3-11 A question of interest to HUD is how far people usually travel to do their major food shopping. A study in one city revealed that urban dwellers' travel distance was normally distributed with a mean of 1.5 miles and a standard deviation of .75 mile. What proportion of the people traveled

(*a*) More than 2 miles?

(*b*) Less than 1 mile?

(*c*) The curve suggests that about 2.5 percent of the people traveled less than 0 mile. Obviously this is not true in reality. Can you account for the "error"?

3-12 Refer to exercise 3-11. A similar study of suburbanites revealed that their shopping travel distance was normally distributed with a mean of 2.5 miles and a standard deviation of 1.25 miles.

(*a*) What proportion of the urban dwellers traveled more than the mean suburban distance?

(*b*) What proportion of the suburban dwellers traveled more than the mean urban distance?

(*c*) In a recent press interview, a city official claimed: "Food markets are no longer readily available to city folk. In fact, a large number of them must travel farther than most suburbanites to get to the food stores." Comment on his statement, noting (1) whether a "large number" do so; and (2) if this means they *must* do so.

Project problems

3-13 In the 1980 American census, find two questions that generate random variables, and two questions whose answers cannot meaningfully be represented by random variables.

Christopher McKenna,
Quantitative Methods for
Public Decision Making

CHAPTER
FOUR

DECISION THEORY: A FRAMEWORK FOR DECISION MAKING

Decision theory provides a method for rational decision making when the consequences are not fully known. Decision makers apply various criteria to decision situations. For specific criteria, decision theory identifies the best alternative. When the criteria are vaguely defined, or not unanimously held in group situations, decision theory provides a framework for evaluating alternatives. In this chapter we explore various specific criteria. So that our attention will not be diverted, the context is one with easily compared dollar consequences. Many decisions, especially in the public sector, have consequences that are less easily measured or compared. The use of decision theory in such a setting is shown in a health-planning application. Other applications are considered in the exercises.

DECISION THEORY AND THE DECISION-MAKING PROCESS

A decision involves choosing one alternative from among a set of alternatives. The decision situation often involves uncontrollable factors and many consequences. Two brief examples will illustrate the essential features of a decision.

Examples

1. A state has to choose from among many bids to drill for natural gas on certain state-owned properties. The amount of natural gas present is an uncertainty. The consequence for the state, the full-bid payment, depends on

the alternative chosen, the particular bid, and on the uncontrollable, the volume of recoverable gas.

2. The federal government through its Department of Health has to decide whether to support an influenza vaccination program for the entire nation, or for a selected portion of the population, or not at all. The outbreak of an epidemic is a possibility, but not a certainty. The consequences for the nation, illness, death, avoidance of illness, and reaction to the vaccination, will depend on the alternative program chosen by the government and on the uncontrollable possibility of an outbreak of influenza.

The role of decision theory in decision making is twofold: first it provides a framework for better understanding the decision situation, and second, it can furnish a way to evaluate alternatives in light of the uncertainty. The relationship between decision theory and decision making is depicted in Figure 4-1. If a decision is to be made on the basis of one of the decision criteria to be examined here, then the selection of an alternative is a direct effect of decision theory. If alternatives are to be compared relative to considerations other than the so-called pure criteria, then decision theory aids in the evaluation of alternatives. Decision theory can indirectly assist in defining the problem and in identifying alternatives, while directly helping to evaluate the alternatives.

Decision situations can involve one objective or a number of objectives. The latter is more realistic, especially in the pluralistic setting of public decision making. We first examine a decision-making framework for a single objective. While some of the particulars of the single-objective situation can be transferred only with difficulty to multiple-objective situations, the framework can often be used with benefit. Later in this chapter we shall illustrate just such a decision situation. In Chapter 6 we will consider an extension of the decision framework to cover multiple-objective situations more appropriately.

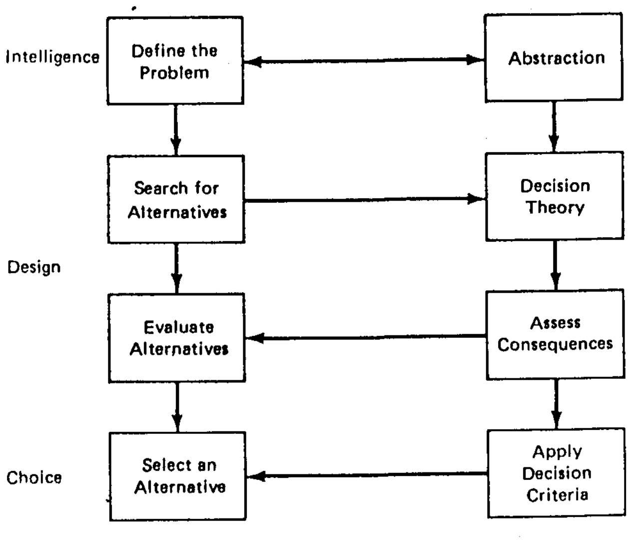

Figure 4-1 The role of decision theory in decision making.

THE ELEMENTS OF A DECISION

Assuming there is some underlying value system and a set of objectives, the set of decision alternatives is the prime ingredient of decision making. Some examples of decision alternatives are:

1*a*. To establish a graduated local income tax
1*b*. To establish a flat-rate local income tax
1*c*. To establish no local income tax at all
2*a*. To repair the potholes in a roadway
2*b*. To repave the entire roadway
2*c*. To construct an alternate parallel roadway
3*a*. To establish a system of completely government-supported medical care
3*b*. To establish a user-supported national health insurance program
3*c*. To avoid direct government support of medical care

If there were but a single possibility that could be chosen, there really would be no decision to make; the "decision maker" would simply go along with the inevitable.

In most decision situations there are many factors beyond the control of the decision maker. These uncontrollable factors are often referred to as "states of nature." This name comes from making a decision in the face of such uncontrollable natural factors as weather, the presence of oil, and the extent of viral growth. The name continues in use even though it is recognized that there are many uncontrollable factors that are not simple natural occurrences or "acts of God." Some examples of states of nature or uncontrollable factors are:

1. The future employment level
2. The future traffic patterns in a particular area
3. The level of demand for medical care

The uncontrollable factors can be studied for a better understanding of the decision situation, but they can neither be controlled nor chosen by the decision maker.

