menu, arrow down to the [T-Interval] option and press the [ENTER] key. Arrow over to the [Data] menu and press the [ENTER] key. Type in the List: L_3, Freq:1, and the confidence level. Arrow down to [Calculate] and press the [ENTER] key. The calculator returns the confidence interval, $\bar{x} = \bar{d}$ and $S_x = S_D$.

2 Proportions

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

Test statistic $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)_0}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$, (usually $(p_1 - p_2)_0 = 0$) where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$, $\hat{q} = 1 - \hat{p}$

- Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [2-PropZTest] and press the [ENTER] key. Type in the X_1, n_1, X_2, n_2, arrow over to the ≠, <, > sign that is in the alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the z-test statistic and the p-value. Note X_1 and X_2 need to be a whole number, not a decimal.

Confidence Interval: $(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ where $\hat{p}_1 = \dfrac{x_1}{n_1}$, $\hat{p}_2 = \dfrac{x_2}{n_2}$

- Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [7:2-PropZInterval] and press the [ENTER] key. Type in the X_1, n_1, X_2, n_2, the confidence level, then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the confidence interval.

2 Variances or Standard Deviations

$H_0: \sigma_1^2 = \sigma_2^2$ or $H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1^2 \neq \sigma_2^2$ $\quad$ $H_1: \sigma_1 \neq \sigma_2$

Both use test statistic $F = \dfrac{s_1^2}{s_2^2}$ with dfN = $n_1 - 1$ and dfD = $n_2 - 1$.

- Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [2-SampFTest] and press the [ENTER] key. Arrow over to the [Stats] menu and press the [Enter] key. Then type in the s_1, n_1, s_2, n_2, arrow over to the ≠, <, > sign that is the same in the problems alternative hypothesis statement then press the [ENTER] key, arrow down to [Calculate] and press the [ENTER] key. The calculator returns the test statistic F and the p-value.

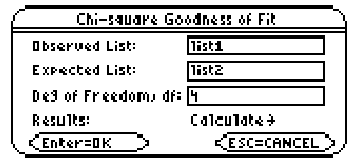
Goodness of Fit Test

"There is no preference" w/ 4 groups H_0: p_1=.25, p_2=.25, p_3=.25, p_4=.25 "There is not a preference"

H_1: At least one proportion is different. "There is a preference"

Proportions are 1/k or different percentages for each group given in the problem. If %'s are given use those decimals for each p_i. Expected values are found by taking each group's proportion times the sample size ($p_i \times n$), df=k–1. Test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E}$

- For the TI-84: Enter the observed frequencies in List1, the expected frequency in List2, press [STAT] key, arrow over to [TESTS] menu, arrow down to [χ^2GOF–Test] (not available on the TI-83 and some TI-84 calculators)
- For the TI-89: Hypothesis test for three or more proportions (goodness of fit test). Go to the [Apps] **Stat/List Editor**, then type in the observed values into list 1, and the expected values into list 2. Select 2nd then F6 [Tests], then select **7: Chi-2GOF**. Type in the list names and the degrees of freedom (df = k-1). Then press the [ENTER] key to calculate. The calculator returns the χ^2-test statistic and the p-value.



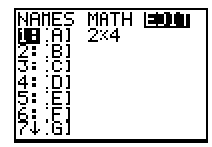
Test for Independence

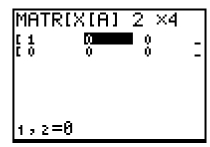
H0: <u>Variable 1</u> is independent of <u>Variable 2</u>

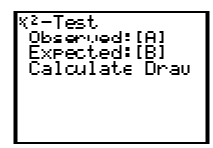
H1: <u>Variable 1</u> is dependent of <u>Variable 2</u>

Expected values are found by taking $\frac{(\text{Row Total})\cdot(\text{Column Total})}{\text{Grand Total}}$ for each cell.

