Christopher K. McKenna, Quantitative Methods for Public Decision Making (McGraw-Hill: New York, 1980)

CHAPTER
TWELVE

PROJECT MANAGEMENT: A NETWORK MODEL

Projects that are undertaken and completed many times over are those that best lend themselves to tight control. The time, personnel, and other resources needed for completion are fairly well understood from past experience. Projects that are one-time-only undertakings are more difficult to plan and control. Many activities in the public sector are of this variety. The *overall project* may be a first, but the *components* or *tasks* of the project have been done many times before in similar projects. Some examples are: construction of a highway or a hospital; preparing for a municipal bond issue; development and implementation of a management information system; transferring operations from one site to another, as from one hospital building to a new one; and conducting a major research effort. A model that is particularly well suited to the control of such major projects is the subject of this chapter.

While various diagrams, often informal, have long been used to represent sequences and interrelationships that characterize multiactivity projects, it was not until the late 1950s that formal network methods were proposed to plan, schedule, and control such projects. The two different network models that were developed concurrently and independently wound up being amazingly similar. Except for minor structural and notational differences, each model had one main ingredient that was absent in the other. The PERT model (performance evaluation review technique or program evaluation review technique) originally permitted three different time estimates for each individual task, while CPM (critical path method) permitted only a single time estimate. On the other hand, CPM identified time-cost trade-off points, whereas PERT did not. During the years since their origin the two methods have essentially been merged, with the current product incorporating the advantages of each model as well as many later developments. In our discussion of project management we will make no distinction between PERT and CPM.[1]

CONTENTS OF PERT/CPM

To plan and to control a project require two pieces of basic information about each task:

1. The length of time to complete each task
2. The tasks that must be completed before another task can begin

We will present these and the other fundamental ideas of PERT/CPM within the framework of a seminar development project, a commonly occurring one in any service agency.

A Seminar Development Project: An Illustration

Suppose the state council on mental health and mental retardation has developed an information system that it expects will provide the council with state-wide program information and the many county agencies with local program and client information. Realizing that such an information/evaluation system requires user understanding and cooperation, the council is planning to conduct a number of 3-day seminars at key locations throughout the state.

Ann Maxall is responsible for coordinating the seminars. She has identified the activities that must be completed prior to the seminars themselves. She has also identified which activities must be completed prior to any one activity and has estimated the number of weeks needed to perform each activity. With these

Table 12-1 Activities, predecessors, and time estimates in the seminar planning project

Activity	Description	Immediate predecessors	Estimated time (weeks)
A	Plan seminar content	—	2
B	Obtain speakers	*A*	1
C	Select seminar sites	—	2
D	Prepare and mail flyer/invitations	*B*, *C*	3
E	Accept reservations	*D*	3
F	Notify press	*D*	1

[1] Actually we will be using the terminology of PERT and the network analysis of CPM.

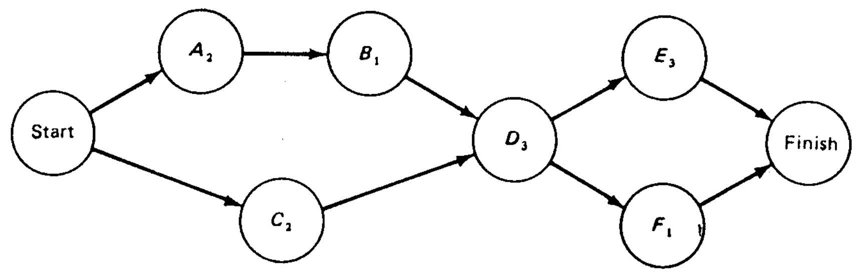

Figure 12-1 PERT diagram of the seminar planning project with activity completion times.

two pieces of basic information, the list of immediate predecessors and the estimated time for each activity, the planning project for the seminars is described as in Table 12-1.

Note that the circles (nodes) are used to represent the activities in this formulation, not the arrows.

Project Network

The immediate predecessor of activity *B* is activity *A*; this means that activity *A*, "plan seminar content," must be completed before activity *B*, "obtain speakers," can be started. In using the network diagram to illustrate the project, activities are represented by circles and sequencing is represented by arrows. For example, the arrow from activity *A* to activity *B* shows that activity *A* is the immediate predecessor of activity *B*. Once the network is complete, the estimated time of each activity can be inserted in the circle representing that activity. Figure 12-1 is the network diagram of the seminar planning project.

Note in Table 12-1 that activities *B* and *C* are both predecessors of activity *D*; in the diagram this is represented by arrows drawn from both *B* and *C* to *D*. Similarly, since activity *D* is the immediate predecessor of both *E* and *F*, arrows are drawn from *D* to both *E* and *F*. The activities represented by the "start" circle and "finish" circle are not real activities but merely signify the beginning and end of the project. The estimated time for each activity can be used to estimate the time for the whole project; in particular the time estimates will lead us to the *critical path* for the project.

THE CRITICAL PATH

A series of connected activities is called a *path*. In the seminar planning project, one path consists of activities *A*, *B*, *D*, *E*; another path consists of *C*, *D*, *F*. There are two other paths through the entire project from start to finish (can you find them?). The length of time associated with the path is the sum of the time estimates associated with each of the activities on the path. For instance, *ABDF* has a time estimate of 7 weeks, and *CDE* has a time estimate of 8 weeks. The path with the greatest time estimate is called the *critical path*. The critical path in our illustration is *ABDE*, which requires 9 weeks. The time required to complete the critical path is the shortest amount of time in which the entire project can be completed. Delays along the critical path result in a delay of the project; if the project is to be completed in less than the time estimate of the critical path, then one or more activities on the critical path must be hurried or *crashed* so that their combined completion time becomes less than the original estimate. On the other hand, crashing an activity not on the critical path will provide no benefits; for example, if activity *C* could be hurried to require 1 week instead of 2, activity *D* still could not start until after the completion of sequence *A* and *B*, which together require 3 weeks. Thus, there would be no benefit to crashing activity *C*. Furthermore, a delay in a noncritical activity will have no effect on the project finish time; for example, activity *F* could be delayed up to 2 weeks with no consequent delay in the completion of the project.