The alternative that is chosen by the decision maker is not usually chosen for its own sake, but rather for what will occur as a consequence. In fact the purpose for a decision choice can be found in the envisioned consequences, whether they be realistic or fantastic, beneficial to many or a select few, essential or cosmetic. Some examples of consequences of a decision act and a state of nature are:

1*a*. A continuous budget deficit
1*b*. A depression of new industry
2*a*. Accelerated deterioration of a roadway
2*b*. The public generally avoiding a roadway
3*a*. Deterioration of the health of a significant portion of the population
3*b*. Huge budget deficit

Consequences occur as the result of both the decision act that is chosen and the uncontrollable factors that occur. Hence, in analyzing a decision situation, the decision maker tries to identify the possible uncontrollable factors or *states of nature* and the consequences associated with each factor and each decision alternative. The extent of the knowledge one has about the uncontrollable factors is the basis for different approaches to decision making.

THE CONTINUUM OF UNCERTAINTY IN DECISION MAKING

The decision-making environment is characterized by the extent of knowledge one has about the states of nature. At one extreme we have:

Decision making under ignorance The decision maker knows absolutely nothing about the states of nature, including not even knowing the possibilities.

This extreme is seldom if ever presented so openly. A slightly more subtle presentation of it has been seen, however.

Example After discussing the future needs of a school district's youngsters, school planners decide on a particular architectural plan from the many considered. The plan chosen holds the most promise for providing the greatest opportunity for a sufficiently large portion of the future school population. Just as the cornerstone is laid, however, the declining birth rate of 6 years ago has its impact on the school population. The decision makers realize they were concerned with the wrong states of nature. Their concentration on the specific needs of future students allowed them to ignore the more basic uncontrollable, namely, the size of the school population. Perhaps the new building can serve as an adult education center, or be sold to the state university for use as a satellite campus to recoup some of the public money. Nonetheless, as far as the actual states of nature are concerned, the decision makers were operating in a relatively knowledge-free environment.

Although the "ignorance" in the heading "decision making under ignorance" refers to the lack of knowledge of the states of nature, in such an environment it is just as applicable to the consequences, and, more than likely, to the decision alternatives as well.

At the other extreme of the continuum we have:

Decision making under certainty The decision maker knows exactly the state of nature that will occur.

This rather blissful situation allows the decision maker to examine the available choices and the single consequence of each. The most desirable consequence dictates the decision alternative to be chosen.

Example If the nutrient content of various foods does not vary from day to day, then the nutritionist can prescribe a particular diet in order to maintain desired levels of nutrient intake. If there are many combinations that will yield the desired levels of nutrition, then the nutrition decision maker can choose the alternative that is the optimum according to some characteristic. For example, the dietician might choose the diet on the basis of providing the greatest variety of foods to patients, whereas the zoo keeper might choose the diet on the basis of lowest cost. In either case, the nutrition content is assumed to be completely known so that the decision can be made on the basis of the most desirable assured consequences.

Between the two extremes are decision-making environments in which there is incomplete knowledge of the states of nature:

Decision making under uncertainty More than one state of nature is known to exist, but the probability of any state is unknown.

Decision making under risk More than one state of nature is known to exist, and the probability of each state is known.

There is more information available in the latter category, but neither situation is completely known. These categories are the real subject matter for decision theory.

The four decision-making environments can be depicted on a continuum of knowledge about the states of nature, as in Figure 4-2. The first and the last environments are dismissed from further consideration here, albeit for different reasons. If the decision maker realizes his ignorance, attention can be given to moving to one of the other environments. If the decision maker possesses certain knowledge of the state of nature, attention can be directed to some other facet of the decision process. For example, if it is known that there will be an outbreak of influenza, then attention can be directed to estimating the cumula-

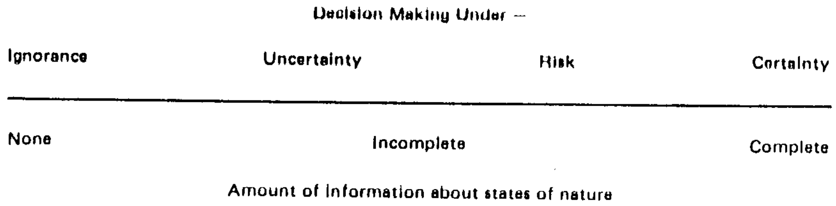

Figure 4-2 Knowledge of the states of nature on a continuum.

tive effects of any vaccination decision. The two environments of incomplete knowledge will be presented in a prolonged illustration.

DECISION MAKING UNDER UNCERTAINTY

Natural resources are ordinarily considered to be the property of the owner of the land in or on which the natural resources are present. Exceptions to this occur when the agreement, by which ownership of land is transferred from one party to another, contains a clause indicating that mineral rights are retained by the former owner. When the minerals are present on federal- or state-owned lands, they are considered to be publicly held resources. The public at large cannot undertake the actual resource recovery, and the government, whether federal, state, or other, is not in the business of mineral recovery. The publicly held resources are recovered through a cooperative venture between the public, represented by the responsible officers, and a private operator who has the technical capability to recover the minerals and transform them into usable products. Under the cooperative venture, the government transfers the right to recover the minerals to the private operator in exchange for a share in the ultimate profits of the venture. The profits are therefore the public's share in the mineral recovery operation and as such are considered like other public money, to be used for public purposes. The mineral profits due to the people through their government depend on the value of the minerals recovered. The government may use the money to replace funds that would otherwise come from taxation or may use the money for some specific purpose that would not otherwise be served from the general fund, for instance, housing subsidies to the elderly.

Natural gas is currently considered to be a rather valuable resource. As with many other natural resources that are valuable and of restricted occurrence, the region that finds and develops the natural gas thereby generates for itself a fair supply of the national currency. Suppose that a geologist has discovered, on state-owned land, certain types of rock formations that suggest the possibility of a natural gas field. There is believed to be a reasonably good chance that a gas recovery operation would be successful.

Since the state itself is not in the gas recovery business, it enters into a cooperative arrangement with private industry to recover the available gas and transform the gas in the ground into gas in the pipes and money in the till. One way to enter into such an arrangement is for the state government, through its appropriate contracting agency or administration (a general services agency, for example), to invite bids from individual private developers or groups of private developers. In so doing the state would stipulate its requirements and conditions, perhaps indicating that the land shall remain the possession of the state, that the operation shall take place during a specific number of years, and that afterward the use of the land will revert to the discretion of the state without further obligation to the private contracting party.