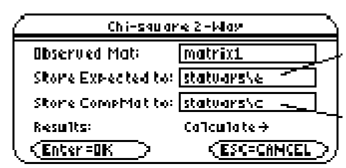
Test statistic: $\chi^2 = \sum \frac{(O-E)^2}{E}$







- For the TI-83 or TI-84: Press the [2nd] then [MATRX] key. Arrow over to the EDIT menu and 1:[A] should be highlighted, press the [ENTER] key. For a m X n contingency table, type in the number of rows(m) and the number of columns(n) at the top of the screen so that it looks like this MATRIX[A] m X n. For example a 2 X 3 contingency table, the top of the screen would look like this MATRIX[A] 2 X 4, as you hit [ENTER] the table will automatically widen to the size you put in. Now enter all of the observed values in there proper positions. Then press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [χ^2-Test] and press the [ENTER] key. Leave the default as Observed:[A] and Expected:[B], arrow down to [Calculate] and press the [ENTER] key. The calculator returns the χ^2-test statistic and the p-value. If you go back to the matrix menu [2nd] then [MATRX] key, arrow over to EDIT and choose 2:[B], you will see all of the expected values.
- TI-89: First you need to create the matrix for the observed values: Press: [Home] to return to the Home screen, press [Apps] and select **6:Data/Matrix Editor**. A menu is displayed, select **3:New**. The **New** dialog box is displayed. Press the right arrow key to highlight **2:Matrix,** and press [ENTER] to choose **Matrix** type. Press the down arrow key to highlight **1:main**, and press [ENTER], to choose **main** folder. Press the down arrow key, and then enter the name **o** in the **Variable** field. Enter **3** for **Row dimension** and **2** for **Column dimension**. Press [ENTER] to display the matrix editor. Enter **4** , **9** , **5** in **c1** and **7** , **2** , **3** in **c2**. Press ♦ [Apps] [ENTER] to close the matrix editor and return to the list editor. If you have more than one Application loaded, press ♦ [Apps], and then select **Stats/List Editor** .
  To display the **Chi-square 2-Way** dialog box, press 2nd then F6 [Tests], then select **8: Chi-2 2-way**. Enter in in the Observed Mat: **o** ; Store Expected to: **statvars\e** ; Store CompMat to: **statvars\c** . This will store the expected values in the matrix folder statvars with the name e, and the $(o-e)^2/e$ values in the matrix c. Press the [ENTER] key to calculate. The calculator returns the χ^2-test statistic and the p-value. If you go back to the matrix menu you will see all of the expected and $(o-e)^2/e$ values.



Correlation and Regression

$H_0: \rho = 0$

$H_1: \rho \neq 0$

Test statistic for correlation: $t = r\sqrt{\left(\frac{(n-2)}{(1-r^2)}\right)}$ with df= n – 2

- In the [STAT] editor enter the x values into list one and the y values into list two. In the [TESTS] menu, arrow down to the option [LinRegTTest] and press the [ENTER] key. The default is Xlist: L_1, Ylist: L_2, Freq:1, β and ρ :$\neq 0$. Arrow down to

Calculate and press the [ENTER] key. The calculator returns the t-test statistic, the y-intercept a, slope b, the standard error of estimate $s=s_{est}$, the coefficient of determination R^2, and the correlation coefficient r.

Prediction Interval for a predicted value of y: $\hat{y} \mp t_{\alpha/2} \cdot s_{est}\sqrt{1+\frac{1}{n}+\frac{n(x-\bar{x})^2}{\left(n\cdot\Sigma x^2-(\Sigma x)^2\right)}}$ with df = n – 2.

$$R^2_{adj} = 1-\left(\left(1-R^2\right)\left(\frac{n-1}{n-p-1}\right)\right)$$

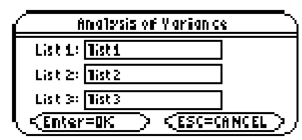
One-Factor ANOVA table   k=#of groups, N=total of all n's

Source	SS	df	MS	F (Test Statistic)
Between (Treatment or Factor)	$SS_B=\Sigma n(\bar{x}-\bar{x}_{GM})^2$	k–1	$MS_B = SS_B / df_B$	$F = MS_B / MS_W$
Within (Error)	$SS_W=\Sigma(n-1)s^2$	N–k	$MS_W = SS_W / df_W$	
Total	SS_T	N–1		

H_0: $\mu_1=\mu_2=\mu_3=\ldots=\mu_k$

H_1: At least one mean is different   CV: Always right-tailed F, use $dfN=df_B$ and $dfD=df_W$