For a network as simple as Figure 12-1 the critical path can be determined by sight. In a more complex project, however, a more formal method of determining the critical path is needed. Such a systematic method is now considered.

Early Time, Late Time, and Slack

In order to find the critical path of a network, we first determine the *early start time* (ES) and *late start time* (LS) for each activity. ES is the earliest that an activity can start after all preceding tasks are completed. LS is the latest that an activity can start without delaying the entire project.

The difference LS − ES is called *slack;* it represents the amount by which an activity can be delayed without delaying the project. An activity *that has no slack is called a critical activity; the path of critical activities is the critical path.*

Find ES and EF by passing through the network from start to finish as follows:

1. The early start time of an activity with no predecessors is zero.
2. The early finish time (EF) of each activity is the sum of its early start time plus the time required to complete it.
3. The early start time of any other activity is the latest of its immediate predecessors' early finish times.

The early start time (ES) and the early finish time (EF) for each activity of Figure 12-1 are determined in the following way.

The ES for activity *A* (ES_A) is 0 since nothing has to happen before activity

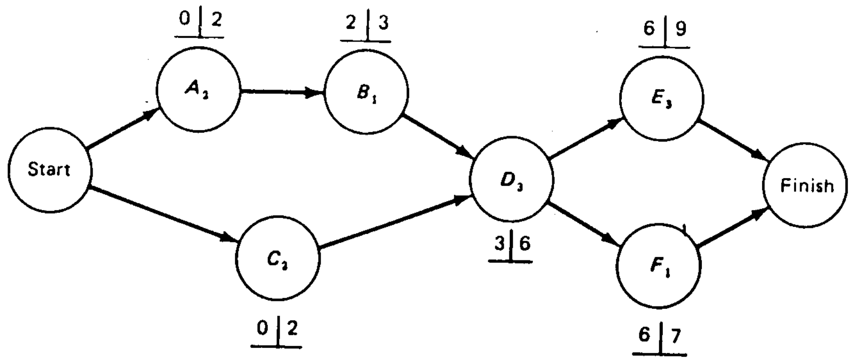

Figure 12-2 PERT diagram of the seminar planning project with early start (ES) and early finish (EF) times written as ES|EF.

A starts. The early finish time for activity *A* (EF_A) is:

$$ES_A = 0 \qquad EF_A = ES_A + t_A = 0 + 2 = 2$$

Similarly,

$$ES_C = 0 \qquad EF_C = ES_C + t_C = 0 + 2 = 2$$

The early start time of any other activity is equal to the largest of the early finish times of all of its immediate predecessors. ES_B equals EF_A since *B*'s only predecessor is *A*; that is,

$$ES_B = EF_A$$

D has two immediate predecessors, *B* and *C*:

$$EF_B = 3 \quad \text{and} \quad EF_C = 2$$

and so

$$ES_D = 3$$

Figure 12-2 contains ES and EF of all activities in the network. Note there that since activity *D* requires both *B* and *C* to be completed before it can begin, ES_D equals the larger of EF_B and EF_C.

The next step in finding the critical path is to determine the late start (LS) and late finish (LF) for each activity. The *late start time* for any activity is the latest that that activity can commence without delaying the whole project. The *late finish time* of any activity is the latest that that activity can finish without delaying the whole project. In order to determine LS and LF of each activity, pass through the network backwards, from finish to start as follows:

1. The late finish time of an activity with no successors equals the project's finish time.
2. The late start time of any activity is its late finish time minus the time to complete it.
3. The late finish time of any other activity is the smallest of its successor's late start times.

The last activities in our network are E and F. The early finish times for those activities are 9 and 7; thus the earliest that the entire project can finish is 9, which becomes the finish time for the project. The LF of the last activity on any path is equal to the finish time of the project. Thus

$$LF_E = 9 \quad \text{and} \quad LF_F = 9$$

The LS for any activity is its LF minus its completion time.

$$LF_E = 9 \qquad LS_E = LF_E - t_E$$
$$= 9 - 3$$
$$= 6$$

Similarly,

$$LF_F = 9 \qquad LS_F = LF_F - t_F$$
$$= 9 - 1$$
$$= 8$$

Activity D is the predecessor of more than one activity. The LF of such an activity is the smallest of the LS values of its successors.

$$LS_E = 6 \quad \text{and} \quad LS_F = 8$$

and so

$$LF_D = 6$$

Figure 12-3 gives LS and LF as well as ES and EF of all the activities in the network.

Slack is the amount of time an activity can be delayed without delaying the entire project. For example, activity C can begin as early as week 0; it can also begin as late as week 1 without delaying the project. Hence, activity C can be delayed 1 week without delaying the project; that is, it has 1 week of slack. In general, the slack for an activity is found by getting the difference between its LS and its ES. (Equivalently, this is also the difference between its LF and EF.) In our illustration F has two weeks of slack and C has one week of slack.

Shared slack is shared with the other activities in sequence with it. In Figure 12-5, activity B and activity D each have 3 days slack; however, a delay in activity B reduces D's slack time. *Free slack* is the amount of time an activity can be delayed without delaying or reducing the slack of any other activity. Exercise 12-5 focuses on the distinction between the two kinds of slack.

Activities with 0 slack are termed *critical activities*. A, B, D, and E all have 0 slack, and are thus critical activities of Figure 12-3. A delay in any critical

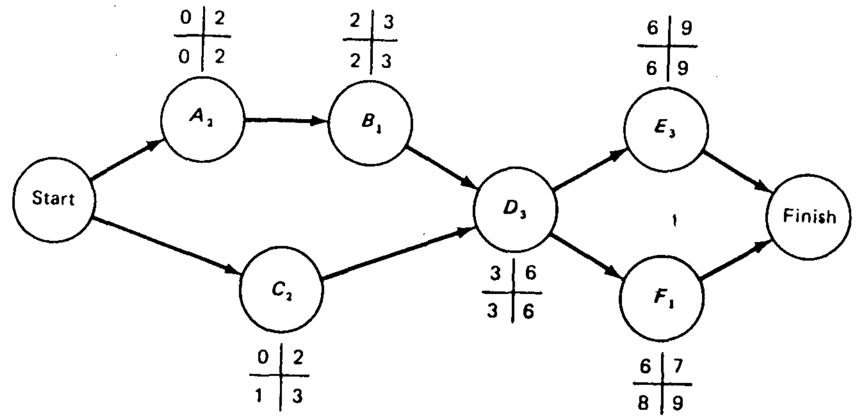

Figure 12-3 PERT diagram with ES|EF over LS|LF

activity will delay the entire project. *A sequence of critical activities forms a critical path*.