It is not entirely unrealistic to assume that the state invites bids in each of three categories:

1. A fixed price for the length of the lease, regardless of the outcome of the gas recovery process
2. A fixed percent of the profits, in which the eventual lease amount depends on the profitability of the recovery operation
3. A combination of fixed amount plus a variable amount dependent on the success of the venture

Upon receipt of the sealed bids in each of the three categories, the state can choose any bid that was best in its category. Thus, the state can reduce the set of alternatives to three.

Assume further that the results of the gas recovery venture can be expressed as complete success, moderate success, or failure. A team of geologists and natural resource economists has estimated the quantity and value of the gas that would be present for each of these three gas recovery possibilities. The gas would be recovered over a long period of time. During each year of the operation, a certain volume of gas, and therefore a certain value of gas, would be recovered. Monetarily speaking, the operation is represented by a stream of dollar amounts in successive years. From that stream one can determine what is called the *present value* of the operation. (The actual determination of the present value of future amounts of money is a topic to be considered in Chapter 7.)

Note that the values involved in this decision situation are very much dependent on a number of estimates. The quantity of gas actually present in the gas field is estimated. Whether the actual venture will be successful is estimated; the speed of recovery is estimated. The future value of natural gas is estimated. Since there are so many estimates, the decision maker should be very cautious in accepting recommendations based on a single analysis. "What if" questions should be posed so that the simple decision model will be the subject of sensitivity analysis, in order to determine the effect of modifications in the estimates on the ultimate decision. We explore such sensitivity analysis in extension exercises 4-1, 4-2, and 4-3.

Table 4-1 Consequences (payoffs) of lease alternatives (millions $)

State of nature	Decision alternatives		
	Percent of profit	Fixed lease	Combination
Complete success	2000	600	1200
Moderate success	600	600	540
Failure	60	600	300

The present values of the lease for each alternative and each state of nature are presented in the decision matrix or payoff table, Table 4-1. When consequences are money amounts, they are usually referred to as *payoffs*.

Although the value of the lease to the state for each of the decision alternatives for each of the possible states of nature is completely specified in the table, note that the decision must be made in the face of uncertainty. The consequence of the decision is not known at the time the decision must be made.

For the same set of alternatives and the same states of nature, different decision makers might very well make different decisions. In effect, the decision makers are making use of different decision-making philosophies or different decision-making criteria. We now explore some of these criteria.

Maximax Decision Criterion

The maximax decision criterion is the one used by the optimistic decision maker. Underlying the maximax philosophy is either of two implicit thoughts. The first is that no matter which decision he makes, the best possible consequence will follow, and so he might as well choose the alternative that has the highest possible value associated with it. The second is that no matter what decision he makes, a poor consequence will not be very disconcerting. In either case, the maximax decision maker will choose the alternative with the highest possible payoff. For the current problem the highest payoff is $2 billion, a possible consequence of the *percent-of-profit* alternative. Hence, the maximax decision maker will choose as his decision alternative the percent-of-profit contract. *Maximax is the criterion used by a decision maker who chooses the act which makes possible the maximum payoff.*

If the payoff table contains losses rather than profits, the maximax decision maker would choose the act which would *make possible the minimum loss*. Here again the decision maker would be choosing the act which could yield the best possible consequence.

Example Most people use the maximax decision criterion in specific situations. An example is the purchase of a lottery ticket. Table 4-2 presents the payoffs associated with the two decision alternatives concerned with purchasing a ticket and the two possible states of nature concerning whether

Table 4-2 Lottery-ticket decision matrix

State of nature	Decision alternatives	
	Purchase ticket	Do not purchase ticket
Number not drawn	($1.00)	$0
Number drawn	$1000	$0

the purchased ticket's number is drawn. If the decision is to purchase the ticket and if the number is not drawn, there is a loss of $1.00 (represented in the table in parentheses, the usual accounting symbolism for a negative quantity). For that same decision, the state of nature "number drawn" generates a positive payoff of $1000, the lottery prize.[1]

Minimax Decision Criterion

The decision maker who is more a pessimist than an optimist will choose his decision alternative on the basis of some form of Murphy's law. Murphy's law suggests that no matter which alternative is chosen, the worst that can possibly happen will happen. Hence, the minimax decision maker examines the possible payoffs for each alternative and takes particular note of the smallest payoff for each alternative. He then chooses the alternative that yields the greatest of those minimum payoffs. In the leasing example, the minimax decision maker notes that the percent-of-profit alternative might wind up yielding only $60 million, the *fixed lease* has a minimum of $600 million, and the *combination* might have a yield as low as $300 million. The greatest of those minimum payoffs is $600 million, and so the minimax decision maker chooses to accept the contract for the fixed lease worth $600 million. *The minimax decision maker tries to maximize the minimum gain.*

If the payoffs are losses rather than gains, the minimax decision maker identifies the greatest loss associated with each alternative, and chooses the alternative that has the smallest of those maximum losses. In other words, *the minimax decision maker would try to minimize the maximum loss*. His guiding principle is to choose the alternative that has the best of the worst possible payoffs.

Actually, it is not only Murphy's law that urges one to use the minimax decision criterion, but also a conservative decision approach that may generate the same decision-making philosophy.

Example Whenever we purchase optional (not legally required) liability insurance for our automobile or home, we are making use of the minimax

[1] Actually if the lottery prize is $1000, then the positive payoff would be $999 since we should deduct the cost of the lottery ticket from the gross prize to yield a net payoff. In order not to confuse the example, we disregard this small difference.

Table 4-3 Liability-insurance decision matrix

	Decision alternatives	
State of nature	Purchase	Do not purchase
Accident occurs	($100)	($10,000)
No accident	($100)	0

decision principle. Table 4-3 presents the payoffs of purchasing or not purchasing liability insurance with or without the subsequent need for insurance.

If the insurance is purchased (we assume there is no deductible amount in the insurance policy), then whether an accident occurs or not, the cost of the decision is the insurance premium, say $100. Table 4-3 presents this as a negative $100 payoff. If the decision is not to purchase the insurance, and if the insurance is not needed, the payoff is $0. However, if there is an accident and we are held liable, then we may suffer a substantial loss; assume that the loss is $10,000. Note that the greatest loss associated with the "purchase" decision is $100, and the greatest loss associated with the "do not purchase" decision is $10,000. Opting for the smaller of these, the minimax decision maker chooses to purchase the insurance. *The minimax decision maker tries to minimize the maximum loss.*

In the state's leasing example, the minimax decision maker tried to maximize the minimum gain. The insurance purchaser sought to minimize the maximum loss. These two objectives are different expressions of the same philosophy. Using that philosophy, one determines the worst payoff for each alternative, and then chooses the alternative that has the best of the worst payoffs. If the payoffs are gains, one seeks to maximize the minimum gain. If the payoffs are losses, one seeks to minimize the maximum loss. In either version, the criterion is referred to as the minimax criterion.