- For the TI-83 or TI-84: Note you have to have the actual raw data to do this test on the calculator. Press the [STAT] key and then the [EDIT] function, type the three lists of data into list one, two and three. Press the [STAT] key, arrow over to the [TESTS] menu, arrow down to the option [F:ANOVA(] and press the [ENTER] key. This brings you back to the regular screen where you should now see ANOVA(. Now hit the [2nd] [L_1] [,] [2nd] [L_2] [,][2nd] [L_3][)] keys in that order. You should now see ANOVA(L_1, L_2, L_3), if you had 4 lists you would then have an additional list. Press the [ENTER] key. The calculator returns the F-test statistic, the p-value, Factor (Between) df, SS and MS, Error (Within) df, SS and MS. The last value Sxp is the square root of the MSE.
- For the TI-89: Go to the [Apps] **Stat/List Editor**, then type in the data for each group into a separate list, (or if you don't have the raw data, enter the sample size, sample mean and sample variance for group 1 into list1 in that order, repeat for list2, etc). Select 2nd then F6 [Tests], then select **C:ANOVA**. Select the input method **data** or **stats**. Select the number of groups. Press the [ENTER] key to calculate. The calculator returns the F-test statistic, the p-value, Factor (Between) df, SS and MS, Error (Within) df, SS and MS. The last value Sxp is the square root of the MSE.



When you reject H_0 for a one-factor ANOVA then you should do a multiple comparison. For example for 3 groups you would have the following 3 comparisons. (4 groups would have ${}_4C_2$=6 comparisons)

$H_0:\mu_1=\mu_2$ $H_0:\mu_1=\mu_3$ $H_0:\mu_2=\mu_3$

$H_1:\mu_1\neq\mu_2$ $H_1:\mu_1\neq\mu_3$ $H_1:\mu_2\neq\mu_3$

- Scheffé Test Statistic (unequal n): $F_S = \frac{(\bar{x}_i-\bar{x}_j)^2}{\left(MSE\left(\frac{1}{n_i}+\frac{1}{n_j}\right)\right)}$ Critical Value: (k–1)×(CV from original ANOVA)
- Bonferroni Test $t = \frac{(\bar{x}_i-\bar{x}_j)}{\sqrt{\left(MSE\left(\frac{1}{n_i}+\frac{1}{n_j}\right)\right)}}$ with df = N – k and multiply the p-value by ${}_kC_2$ groups. For example if you use the tcdf in your calculator to find the area in both the tails and you have 4 groups you would multiply the tail areas by ${}_4C_2$=6.

Two-Factor ANOVA table   a=# or row factors, b=# of column factors, n=sample size in one group, N=total

Source	SS	df	MS	F (3 Test Statistics)
Row (Factor A)	SS_A	a–1	$MS_A = SS_A / df_A$	$F_A = MS_A / MS_E$
Column (Factor B)	SS_B	b–1	$MS_B = SS_B / df_B$	$F_B = MS_B / MS_E$
Interaction (A × B)	$SS_{A\times B}$	(a–1)(b–1)	$MS_{A\times B} = SS_{A\times B} / df_{A\times B}$	$F_{A\times B} = MS_{A\times B} / MS_E$
Error (Within)	SS_E	ab(n–1)	$MS_E = SS_E / df_E$	
Total	SS_T	N–1		

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Row Effect (Factor A): H_0: The <u>row variable</u> has no effect on the average ___.
H_1: The <u>row variable</u> has an effect on the average ___.

Column Effect (Factor B): H_0: The <u>column variable</u> has no effect on the average ___.
H_1: The <u>column variable</u> has an effect on the average ___.

Interaction Effect (A × B): H_0: There is no interaction effect between <u>row variable</u> and <u>column variable</u> on the average _____.
H_1: There is an interaction effect between <u>row variable</u> and <u>column variable</u> on the average ___.

Nonparametric Statistics

Sign Test For the p-value use binomcdf(n,p,x) where n is the sample size after zeros are removed, p = 0.5, and x is the test statistic which is the smaller of plus or minus signs. Multiply by two if you have a two-tailed test.

Wilcoxon Rank Sum Test $z = \frac{R - \mu_R}{\sigma_R}$ where R = sum of the ranks for the smaller sample size (n_1),

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}, \ \sigma_R = \sqrt{\frac{n_1 \cdot n_2(n_1 + n_2 + 1)}{12}}$$

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