The above method generates ES and LS for each activity, and so will always find the critical path. It may happen that there is more than one critical path. In Figure 12-3, if the estimated time of F were 3 weeks rather than 1 week, then $ABDF$ as well as $ABDE$ would be a critical path, and a delay in either E or F would delay the complete project. As one can imagine, complex projects generate rather complex networks. There are many computer programs available for computing the slack and finding the critical path, provided one knows the sequence and estimated times of activities.

Having found the project completion time, one can then determine when the project must start in order for it to be completed on time. Suppose in our illustration the seminars are to be started on Monday, May 12; then the planning, which requires 9 weeks, must start on Monday, March 10. If it is already March 17, then it is quite reasonable to ask which activity times can be shortened or crashed, in order to speed up the project. We now consider this aspect of project management.

CRASHING ACTIVITY TIMES

The costs of a project are of three basic kinds:

1. Direct costs of activities
2. Indirect costs of supervision, administration, and so forth
3. Opportunity costs including late penalties and forfeited early bonuses

Generally, as the overall length of a project increases, the direct costs decrease

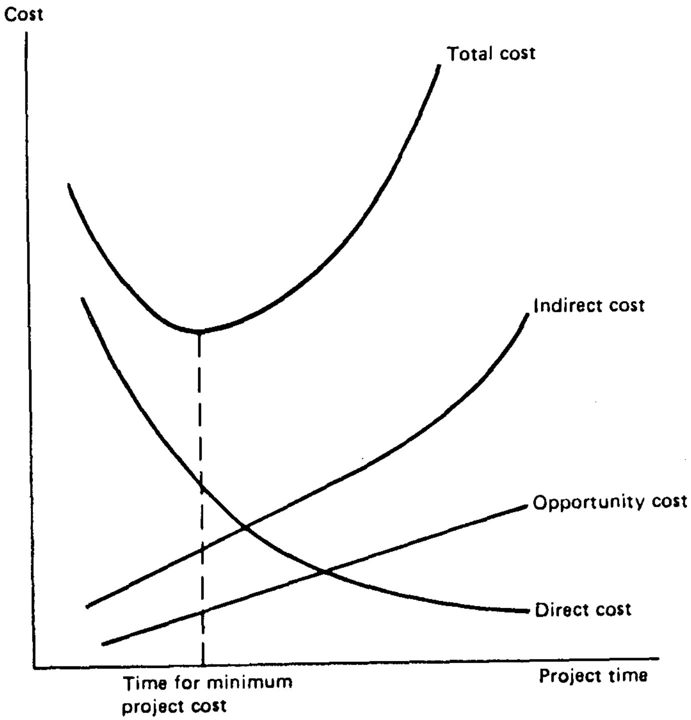

Figure 12-4 Cost-time relationship for a typical project.

but the indirect and opportunity costs increase, as depicted in Figure 12-4. Consequently, crashing activities may decrease *total* project costs while increasing *direct* costs. Costs here include monetary outlays and nonmonetary effects.

Estimated activity times are ordinarily based on normal working conditions. The length of time required to complete an activity can sometimes be shortened, at a price. In some instances that price may be a dollar cost; for instance, additional personnel may have to be hired temporarily or regular personnel may be asked to work overtime. In other instances, crashing the activity may lessen the quality of the activities; for instance, the surface of a highway

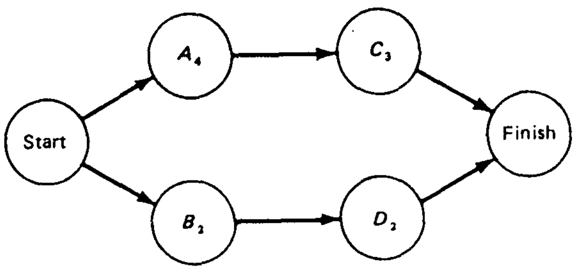

Figure 12-5 PERT network with critical path *AC*.

Table 12-2 A simple network with crash times

	Time (days)		Cost (dollars)	
Activity	Regular	Crash	Regular	Crash
A	4	2	$ 400	$ 800
B	2	1	300	450
C	3	2	500	600
D	2	1½	300	350
			$1500	$2200

may not be quite smooth if the smoothing operation has been rushed, or an agency report may be less informative if charts have to be omitted in the rush, or, in our illustration, the proper equipment may not be available if the sites have to be hurriedly selected. If it is necessary to speed up the project, then it is worthwhile determining (1) which activities are worth crashing and (2) which of those activities can be crashed at least cost, either monetary or otherwise.

We will first illustrate the time-cost trade-offs with a simple dollar example. Then we will return to the seminar planning illustration in which the costs may be nonmonetary.

Consider the simple PERT network of Figure 12-5. The regular time and cost, and the crash time and cost, are as in Table 12-2.

Consider critical activities *A* and *C*; we determine the marginal cost per day crashed so that crashing can be done as cheaply as possible. Here we assume that the cost of crashing *A* from 4 to 3 days is the same as the cost of crashing it from 3 to 2 days; that is, we assume the time-cost trade-off to be linear. If this were not the case, the logic would be the same but the computation would be a bit more involved. Table 12-3 focuses on the incremental cost per day for crashing critical activities.

If it is necessary to crash the project by 1 day, then *C* should be crashed. Its crash time is only $100 for the day, whereas crashing *A* one day would cost $200. If the project has to be crashed 2 days, then *C* should be crashed 1 day, and *A* should be crashed 1 day, resulting in a crash cost of $300. Finally if the project has to be crashed 3 days, then both *A* and *C* would be crashed completely. The crash cost would be $500 above the regular project cost.