Minimax Regret Decision Criterion

The presence of a second guesser is usually made painfully obvious. Such a presence becomes all the more painful when the second guesser has the power, either political, supervisory, or psychological, to punish an errant decision maker. Such a decision maker may find himself or herself in the position of making a decision based not on the payoff associated with the decision but rather on the regret associated with making the wrong decision. For example, suppose the decision maker in the gas recovery problem decided to accept a fixed-lease contract, and suppose further that the result of the gas recovery operation was a complete success. The actual payoff to the state would be $600 million; had the decision been made to choose the percent-of-profit contract,

Table 4-4 Regret, or opportunity loss, of lease alternatives (millions $)

	Decision alternatives		
State of nature	Percent of profit	Fixed lease	Combination
Complete success	0	1400	800
Moderate success	0	0	60
Failure	540	0	300

then the payoff to the state would have been $2 billion. In this situation the decision maker would regret having made the wrong decision, regret in the sense that the actual payoff was less than the payoff for what turned out to be the right decision. The acceptance of the fixed lease contract followed by a completely successful gas recovery operation would generate a regret of $1400 million. In economic terms, this is commonly referred to as the *opportunity loss*, which can be described as the *difference between the payoff following the right decision and the payoff following the actual decision.*

Table 4-4 presents the regret, or opportunity loss, associated with each alternative for each of the possible states of nature. Choosing the percent of profit alternative may generate a regret of $540 million if the gas recovery venture turns out to be a failure. That is, the payoff of $60 million for percent of profit is $540 million less than the payoff would be for fixed lease with the "failure" state of nature. The fixed-lease contract may generate a regret as big as $1400 million while the combination contract may generate a regret as big as $800 million. The smallest of the three maximum regret values is $540 million. Hence a minimax regret decision maker would choose the percent-of-profit decision alternative. Essentially *the minimax regret decision maker tries to minimize the maximum regret;* if he is to be wrong, he does not want to be "too wrong."

Average Payoff Decision Criterion

A kind of middle-ground approach considers all the payoffs for each alternative. One such approach takes the average of the payoffs for each alternative. In finding and using the average payoff, one tacitly assumes that all states of nature are equally likely. *The average payoff criterion says that the alternative with the best average payoff should be chosen.*[2]

For the leasing problem, the average payoffs as in Table 4-1 Revised, are: $886.67 million, $600 million, and $680 million; the average payoff criterion points to the percent-of-profit alternative. If there is reason to believe that not all states of nature are equally likely, then it is reasonable to make use of a

[2] The average payoff criterion is based on the principle of insufficient reason, which assigns equal likelihood to all possible states of nature.

Table 4-1 Revised Payoff (present value) of lease alternatives (in millions $)

	Decision alternatives		
State of nature	Percent of profit	Fixed lease	Combination
Complete success	2000	600	1200
Moderate success	600	600	540
Failure	60	600	300
Average payoff	886.67	600	680

weighted average, with the more likely states of nature more heavily weighted. Knowing the probability distribution of the states of nature distinguishes "risk" from "uncertainty." The criteria just described are appropriate for decision making under uncertainty. Very frequently probability values for the states of nature can be assigned, if not objectively then at least subjectively. Knowing or estimating probability values allows the use of other decision criteria.

DECISION MAKING UNDER RISK

After the states of nature have been identified, the decision maker tries to determine the likelihood of each. There is usually some basis for believing that one state of nature is more likely to occur than another. For this reason rarely must one rely entirely on the decision criteria for uncertainty; rather one incorporates the likelihood in selecting an alternative. As described in Chapter 2, the probabilities can be objective or subjective.

Objective probabilities are preferable but not always possible. Seldom is there a theoretical or a priori basis for assigning probabilities in a decision situation. Records of past performance are useful in finding empirical objective probabilities. Such records would be useful, for example, in estimating which of two bidding vendors is more likely to deliver contracted supplies by the contract's required date. If neither kind of objective probability is possible, subjective estimates are used. The following procedure can be used to develop a subjective probability distribution:

1. Rank the states of nature in order of their likelihood of occurrence.
2. Assign a probability to the most likely state of nature.
3. Assign a probability to every other state of nature by comparing its likelihood to that of the most likely state.
4. Make sure the probabilities total 1, adjusting them up or down as necessary.

Example The geologists in the gas recovery venture do the following:

1. Rank the states: failure, complete success, moderate success.
2. Believe that a failure is a bit more likely than a success of any kind, and so they assign a probability of .55 to failure.
3. Believe that a complete success is slightly more likely than a moderate success. Realizing that success of either kind has a probability of 1 − .55 = .45, they assign .25 to a complete success and .20 to a moderate success.
4. Because of the way they assigned probabilities in step 3, the probabilities total 1; no adjustments are needed.

There are a few basic decision criteria that use probabilities of the states of nature.

Expected Value Decision Criterion

If there is some basis for believing that one state of nature is more likely than the others, a weighted average of payoffs is preferable to a straight average. The weighted average, in which the probabilities are the weights, is called the expected value of the payoff or simply the *expected payoff.* The expected payoff is the sum of the products of probability times payoff for each of the decision alternatives. *The expected value decision maker chooses the alternative with the greatest expected payoff.* The underlying rationale is that for repetitive decision situations this criterion provides the best long-run average payoff. In nonrepetitive situations the criterion leads to the best overall decision making provided there are many similar decisions. With a strictly one-time occurrence, the expected payoff criterion is a mechanism for incorporating into the decision-making process all payoffs and probabilities for each alternative.

Example The calculation of the expected payoff for each of the oil recovery lease alternatives is presented in Table 4-5. The expected value decision maker would choose the percent-of-profit alternative since its ex-

Table 4-5 Expected payoffs of lease alternatives (millions $)

	Decision alternatives		
State of nature	Percent of profit	Fixed lease	Combination
Complete success	.25 × 2000 = 500	.25 × 600 = 150	.25 × 1200 = 300
Moderate success	.20 × 600 = 120	.20 × 600 = 120	.20 × 540 = 108
Failure	.55 × 60 = 33	.55 × 600 = 330	.55 × 300 = 165
Expected payoff	653.0	600	573

pected payoff, $653 million, is greater than either $600 million or $573 million.

