Limnology 2009 Kalff Chapter 12

Diffusion: Molecular and Turbulent

## A. Molecular diffusion

Because of the thermal motion, molecules of water and molecules of substances dissolved in water move about randomly. As a consequence, dissolved substances gradually move from an area of high concentration to an area of low concentration. This transport may be described by Fick's law (using oxygen as an example):

$$d(O_2)/dt = -K A d(O_2)/dz$$

where:  $d(O_2)/dt$  = the rate of oxygen transport

K = the molecular diffusion coefficient, and =  $(1.98 \times 10^{-5} \text{ cm}^2/\text{sec})$  for oxygen dissolved in water,

A = the area through which the substance is diffusion (often set to unit area, 1),

 $d(O_2)/dz =$  the (vertical) gradient of oxygen concentration.

Because the value of K is so small, molecular diffusion is much too slow to explain the actual vertical transport of oxygen and other substances in the water column (except at very short distances, on the order of a few microns).

Hutchinson constructed the following example to illustrate that molecular diffusion alone could not be responsible for the vertical diffusion of dissolved substances observed in lakes.

Consider a lake which is 11.0 mg  $O_2/L$  throughout, which is brought to 12.6 mg  $O_2/L$  at the surface by equilibration with the atmosphere. At a depth of 10 meters, the oxygen concentration would increase from 11.0 to 11.6 mg/L in "only" 638 years if molecular diffusion were to act alone.

B. Turbulent diffusion

1. Molecular diffusion is entirely insufficient to produce the rate of diffusion observed in natural waters. The difference between the observed rate of diffusion and the rate of diffusion expected from molecular diffusion alone is the result of **turbulent diffusion**. Turbulent diffusion is of great importance in natural waters. Some definitions of turbulence:

(a) Stewart (1959) J. Geophys. Res. 64:2112-2115: "The natural occurrence of turbulence."

"...a fluid is said to be turbulent if each component of the vorticity is distributed irregularly and aperiodically in time and space, if the flow is characterized by a transfer of

energy from larger to smaller scales of motion, and if the mean separation of neighboring fluid particles tends to increase with time.

(b) L.F. Richardson"Big swirls have little swirls, that prey on their velocity, and little swirls have lesser swirls, and so on...to viscosity."

(c) Nauman and Pierson, Principles of Physical Oceanography

"Turbulence, stirring, mixing, and diffusion encompass an extremely difficult area of both oceanography and meteorology. A definition of turbulence that is satisfactory to everyone has not yet been offered. It is even difficult to decide what the essential features of turbulence ought to be and what scales are to be assigned to turbulent and non-turbulent motion."

2. When is a fluid turbulent?

The **Richardson number**, as noted previously (Section 8) may be used to predict when a stratified water column will become turbulent and thereby enhance vertical mixing. (see Kalff, p183).

 $Ri = [g(d\rho/dz)]/[\rho(du/dz)^{2}]$ 

where: g is the acceleration of gravity,  $d\rho/dz$  is the vertical density gradient,  $\rho$  is the density of water, and du/dz is the vertical gradient of horizontal velocity.

If the Richardson number is *less* than about 0.25, turbulent mixing develops. In short, if density stratification is weak, or if horizontal currents are strong, turbulent mixing develops.

3. When is the motion of an organism rapid enough to experience turbulence?

Flow around an object can be laminar, where flow lines are smooth and parallel



Or flow can be turbulent, where flow lines are chaotic, forming eddies



The type of flow is determined by the size of the object, the density and viscosity of the fluid and the velocity of the fluid relative to the object.

The **Reynolds number** describes the relative significance of inertial forces and viscous forces on the motion of an organism. (Kalff, p180).

$$\text{Re} = \rho v l/\mu$$

where:  $\rho$  is the density of fluid, v is the velocity of the organism relative to the fluid, l is a characteristic length term (such as the length of the organism),  $\mu$  is the coefficient of viscosity

Note that if fluid under consideration is water the density is 1 and Re=  $vl/\mu$ , which is the same as EQ. 12.1 in Kalff

If Re is large ( $>\sim$  1000), the motion of the organism is subject primarily to inertial forces that produce vortices, eddies and turbulence.

If Re is small (less than 500, Kalff, p180), the organism will be primarily subject to viscous forces and smooth, constant fluid motion.

Given the sizes of organisms and their swimming velocities, it is apparent that plankters (smaller than a few millimeters) live in a primarily viscous environment. Fish live in a world in which inertial forces dominate.

Typical Reynolds numbers for different organisms:

Human	Re~10 <sup>6</sup>
Large fish	Re~10 <sup>4</sup>
Paramecium	$Re \sim 10^{-1}$
Small phytoplankter	$Re \sim 10^{-4}$
Bacterium	$Re \sim 10^{-5}$

At high Reynolds numbers locomotion is depended on lift generation with focus on local fluid circulation. As the Reynolds number decreases this streamlining becomes less important and the shape of the organism depends on other factors such as skin friction. This consequently makes for an endless design of body shapes in the dinoflagellate world.



Sida chrystallina



Myliobatis californica



4. What are typical values for turbulent diffusion? (Kalff, p186ff)

As Kalff points out, temporal scale is an important consideration in application of diffusion coefficients. Biological limnologists are interested in how movement of substances influences biological communities.



*Figure* 12–7 Mean values of average vertical diffusion coefficients during stratification in the upper hypolimnia of various lakes as a function of lake surface area. Bars indicate the observed range of variation. Solid circles represent older data summarized by C. H. Mortimer (1942). *(After Maiss et al. 1994.)* 

(These values are about  $10^{10}$  times greater than the coefficient of molecular diffusion!)

Generally, vertical turbulent diffusion is much less than horizontal turbulent diffusion (in a stratified lake) but nevertheless much greater than molecular diffusion. The ratio of the vertical diffusion coefficient to the horizontal coefficient is commonly very small:

$$10^{-7} < A_z/A_x < 10^{-6}$$

5. Methods to observe Austausch coefficients.

Kalff (p186) points out that changes in temperature have been used to estimate the magnitude of vertical mixing, but that direct measurement of mass transfer, using tritium as a tracer, demonstrate that the transfer of mass may be a good deal less than the transfer of heat. Nevertheless, the measurement of heat transfer, via temperature measurements is the often the only practical means of estimating turbulent diffusion.

See Jassby and Powell. 1975. Limnology and Oceanography 20: 530-543 for an example of eddy diffusivity calculations in a lake. For a qualitative derivation of a turbulent diffusion coefficient see Nauman and Pierson, Physical Oceanography, p395ff).