If the critical path were crashed as much as possible, its completion time would be 4 days. Since the regular completion time of path *BD* is 4 days, it

Table 12-3 Crashing critical activities

Activity	Days shortened	Additional cost	Marginal cost per day
A	2	$400	$200
C	1	100	100

Table 12-4 The simple network revised

	Time (days)		Cost (dollars)	
Activity	Regular	Crash	Regular	Crash
A	4	2	$ 400	$ 800
B	2	1	300	450
C	3	2	500	600
D	3	2	300	400
			$1500	$2200

makes no sense to crash *B* and *D*. Ordinarily only the activities on the critical path are likely candidates for crashing. An exception occurs when the critical path crash time is less than the noncritical path regular time; then we consider crashing noncritical activities. Suppose activity *D* had a regular time of 3 days and a crash time of 2 days; the revised network is as in Table 12-4. The regular completion time of path *BD* would then be 5 days, whereas its crash time would be 3 days. If it is necessary to crash the project completion time from the regular time of 7 days to the crash time of 4 days, then path *BD* must also be crashed by 1 day. Since it is cheaper to crash *D* than *B*, *D* would be crashed by a day. When the crash time of the critical path is less than the regular time of a noncritical path, then noncritical activities may also have to be crashed.

Crashing with Nonmonetary Costs

We return now to the problem facing Ann Maxall, who is responsible for coordinating the forthcoming seminars for the state mental health and mental retardation (MH-MR) council. The seminars are to begin May 12, just 8 weeks from today, March 17. After developing the project network, she realizes that the regular completion time of the planning project is 9 weeks. Hence, she has to try to save a week somewhere. Her reasoning is as follows.

Ann estimates that if she reduces the time spent on planning the content of the seminar, it is quite likely that some important aspects of the information/evaluation system will be left out. She may think of it later, but then it may be too late to get the right speaker for that topic. She does not believe she can reduce the time to obtain speakers below the regular time of 1 week.

Preparing and mailing the flyer/invitations is an important step because the flyer will hopefully attract the county MH-MR personnel to the seminar. The state council is trying to avoid the "big stick" approach, by encouraging agency personnel to participate. Having it poorly prepared or short-cutting the mailing phase would run the risk of not getting "the word" to all the appropriate agency people. The consequence would be to have fewer participants at the seminar and subsequently have MH-MR personnel improperly using the information system.

Finally, the period for accepting reservations could be reduced from [illegible] weeks at the risk of not giving people sufficient advance notice, and hence getting the system off on the wrong foot by creating a "rush job" image. Alternatively, Ann could accept reservations for 2 weeks and then permit desk registration at the seminar. This would mean she would not have a good estimate of the number of participants, which she needs to be able to choose rooms of the right size, to have sufficient supplies, and to notify the seminar sites of the numbers for lunch and dinner.

Ann realizes that it is not impossible to crash most of the activities, but there is a price for any hurrying. Time saved in crashing *C* or *F* will have no effect on the project completion time, and so she has not even considered crashing them. To aid her own choice and to get counsel from her colleagues, she has summarized the crashing possibilities and their consequences. The summary is presented in Table 12-5.

After some thought and discussion, Ann finally arrives at a priority order for crashing the critical activities. Activity *D* is given top priority (it should be the last one to be crashed), since the seminar cannot possibly be successful if the county agency personnel do not attend. The second priority is given to activity *A*; if the personnel are not sold on using the system and do not learn how to use it properly, then the system has little chance of ultimate success. The last priority is given to activity *E*; the poor image can be avoided by encouraging but not demanding agency personnel to respond. Not knowing exactly how many will participate does pose some problem, but Ann feels she can easily have more supplies on hand than might be needed. Not knowing how many meals to serve will be a nuisance to the food service, will probably generate criticism of those running the seminar, but—this is important to Ann and the state council—will not involve the participants.

Accordingly, activity *E* is the one chosen to be crashed. Planning for the seminars now goes forward as originally outlined except for the shortened participants' response period. The seminars should be ready to start May 12.

Table 12-5 Crashing critical activities

	Time (weeks)		
Activity	Regular	Crash	Consequence of crashing
A	2	1	Important topics omitted; improper selection of speakers.
B	1	1	Crashing is infeasible.
D	3	2	Flyer not well prepared; some county agency personnel not on mailing list.
E	3	2	Rush-job image or unknown number of participants.

Determining time-cost trade-offs in PERT/CPM is analogous to sensitivity analysis in linear programming and, more accurately, to postoptimality analysis in goal programming. The time-cost trade-off of network management is a contribution specifically made by CPM. The contribution made particularly by PERT relaxes the requirement that there be a single time estimate to be treated as a constant and allows the network to be based on variable activity times. We will consider this aspect of project management in our next section.

VARIABLE ACTIVITY TIMES

The time required by most activities, especially those depending on human performances, is rarely a constant. Rather than make a single estimate of the activity time, three estimates are made. The three different estimates and their symbols are:

- a *The optimistic time.* This is the shortest amount of time the activity would need under the most favorable circumstances short of a miracle. The probability is very small that the activity could actually be completed in this amount of time.
- m *The most likely time.* This is the amount of time that the activity is most likely to require.
- b *The pessimistic time.* This is the amount of time the activity needs under the most unfavorable circumstances short of an act of God or some man-made catastrophe. The probability is small that this amount of time would actually be needed by the activity.

From these three estimates of activity time the mean, or *expected activity time,* is found by getting a weighted average of the three estimates. Intuitively it seems reasonable that the most likely time m should be given a greater weight than either the optimistic time a or the pessimistic time b. The equation for the expected activity time supports this notion; it is[2]:

$$t = \frac{a + 4m + b}{6}$$

[2] The equations for the expected activity time and standard deviation of activity time actually come from a *beta* probability distribution. The beta distribution is continuous over a finite range of values from a to b with mode m. We simply point out that the equations used have not been pulled from a hat; they come from the theoretical beta distribution, which repeated applications have shown to be an appropriate description of the activity time. It is beyond the scope of this book to examine the details of the beta distribution, how well it describes actual activity times, or the derivation of the equations for expected activity time t and standard deviation of activity time σ.