Maximum-Likelihood Decision Criterion

There is another decision philosophy which makes use of the probabilities. *Using the maximum-likelihood criterion, the decision maker chooses the alternative that is best for the most likely state of nature.* While this criterion has the advantage of simplicity, strict adherence to it requires ignoring substantive information related to the less likely states of nature.

Example In the lease illustration failure is more likely to occur than either complete success or moderate success. Consider the payoffs associated just with failure. For the state of nature failure, percent of profit yields $60 million, fixed lease yields $600 million, and combination yields $300 million. The maximum-likelihood decision maker would thus choose the fixed-lease alternative. However, failure is not much more likely than some kind of success:

$$P(\text{failure}) = .55 \quad \text{and} \quad P(\text{success}) = .45$$

One would be ill advised to use this criterion under such circumstances.

Applying the maximum-likelihood criterion is of questionable wisdom whenever the most likely state of nature is just slightly more probable than the others. Its use has better support when one alternative has a rather high probability, say .90.

The last criterion we examine applies the notion of expected value to the concept of regret or opportunity loss.

Expected Opportunity Loss Decision Criterion

A decision maker using this decision criterion will determine the regret or opportunity loss associated with all states of nature for each alternative. The expected value of these opportunity losses is referred to as the *expected opportunity loss*. Since the expected opportunity loss represents the expected difference between the payoff of the right decision and the payoff of the actual decision, *the expected opportunity loss decision maker will choose the alternative that has the smallest expected opportunity loss.*

Example Table 4-4 Revised presents the opportunity losses, together with the expected opportunity loss, for each alternative in the oil recovery lease illustration. Using this criterion, one would choose the percent-of-profit alternative, since its expected opportunity loss is the smallest.

Table 4-4 Revised **Regret or opportunity loss of lease alternatives**

		Decision alternatives (millions $)		
State of nature	Probability	Percent of profit	Fixed lease	Combination
Complete success	.25	0	1400	800
Moderate success	.20	0	0	60
Failure	.55	540	0	300
Expected opportunity loss		297	350	377

A comparison of the expected opportunity losses in Table 4-4 Revised with the expected payoffs in Table 4-5 points out that the same decision is made under those two criteria. This does not come as a great shock when one considers that the opportunity loss is the difference between the payoff associated with the best decision and the payoff associated with the actual decision. Hence, the alternative with the highest expected payoff will be the alternative with the lowest expected opportunity loss, and vice versa. In summary, the expected payoff and expected opportunity loss are not distinct decision criteria but are rather two different ways of looking at the same decision philosophy.

HEALTH AND FACILITIES PLANNING: AN APPLICATION[3]

The planning council for a large portion of southeast Texas was recently faced with the task of setting a limit on the number of hospital beds in the region. About 5500 new beds were planned, but it was not unanimously believed that all these beds were needed. The planning period was 7 years.

In an attempt to identify the appropriate bases for deciding on the bed limit, a decision model was developed. The decision would ultimately be stated in terms of a reduction or increase in the number of planned beds. The uncontrollables were many, but using specific population forecasts and accepted occupancy rates, the states of nature were able to be expressed as the level of "use rate."

Different levels of future use rate were forecast on the basis of the use rates for different numbers of previous years. The projections and the number of years on which they are based are as follows:

[3] This illustration is taken from an article, "Use of Decision Theory in Regional Planning," by Richard Grimes et al., which appeared in *Health Services Research*, Spring 1974, pp. 73–78.

Projected use rate	Number of previous years
Medium-low	1
Low	2
High	4
Medium-high	7

The critical consequences were that constructing too few beds would leave a portion of the sick of the community without adequate care, while constructing too many beds would place an unnecessary financial burden on the community.

The simply stated and widely recognized objectives conflicted: Provide adequate health care for all in need in the community, and keep health care costs to, if not a minimum, at least a bearable level. Attempting to use a stand-in measure to represent both health care and costs would unnecessarily conceal the impact of any decision on the two relevant factors. Although our decision framework has assumed a single objective or at least a single measure, the decision table format for presenting alternatives, states of nature, and consequences can be used to provide a good synopsis of the various relationships even in a multiple-objective decision situation. Rather than use an artificial factor to convert too few beds into dollars, that consequence was left in terms of beds. Excess beds could be realistically converted into dollars with the following rationale. There was a projection for the cost of occupied beds, and an accepted estimate for the relationship between the costs of occupied and unoccupied beds. Hence, the total operating cost for each excess bed for the 7-year planning period was estimated to be about $202,000.

The decision table for the problem is presented in Table 4-6. If the council

Table 4-6 Decision table for southeast Texas hospital planning*

	Decision alternatives			
State of nature	Major reduction in beds (I)	Slight reduction in beds (II)	Slight increase in beds (III)	Major increase in beds (IV)
Low use rate	OK	373 eb $75.3M	1095 eb $221.2M	1356 eb $273.9M
Medium-low use rate	373 bbn	OK	722 eb $145.8M	983 eb $198.6M
Medium-high use rate	1095 bbn	722 bbn	OK	261 eb $52.7M
High use rate	1356 bbn	983 bbn	261 bbn	OK

* eb = excess beds; bbn = beds below need; M = million.

chose alternative II, then the potential cost of overage would not exceed $75.3 million, but there could be as many as 983 beds too few. If the council decided to minimize the bed shortage, then they would choose alternative IV which could cost the region as much as $273,900,000 in excess beds over the 7-year planning period.

The council wanted to have some way of incorporating all the possible consequences of each alternative. Since there was no past experience that suggested any one use rate was more likely than any other, the four states of nature were assumed equally likely for the sake of making an aggregate comparison. Beds and dollars cannot be added, of course, and so the expected payoff contains both beds and dollars. The decision table with the average payoffs is presented in Table 4-7.

Although the assumption of equal probabilities left some of the council members a little uneasy, they saw the new payoff table as useful in comparing various alternatives. One comparison showed that reducing the expected bed shortage from 706 beds to 426 beds would bring with it an expected cost of $18,800,000.

As the discussion about the likelihood of projected use rates continued, there emerged a sense—albeit intuitive—that projections based on longer periods were more likely to hold over the relatively long 7-year planning period. Hence the council agreed to consider new probabilities based on length of trend to see if the bed shortage–cost relationship would be substantially altered. Each projection was given a rating equal to the length of the trend

Table 4-7 Decision table with average payoffs for southeast Texas hospital planning*

		Decision alternatives			
State of nature (use rate)	Probability	Major reduction in beds (I)	Slight reduction in beds (II)	Slight increase in beds (III)	Major increase in beds (IV)
Low	.25	OK	373 eb $75.3M	1095 eb $221.2M	1356 eb $273.9M
Medium-low	.25	373 bbn	OK	722 eb $145.8M	983 eb $198.6M
Medium-high	.25	1095 bbn	722 bbn	OK	261 eb $52.7M
High	.25	1356 bbn	983 bbn	261 bbn	OK
Average payoffs		706 bbn	426 bbn and $18.8M	65 bbn and $91.8M	$131.3M

* eb = excess beds; bbn = beds below needs; M = million.