Example If

$$a = 2$$
$$m = 2\tfrac{1}{2}$$
$$b = 6$$

then

$$t = \frac{2 + 4(2\tfrac{1}{2}) + 6}{6} = 3$$

With a and b defined as they were, we can estimate the standard deviation of activity time. Assuming that the range $b - a$ covers six standard deviations,[3] the equation for the standard deviation is:

$$\sigma = \frac{b - a}{6}$$

Example Suppose that the simple network of Figure 12-5 has multiple time estimates as given in Table 12-6, columns headed "a," "m," and "b."

The mean or expected completion time of activity C is found by

$$t_c = \frac{a + 4m + b}{6} = \frac{1 + (4)2\tfrac{1}{2} + 7}{6} = \frac{18}{6} = 3$$

The standard deviation of the completion time of activity C is given by

$$\sigma = \frac{b - a}{6} = \frac{7 - 1}{6} = 1$$

The mean and standard deviations are found in like manner for the other activities.

[3] In the normal distribution 99.73 percent of the distribution falls within the six-standard-deviation range; regardless of its form, at least 89 percent of any distribution falls within this range.

Table 12-6 Multiple time estimates

Activity	a	m	b	$t = \frac{a + 4m + b}{6}$	$\sigma = \frac{b - a}{6}$	σ^2
A	2	4	6	4	.67	.45
B	1	2	3	2	.33	.11
C	1	2½	7	3	1.0	1.0
D	1	1½	5	2	.67	.45

Expected Time and Standard Deviation of a Path

A path is a sequence of separate tasks or activities. The time to complete a path is the sum of the completion times of the activities on the path. To find the expected completion time and the standard deviation of the completion time for a path, we use two results from probability theory:

1. The expected value of a sum of variables is the sum of the expected values.
2. The variance of a sum of variables is the sum of the variances, provided the variables are statistically independent.

Example The expected completion time of path AC is

$$t_{AC} = t_A + t_C$$
$$= 4 + 3 = 7$$

To find the standard deviation of the completion time of AC, we first find its variance. The variance of AC, assuming its activity times are independent, is given by:

$$\sigma^2_{AC} = \sigma^2_A + \sigma^2_C$$
$$= .45 + 1.0 = 1.45$$

The standard deviation of the path completion time is

$$\sigma_{AC} = \sqrt{1.45} = 1.20$$

One finds expected completion time and standard deviation of completion time of any path in the same way. With these two pieces of information probability statements become meaningful. Of particular interest are probability statements about the critical path and about the whole project.

Completion Time and Probabilities

To be able to make a probability statement about a path's completion time (for example, the probability of completing the path in 4 weeks is .90), its probability distribution must be known. Since most projects have a large number of

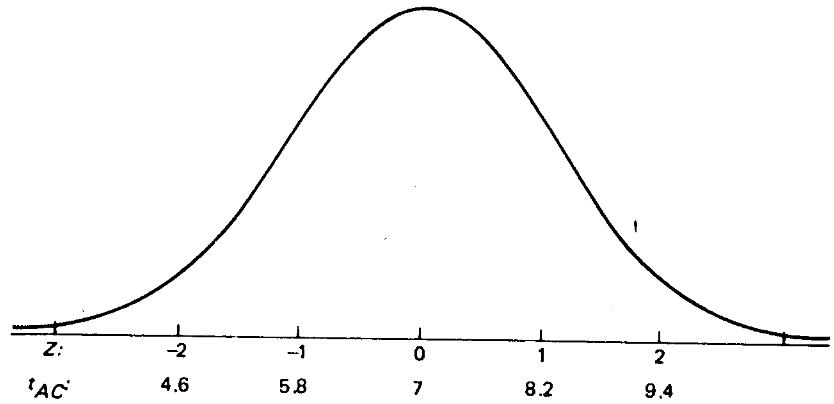

Figure 12-6 Normal probability of path completion time, with $t_{AC} = 7$ and $\sigma_{AC} = 1.2$.

activities, we can focus on those without much loss. If a path has many activities, then its completion time will be approximately normally distributed.[4]

Example With only two activities on the critical path the assumption of normality is a rather coarse approximation. Assuming for the moment, however, that the critical path completion time is approximately normally distributed with a mean of 7 days and a standard deviation of 1.2 days, the critical path completion time follows the probability distribution illustrated in Figure 12-6. Assuming a normal distribution, we can find the probability that the project will be completed within any given number of days.[5] The probability that path AC will be completed within 8.2 days is given by

$$P(t_{AC} \leq 8.2) = P(Z \leq 1) = .8413$$

The probability that the project will be completed within 9 days is found as follows:

$$P(t_{AC} \leq 9) = P\left(Z \leq \frac{9 - 7}{1.2}\right) = P(Z \leq 1.67) = .952$$

Finding these probabilities assumes that the critical path completion time is normally distributed, which is more closely adhered to when the path has many activities. Determining the mean and the standard deviation from the three estimates assumes that the activity time is approximately beta-distributed. If either of these assumptions is not satisfied, then the network may best be analyzed by simulation rather than by a direct analysis.

[4] According to the *central limit theorem* of probability theory, the sum of n independent variables tends to be normally distributed as n tends toward infinity.

[5] Finding normal probabilities is described in Chapter 3. The standard normal table is Table 2 of the Appendix to this text.

Finding probabilities of the critical path completion time, to the exclusion of the completion times of other paths, is deficient because a noncritical path may take longer than the critical path because of the variability of the activity times. Hence, especially if noncritical paths have completion times close to the critical path completion time, their expected completion times should be included in the analysis.

Example Suppose the network of Figure 12-7 has the estimated completion times as given in Table 12-7.

Path *ABD* is critical with $t_{ABD} = 9$ days, and path *CE* is almost critical with $t_{CE} = 8$. Here it is important to find the probability of both paths being completed in the specified number of days. Means and standard deviations of path completion times for the two paths are:

Path	t	σ
ABD	9	1.53
CE	8	1.20

To determine the probability that the project will be completed within 10 days, first determine the probability that each path will be completed within 10 days. These probabilities are found by

$$P(t_{ABD} \le 10) = P\left(Z \le \frac{10-9}{1.53}\right) = P(Z \le .65) = .743$$

Similarly, $$P(t_{CE} \le 10) = P\left(Z \le \frac{10-8}{1.20}\right) = P(Z \le 1.67) = .953$$

The project is complete only when *both ABD and CE* are complete.