Table 4-8 Finding subjective probabilities for projected use rates

Projected use rate	Number of previous years	Rating	Probability
Medium-low	1	1	$\frac{1}{14}$
Low	2	2	$\frac{2}{14}$
High	4	4	$\frac{4}{14}$
Medium-high	7	7	$\frac{7}{14}$
		14	

period. The sum of the ratings was 14. Dividing each rating by 14 yielded a probability for each projection. This process is summarized in Table 4-8.

Using these probabilities, a new set of expected payoffs resulted. These expected payoffs are presented in Table 4-9. Once more the council found the table useful for comparisons. Even a relatively small reduction in expected bed shortage would be accompanied by a relatively high dollar cost.

After due deliberation the council chose decision alternative I, not only for the possibly high dollar cost of other alternatives but also because a decision to curtail construction could be altered later if the need arose, whereas a decision to allow further construction could be altered only with great difficulty.

In this instance, the decision table certainly did not dictate the decision nor was any one of the criteria unambiguously preferable. However, decision theory did provide a framework through which the decision makers could identify the relevant factors and grasp the relationships among them. As a result of the decision and the recognized components of the decision situation, bed use rates are being monitored to assure that a greater bed use rate will in fact be responded to in an appropriate manner.

Table 4-9 Decision table with expected payoffs for southeast Texas hospital planning*

		Decision alternatives			
State of nature (use rate)	Probability	Major reduction in beds (I)	Slight reduction in beds (II)	Slight increase in beds (III)	Major increase in beds (IV)
Low	$\frac{2}{14}$	OK	373 eb $75.3M	1095 eb $221.2M	1356 eb $273.9M
Medium-low	$\frac{1}{14}$	373 bbn	OK	722 eb $145.8M	983 eb $198.6M
Medium-high	$\frac{7}{14}$	1095 bbn	722 bbn	OK	261 eb $52.7M
High	$\frac{4}{14}$	1356 bbn	983 bbn	261 bbn	OK
Expected payoffs		962 bbn	642 bbn and $10.8M	75 bbn and $42.0M	$79.7M

* eb = excess beds; bbn = beds below need; M = million.

DECISION ALTERNATIVES WITH DIFFERENT PROBABILITY DISTRIBUTIONS: AN ILLUSTRATION

The state's Bureau of Management Services is considering entering into a contract with one of two firms that will supply technical assistance in a number of specialized areas. The need for assistance is not continuous within any one specialty; so it is not worthwhile for the state to maintain employee positions to satisfy each of the specialized needs.

The firms' bids indicate that the direct cost to the bureau is the same for either firm. The efficiency and completeness vary between the two firms, however. Any technical assistance must ultimately be transmitted to less technical decision makers. The bureau must prepare its own reports and develop recommendations based on the work that the firm provides. Hence, less efficient and complete assistance means greater in-house costs to the bureau. Both of the bidding firms have had previous contracts with the state, and so there is some empirical basis for estimating how close their actual work will be to the contract indications.

After some searching, the bureau learns that Flash, Inc. has had some extremely capable specialists as well as some trial-and-error devotees on its staff. The bureau estimates that there is a 60 percent chance that Flash, Inc. will do rather complete work, and the bureau will incur in-house costs of only $20,000 to support the specialized activities. If Flash, Inc. does unsatisfactory work, which has a 40 percent chance, then the bureau will spend $60,000 in support. On the other hand, Penser, Ltd. is much more consistent; it is not capable of either extreme. The bureau estimates that there is a 70 percent chance that it will have to spend $35,000 and a 30 percent chance that it will have to spend $50,000 in support of Penser, Ltd.'s technical assistance. Which firm should be awarded the contract?

The information can be summarized in a decision table, which is presented in Table 4-10. This problem statement and the payoff table differ from earlier ones in that the states of nature, satisfactory and unsatisfactory, have one set of probability values for the decision alternative Flash, Inc., and another set for Penser, Ltd.

If the decision maker, the bureau, is an expected value decision maker, then Flash gets the contract. If the decision is made with the more conservative minimax criterion, then Penser gets the contract. The maximax criterion indicates choosing Flash. The state of nature "satisfactory" is more likely for either alternative, but using the maximum likelihood criterion is questionable since the probabilities are not the same.

Table 4-10 Decision table of in-house costs for choosing a technical assistance firm

	Decision alternatives			
State of nature	Flash	Probability	Penser	Probability
Satisfactory	$20K*	.60	$35K	.70
Unsatisfactory	$60K	.40	$50K	.30
Expected cost	$36K		$39.5K	

* K = thousand.

When the state of nature is dependent on the chosen alternative, as it is here, the minimax regret and expected opportunity loss criteria are not appropriate. If Penser is chosen and does a satisfactory job, there is no regret that Flash was not chosen because there is no guarantee that Flash would also have done a satisfactory job. Hence even the idea of an opportunity loss is not applicable when the state of nature depends on the alternative.

SUMMARY

Decision theory provides a decision-making rationale when the decision situation is not fully known. The basic elements in a decision theory approach are: identify decision alternatives; identify uncontrollable factors or states of nature; identify the consequences of each alternative for each state of nature.

The decision situation is termed uncertain when the relative likelihoods of the states of nature are not known. Applicable criteria include maximax, minimax, average payoff, and minimax regret. The situation is termed risky when the relative likelihoods are known. Applicable criteria include expected payoff, expected opportunity loss, and maximum likelihood.

We have examined these criteria through simple examples in which the payoffs are not only quantified but are also well represented in simple dollar terms. Not all decision consequences in the public sector are as well represented by dollars, nor are decision criteria always so specifically stated. Often payoffs are better represented, for example, by improved health services or greater recreational opportunity or better educational enrichment. The purpose in developing a decision matrix there is as much to be sure that one has identified the alternatives, states of nature, and consequences, as it is to quantify the consequences. A health planning application typified this purpose.

A final illustration showed which criteria are applicable when the states of nature have different probability distributions for each alternative, and are dependent on the alternative that is chosen.

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EXERCISES

Extensions of chapter examples

4-1 Reconsider the drilling decision situation presented in Table 4-5. A reassessment of a complete success indicates that the payoffs have changed and are as follows:

	Percent of profit	Fixed lease	Combination
Complete success	1700	600	1400

(*a*) Using the expected payoff criterion, what is the preferred alternative?

(*b*) What does this suggest about the sensitivity of the decision to estimates of future revenues?

4-2 Arriving at the probability of each outcome in the drilling problem was in part a subjective process. Although we have numeric estimates, we should be attuned to their subjective origin. The reasoning that generated the probability values .25, .20, and .55 might just as validly have generated the values .21, .20, and .59, which are not all that different. Using this new set of probability estimates and the payoffs of Table 4-1, determine the alternative preferred by the expected value criterion. What does this suggest about the sensitivity of the decision to the estimated probability values?

4-3 Assume you are an expected value decision maker. Noting the results of exercises 4-1 and 4-2, what decision alternative would you choose?

4-4 In the southeast Texas hospital planning problem, the basis for the dollar value of excess beds is the $202,000 operating cost for each excess bed. Suppose the true cost is only $20,200 for each excess bed, or just 10 percent of the original estimate. Reconstruct the decision table and the expected payoffs, using the probabilities of Table 4-9. How might these changes have affected the council's choice of decision alternative 1?

4-5 In problem 4-4, suppose the true cost is $181,800 for each excess bed, or just 10 percent less than the original estimate. How might these changes have affected the council's choice of decision alternative I?

4-6 Assume your answer to problem 4-4 suggests that the council would have chosen an alternative other than alternative I, and your answer to problem 4-5 suggests they would have stayed with alternative I. Determine the level of "operating cost for each excess bed" at which they would change their decision. Discuss.

4-7 Refer to the last section of Chapter 4 and the illustration on choosing a technical assistance firm. It is sometimes claimed that in-house costs to support contractual services are sunk costs. They are costs already incurred and will not change whether the result of the contract is satisfactory or unsatisfactory. What may change is the delay in delivering required studies and the level of complaint and harassment. Reconstruct Table 4-10 with the following changes: The consequences of satisfactory work are no delays for Flash, Inc. and a 1 week delay for Penser, Ltd. The consequences for unsatisfactory work are a 4-week delay for Flash, Inc. and a 2-week delay for Penser, Ltd.

(*a*) What is the numerical expected payoff or consequence?

(*b*) What would the decision be according to the expected value, maximax, and minimax principles?

(*c*) What would your decision be?

Other applications

4-8 The following letter to the Editor appeared in *The New York Times* of February 28, 1971. The writer directed the theoretical division of the Los Alamos Scientific Laboratory from April 1943 to January 1946. Hans Bethe was the 1967 Nobel laureate in physics.

> Yalta: Lack of Communication on Bomb
> To the Editor:
>
> Under the title "The Truth about Yalta," C. L. Sulzberger [column of February 14] discussed the assessment by Ambassador Charles Bohlen of the chief problems that faced President Roosevelt and the U.S. delegation at the time of the Yalta Conference, February 1945. The third point of this assessment reads in part as follows:
>
> "While Roosevelt and a handful of advisers knew about the Manhattan Project, no one could be certain the atomic bomb would in fact explode or how effective a weapon it would be."
>
> This problem looked different as seen from the Los Alamos Scientific Laboratory charged with the development of the bomb. By February 1945, it appeared to me and to other fully informed scientists that there was a better than 90 percent probability that the atomic bomb would in fact explode, that it would be an extremely effective weapon, and that there would be enough material to build several bombs in the course of a few months.
>
> Thus even if the first bomb should have failed, the project was bound to succeed in a relatively short time. Few things in war and even fewer in politics have as good a chance as 90 percent.
>
> That the full flavor of this conviction of the scientists was not transmitted to the decision-makers was a failure of communication—excessive secrecy and the absence of direct channels between scientists and high Government officials were responsible. Because of this failure of direct communication, the U.S. at Yalta urged Russia to participate in the assault on Japan, with grave consequences for the future of the political situation in the Far East. Suppose there had been good communication. Should the U.S. Government have acted on a 90 percent probability of technical success? In my opinion, definitely yes.
>
> Again, in 1958 we had a chance to arrange a ban on the testing of nuclear weapons at a time when the U.S. had a clear advantage over the Soviet Union in weapon design. However, we were afraid of the possibility of clandestine underground Russian tests of small nuclear weapons and insisted therefore on ironclad safeguards. These were unacceptable to the U.S.S.R., and no agreement was reached by 1961.
>
> In 1961, the Russians conducted a series of nuclear weapon tests in which they managed to equal, in most of the important aspects, the performance of U.S. thermonuclear weapons. Thus here again, by insisting on certainty, the U.S. lost a clear advantage.
>
> This letter is not meant to imply that our foreign policy should center on advantage over the U.S.S.R. I merely wish to argue that if and when the seeking of such an advantage is part of our policy, we should act on high technical probability rather than requiring certainty and should have easy communication between the knowledgeable persons and the decision-makers.
>
> Hans Bethe
> Ithaca, New York, February 16, 1971

Comment on the letter,[4] placing emphasis on the decision philosophies

(*a*) Apparently used at top levels of government

(*b*) Suggested by Bethe

(*c*) Recommended by you.

Support your recommendation.

4-9 The highway police patrol the interstate highways in order to prevent accidents and to service them when they do occur. Stopping speeders may be seen as a means to the end. Two types of patrol are available: (1) standard patrol, in which cars combine roadside standing to identify speeders and highway driving to catch them, and (2) cruising patrol, in which cars drive almost constantly at the posted speed limit to set a traffic pace. The operating cost per shift is $600 for the standard patrol and $800 for the cruising patrol. It is estimated that the mean cost to the patrol of servicing an accident is $100. In response to the right leading questions, the troop commander estimates that with the standard patrol, the chances of averaging 0 or 2 accidents per shift are about the same, and either is twice as likely as averaging 6 accidents. With the cruising patrol, averaging 0 accidents is four times as likely as 2 accidents, and averaging 6 accidents just will not happen. Assume no other average is possible.

(*a*) Develop a dollar-cost table for the two decision alternatives and the three states of nature.