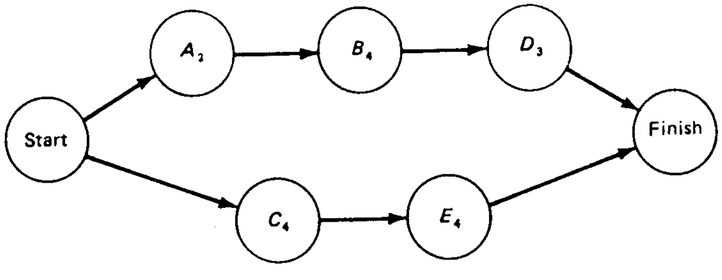

Figure 12-7 A critical path and an almost critical path.

Table 12-7 A critical path and an almost critical path

Activity	a	m	b	$t = \frac{a+4m+b}{6}$	$\sigma = \frac{b-a}{6}$	σ^2
A	1	2	3	2	.33	.11
B	2	3	10	4	1.33	1.77
C	2	3½	8	4	1.00	1.00
D	2	2½	6	3	.67	.45
E	2	4	6	4	.67	.45

Assuming that the paths are independent, the probability multiplication rule (Chapter 2) applies.

$$\begin{aligned} P(t_{\text{project}} \le 10) &= P(t_{ABD} \le 10) \cdot P(t_{CE} \le 10) \\ &= (.743) \cdot (.953) \\ &= .71 \end{aligned}$$

The closer the expected times of a noncritical and a critical path, the more likely it is that the noncritical path will actually wind up taking longer than the critical path. If the paths share resources (personnel or machinery or vehicles, for instance) or if the paths have a common activity (in the seminar planning project, activity *D* was common to paths *ABDE* and *CDF*), then the paths are not independent. In such a case the simple multiplication rule would not apply. If the project is complex with many conditional probabilities, simulating the network would be the wisest approach.

PERT/CPM AND PROJECT DECISION MAKING

Development of an initial project network is time consuming, but it greatly enhances understanding the project. As the project unfolds, and actual times and costs are used to update the network, reassessment of critical activities and paths provides ongoing planning and control. It helps to identify alternative ways of achieving the project management objectives related to time, cost, and quality. A large-scale project may have hundreds or thousands of activities. The corresponding network would most assuredly be computerized, but even so, the voluminous printout requires time, patience, and commitment if the technique is to contribute to project management and control. As with all management aids, the decision of whether to employ it depends on whether the improvement it provides outweighs the resources it consumes. While there are a few extensions of PERT/CPM, one in particular focuses on the cost aspects of project management.

PERT/Cost

PERT/Cost is a cost accounting extension to project management; it groups costs by activities rather than by organizational lines, fitting quite well with project management. PERT/Cost provides information on activity cost-to-date, which can be compared to activity completion-to-date. The mechanism allows for better control of each activity and the project as a whole.

Like any model or mechanism, PERT in any form or extension has its shortcomings.

Limitations

Project management is not without its problems, which include difficulties in estimating completion times, estimating regular and crash costs, the validity of statistical independence of activities, the validity of applying the beta distribution to activity times and the normal distribution to project times. There are also less technical difficulties. The effort required both to develop and to update the network is sometimes seen as extraneous both to the project and to its operational control.

Our discussion of project management will conclude with an application of a project network to a local referendum.

APPROVING A BOND ISSUE: AN APPLICATION[6]

Ordinary revenue is the usual source for expenses associated with day-to-day and year-to-year operations of a municipality or school district. Extraordinary expenses, arising from the construction of a new sanitary system, water purification plant, or a new school building, are often satisfied by grants from a higher authority (federal or state) or from funds invested by the public to be returned with interest at a later date. The latter source, through a bond issue, often requires voter approval. The process of gaining voter approval through a referendum must convince the voters of the real need for the bond issue, identify the time and places of polling, and record and publicize the results of the referendum. Such a project, then, is composed of many activities. Coordinating and planning such a project may benefit from a network representation. Here we present a simplified version of the activities that would comprise gaining voter approval for a school district's bond issue. The activities listed below are identified by number in the project network in Figure 12-8.

1. Establish need within long-range plans.
2. List resources and timed needs.

[6] This application is an adaptation of H. W. Handy and K. M. Hussain, *Network Analysis for Educational Management*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1969, pp. 79–83.

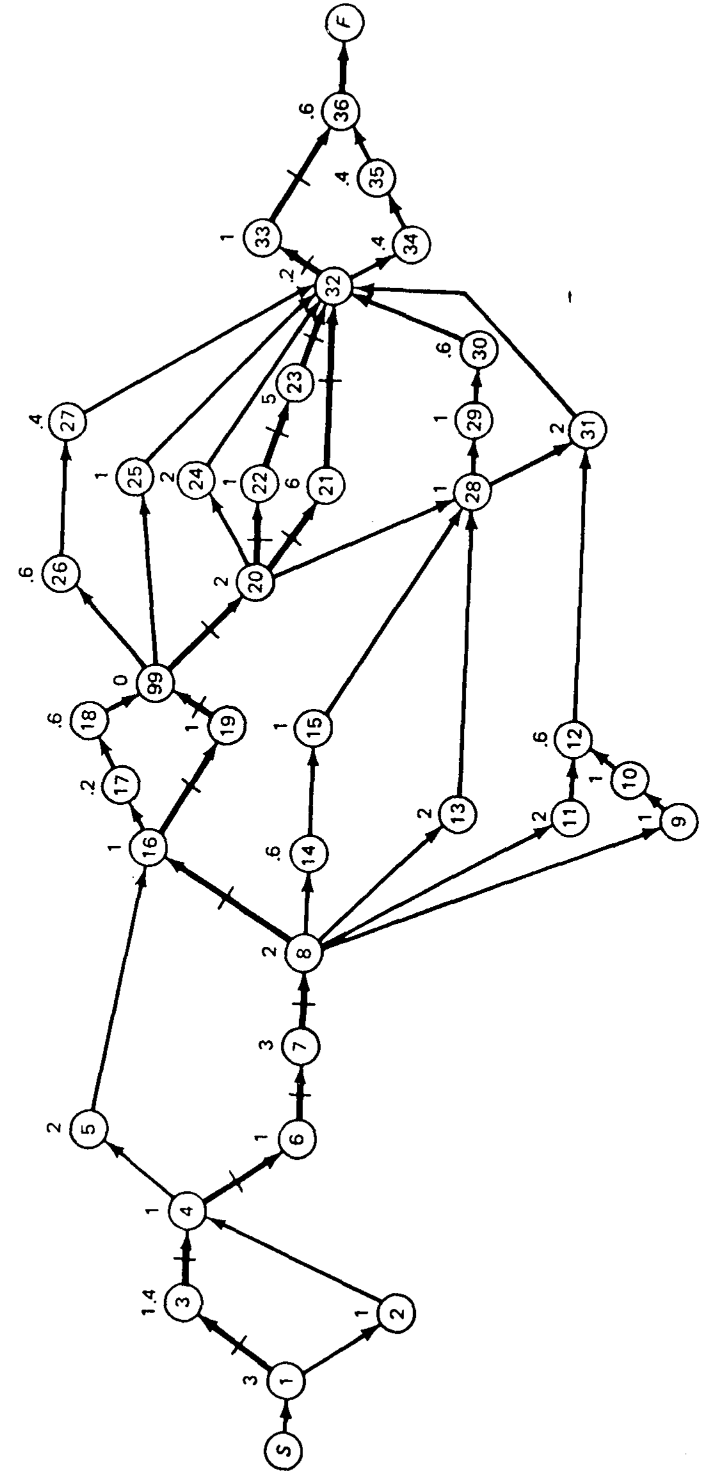

Figure 12-8 School bond issue PERT network (critical path: 23.2 weeks).

3. Form citizens' planning committee.
4. Identify campaign objectives and amount of bond issue.
5. Seek support of board members.
6. Organize information committees.
7. Plan promotion strategy and costs.
8. Develop master chart of dates and events.
9. Activate a speakers bureau.
10. Arrange for speakers and audiences.
11. Prepare campaign literature.
12. Distribute materials.
13. Solicit volunteer workers.
14. Solicit aid of a volunteer worker leader.
15. Organize materials to be given to volunteers.
16. Petition school board for elections.
17. Set election date.
18. Check for conflict with other elections.
19. Verify sufficiency of petition.
20. Determine polling places and district boundaries.
21. Declare election (with 6-week waiting period).
22. Obtain group endorsements of bond issue.
23. Run advertising campaign in media.
24. Conduct PTA voter registration drive.
25. Select election personnel.
26. Print election materials.
27. Deliver election materials.
28. Assign districts to volunteers.
29. Hold house-to-house campaign.
30. Arrange for baby-sitting and transportation.
31. Have speakers make presentation to their groups.
32. Hold election.
33. Apply for no litigation certificate (1-week delay required).
34. Canvas election results.
35. Publicize results.
36. Make official record of proceedings.
99. Dummy activity.

The network requires a minimum of 23.2 weeks for completion.

The estimated time for each activity, in weeks, appears above the activity circle in the diagram. The critical path, indicated by a heavy line, requires 23.2 weeks for completion. Note the parallel critical paths from activity 20 to activity 32, the election itself. A *dummy activity*, 99, has been inserted after activities 18 and 19 to simplify the network. Three activities, 20, 25, and 26, depend on the completion of activities 18 and 19. In such a case, the use of a dummy activity avoids the jumble of many intersecting sequence arrows.

One of the advantages of developing such a project network is that it provides greater opportunity for identifying all the activities, and indicates when each one should start. This helps to avoid omitting a necessary or at least helpful activity, or starting it so late that the result is less than desirable. This is particularly the case in a "soft" project such as the bond issue, where the election will take place even if some of the prerequisite activities have not been completed. For example, the election will be held on the declared day even if sufficient time has not been given to the voter registration drive, and even if the advertising campaign never got into full swing, and even if house-to-house campaigning reached only a few voters. In a "hard" project, such as the construction of a building, prerequisite activities are less easily omitted but may be started late with the ultimate effect of delaying the project.

SUMMARY

Management of projects which are rather unique combinations of activities is appropriately in the realm of a network model. A hybrid of PERT and CPM requires identifying the project activities, the activity times, and the sequencing information. From that, the earliest possible time an activity can start (ES) and the latest it can start without delaying the project (LS) are calculated. The difference LS − ES yields the slack time for each activity. Activities with zero slack are critical activities. A sequence of critical activities forms a critical path; the completion time of the critical path is the project's completion time. Critical activities may be able to be crashed at a price. PERT/CPM helps one find the right activities to crash to keep the cost of crashing to a minimum. Especially when those costs are nonmonetary, network analysis helps to assess the time-cost trade-offs in much the same way that goal programming helps to assess goal priorities.

Where activity times are random variables, one makes three different time estimates:

The optimistic, a
The most likely, m
The pessimistic, b

From these, the expected completion time

$$t = \frac{a + 4m + b}{6}$$

and the standard deviation of completion time

$$\sigma = \frac{b - a}{6}$$

can be estimated.

Assuming that the activity times are independent and the path completion time is normally distributed provides the basis for probability statements about the project completion time. Where any of these assumptions are severely lacking, simulation of the project network may be necessary.

Various extensions of this network technique have been developed which provide ways of building resource constraints into the network and which keep track of costs as well as time. A PERT-type network is not only useful in the planning stage but also helpful in monitoring the project once it is underway.

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EXERCISES

Extensions of chapter examples

12-1 Refer to the seminar planning project. (See p. 247.) How is the network affected in sequence and time if the seminar selection site *C* is dependent on the seminar content *A*?

12-2 Refer again to the seminar planning project. Ann Maxall realizes, as an afterthought, that the single time estimates that she originally made were really the most likely times to complete the activities. Upon further reflection she identifies an *optimistic* and a *pessimistic* time for each activity; they are as follows:

For A: 1 and 4
For B: ½ and 2
For C: 1 and 3
For D: 2 and 4
For E: 2 and 6
For F: ½ and 2

(*a*) What is now the expected project completion time?
(*b*) What is the probability that the project will be completed within the 9 weeks originally planned for?

12-3 Refer to problems 12-1 and 12-2. Suppose the time estimates of problem 12-2 apply to the network in which activity *C* depends on the completion of activity *A*.
(*a*) What is the expected completion time?
(*b*) What is the probability that the project will be completed within 9 weeks?
(*c*) What does this suggest about developing the proper sequence of activities?

12-4 Refer to the referendum application of Figure 12-8. Consider the portion of the network from activity 8 to activity 32, including only the critical paths and activities 13, 14, 15, 28, 29, and 30.
(*a*) Draw the network diagram.
(*b*) Calculate ES, EF, LS, and LF for all activities.
(*c*) Find the slack time of all activities.
(*d*) Find the critical path of the diminished network.

12-5 Refer to exercise 12-4.
(*a*) What is the slack time of activity 13?
(*b*) If activity 13 is 5 days late in getting started, how much slack will activity 28 still have?
(*c*) How much can activity 13 be delayed without delaying activity 28? This portion of the slack time is referred to as *free slack*, the difference between the ES of a successor activity and EF of the activity itself. The remainder of activity 13's slack is shared with activities 28, 29, and 30, those activities in series with it.

Other Applications

12-6 A project consists of the following activities:

Activity	Immediate predecessors	Estimated time (days)
A	—	2
B	—	3
C	*A*	3
D	*B*, *C*	1
E	*D*	4
F	*D*	2
G	*E*, *F*	1

(*a*) Draw a project diagram.
(*b*) Compute the ES, EF, LS, and LF for each activity.
(*c*) What is the project completion time?

12-7 Refer to problem 12-6. If activity *D* is not an immediate predecessor of activity *E*, but *B* and *C* are predecessors of *E*, what change should be made in the network diagram? Revise the project completion time.

12-8 In case it should be necessary to hurry the project of problem 12-6, the crash times of the activities have been identified. At the same time, estimates of costs under regular and crash conditions have also been identified. The time estimates and cost estimates for both regular and crash conditions are shown in the table on page 296.
(*a*) Find the regular project cost.
(*b*) Identify the time-cost trade-offs in the order in which they should be made, depending on the extent of crashing necessary.
(*c*) Find the cost of crashing 2 days.

12-9 Refer to problem 12-8. Suppose activity *D* is no longer a predecessor of activity *E*. Revise the order in which time-cost trade-offs should be taken.

Activity	Time (days)		Cost	
	Regular	Crash	Regular	Crash
A	2	1	$200	$ 300
B	3	2	300	400
C	3	1½	400	700
D	1	—	200	—
E	4	2	700	1000
F	2	1	300	500
G	1	½	200	300

12-10 The State Department of Public Welfare has had separate contracts with data processing firms to service its needs. It has just been decided that the payroll, client status/eligibility, and cash assistance/medical assistance functions will henceforth be handled internally on the department's own new computer system. Besides the authorization to enter into a leasing agreement for the equipment, the department has also been given hiring authority to secure programmers who will develop and maintain the system. Programmers who are hired will be responsible for developing the payroll program (P) and the client status/eligibility (CS/E) program, and for testing and maintaining the cash assistance/medical assistance (CA/MA) program, that will actually be written by the management services division of the Bureau of the Budget (BOB). Certain aspects of the cash assistance/medical assistance program depend on the client status/eligibility program. The director of systems support has attempted to identify the various activities that must be executed to get the complete set of programs running on the system that will be shortly installed. The activities and their expected completion times are described in the following table.

Symbol	Activity	Time (weeks)	Predecessors
A	Compare computer systems; notify manufacturer of choices.	8	
B	Await delivery of computer.	16	A
C	Hire programmers.	4	A
D	Flow chart P program.	6	C
E	Flow chart CS/E program.	10	C
F	Code P program.	3	D
G	Code CS/E.	5	E
H	Consult with BOB staff on CA/MA program.	4	A
I	Flow chart CA/MA program.	8	E, H
J	Code CA/MA program.	8	G, I
K	Test P and CS/E programs.	2	F, G
L	Debug P and CS/E programs.	2	K
M	Install and test new computer system.	2	B
N	Test P and CS/E programs on new computer.	2	L, M
O	Prepare P and CS/E program manuals.	4	K
P	Test CA/MA program on new computer.	2	J, M
Q	Prepare CA/MA program manual.	3	P
R	Implement P and CS/E programs.	2	N
S	Implement CA/MA program.	2	P
T	Thoroughly acquaint hired programmers with CA/MA program.	3	N, P

(*a*) How long will it take for the three programs to be up and running and under the control of department programmers?

(*b*) Which activities are critical in meeting the schedule of part *a*?

(*c*) How long will it be before the department can do an internal audit on client eligibility on the new computer?

12-11 This problem, though greatly simplified, is based on the Kittleson and McCarthy (1973) article, which concludes with, "It is clear that producing a play is a complex systems management problem. What was surprising to the director was the sheer usefulness of PERT in perceiving the interrelationships of the large number of activities".

The Community Theatre starts its Shakespearian Festival on Monday, June 30. It has identified the following tasks necessary for the production.

Activity	Description	Predecessors	Time (weeks)
A	Selection of plays		5
B	Casting	A	6
C	Set design	A	4
D	Costume design	A	5
E	Set construction	C	6
F	Rehearsals	B	5
G	Dress rehearsals	D, E, F	4
H	Printing tickets and programs	B	8
I	Festival presentations	G, H	2

(*a*) On what date must the selection of plays begin?

(*b*) It is now 2 weeks after that date. Propose alternatives for crashing, presenting the cost of crashing each activity.

Project problems

12-12 Pick some project you are or will be involved in, such as a recruiting effort to fill some position(s), the implementation of a new system, the development of some legislation, a research paper, or other undertaking. List its components, or activities, and their predecessors, and estimate times. Construct a project network. Identify the critical path and any almost critical paths.

12-13 The Calvin and Fielding (1975) article presents an application of a modification of PERT in an area that is not always considered the domain of management techniques.

(*a*) In what way is PERT modified in this application?

(*b*) What contributions did PERT make?

(*c*) Can you think of another potential application from an unlikely area?