(*b*) Determine the probabilities of the states of nature based on the given descriptive likelihoods. (Note that they are different for each decision alternative. If a state of nature for the first alternative has 0 probability for the second alternative, it is not considered a state of nature for the second alternative.) For each of the following criteria determine the better alternative:

1. Minimax.
2. Maximax.
3. Expected value.
4. Minimax regret. Comment on the appropriateness of this criterion.

4-10 It is suggested that the revenue that patrols generate in issuing citations to speeders should be taken into consideration. Reassess problem 4-9 if standard patrols generate 8 citations per shift (averaging $25 each) and cruising patrols generate only 2 per shift, because their cruising keeps traffic at a proper speed.

4-11 No matter how many relevant facts are considered, there are usually some that have been neglected. It is further noted that the cost of issuing and adjudicating a citation is about $50. Now reconsider problem 4-10. What other information should be considered in choosing the type of patrol? Should the dollar costs be considered at all?

4-12 Cy Linder is responsible for the medical supply room of a large medical center. Overall he is responsible for having supplies on hand as they are needed; however, he is also accountable for controlling expenses. Medicines acquired by him and subsequently requested for a patient are charged to that patient, and hence do not affect his budget. Supplies lost, stolen, or discarded do affect it. Cy has observed that a widely used but perishable medication has been overstocked for the past few weeks. Records reveal that the demand for the medication over the past 4 years (200 weeks) is as follows:

Vials of medicine used per week	Number of weeks
130	20
140	80
150	70
160	20
170	10
Total number of weeks	200

The medication costs $5 a vial and is ordered and delivered weekly. The shelf life of the medicine is 1 week. (Supplies on hand at the end of the week must be discarded.)

(*a*) Set up a loss table for the possible states of nature and decision alternatives.

(*b*) Keeping in mind Cy's overall responsibilities, what alternative would you recommend?

(*c*) What is his expected dollar loss for that alternative?

4-13 In problem 4-12, if Cy were able to arrange two deliveries per week, would his expected dollar loss be approximately halved? Discuss the overall effects of such an arrangement.

4-14 The U.S. Army Corps of Engineers has proposed to the Congress that the cost of maintaining the inland waterway system should be in part financed by the users of the system. They have also presented four different alternatives for assessing this charge: a tax on the fuel used by the tugs, a lockage fee for each time a barge passes through a lock, a toll on the barges for each ton per mile, and a license fee for each barge. While none of these alternatives will return the complete cost of maintaining the system, they were set at a level that would not significantly reduce the traffic. The corps also predicts that there is a 30 percent chance of a major increase of traffic, a 30 percent chance of a moderate increase, a 30 percent chance of a slight decrease, and a 10 percent chance of a major decrease. The consequences are displayed in the following table.

Decision table for inland waterway revenue*

		Decision alternatives			
State of nature	p	Fuel tax	Lockage fee	License fee	Toll
Major increase	.3	$3.7 B	$3.0 B	$2.7 B	$4.1 B
Slight increase	.3	3.0	2.5	2.3	3.3
Slight decrease	.3	2.4	2.1	2.1	2.4
Major decrease	.1	1.9	1.9	2.0	1.7

* B = billion.

(*a*) Which alternative is preferred according to each of the applicable criteria?

(*b*) Which alternative do you recommend? Why?

(*c*) Inasmuch as the probability for increases and decreases are estimates, what kind of sensitivity analysis would you recommend?

Project problems

4-15 Recent years have seen some wide-scale public efforts to prevent the occurrence of some epidemic that was considered a real possibility. An epidemic, obviously, is a widespread disaster. A public program taken to forestall or prevent it is necessarily a widespread program. A vaccination

program carries with it not only the purported benefit of preventing the illness but also the recognized risk of a severe reaction in certain types of individuals.

Suppose the National Center for Disease Control indicates that there is a possibility of a widespread influenza epidemic within the next few months. Specifically, suppose there is a .02 probability that the epidemic will reach 20 million persons, a .08 probability that it will reach 2 million persons, and a 90 percent chance that there will be only a trace, that is, that the disease will reach 1,000 or fewer persons. Subjective estimates indicate that the influenza will be fatal to 1 in 1,000 persons afflicted. Hence, the death toll due to the influenza could go as high as 20,000 people without any prevention program.

In the face of such a dismal forecast, the federal government is faced with a decision concerning whether to fund a public vaccination program. It has three alternatives: to have no program; to have a select program, under which 20 million high-risk persons could be vaccinated; and to have a widespread vaccination program making the vaccine available to all who desire it. Estimates in this case are that 100 million persons would avail themselves of the vaccination. The cost is estimated to be $5 per vaccination. It is expected that there will be rather widespread reaction to the vaccine. It is believed that this reaction to the vaccine will occur only in those who have not been exposed to the influenza, so that the more widespread the influenza is, the fewer reactions there will be. The reaction is considered to be fatal to one of every 100,000 vaccinated people who are not also exposed.

If a selective vaccination program is undertaken, those considered most likely to get the influenza illness will be among those vaccinated. Since the influenza is believed to affect heart muscles, persons with chronic heart disease will be given the opportunity to be vaccinated.

All of the various estimates, although in part subjective, can be summarized in the following table. The entries for "epidemic = 1,000" and "no program" are derived from the above information. The remaining entries are estimated but are not directly calculable from the given information alone.

Decision table for influenza vaccination program; Number of fatalities due to influenza and vaccination reaction

		Type of program, Number vaccinated, Cost		
Extent of epidemic	Probability	Full program, 100 million, $500 million	Select program, 20 million, $100 million	No program, 0, $0
20 million	.02	1,000 + 200	10,000 + 50	20,000 + 0
2 million	.08	100 + 600	1,000 + 150	2,000 + 0
1,000	.90	0 + 1,000	0 + 200	1 + 0

(*a*) For purposes of comparison, find the alternative that would be chosen according to each of the decision criteria applied to estimated fatalities: maximax, minimax, minimax regret, expected value decision rule, maximum likelihood.

(*b*) What would your decision be and why? Did you consider the dollar cost in your decision?

4-16 Refer to problem 4-15. There are many subjective estimates in this decision situation and, hence, sensitivity analysis of the estimates is called for.

(*a*) If the probability estimates were corrected to be .01, .09, and .90 respectively, how would the expected value change for each alternative?

(*b*) Which alternative do you recommend?

(*c*) Did you make use of the cost in choosing your alternative?

(*d*) Would a decrease in cost to $1 per vaccination or an increase to $100 per vaccination have any effect on your decision choice? Is there any unit cost per vaccination at which you would include costs as a consideration?

(*e*) If you were a decision maker, or someone charged with making recommendations to the decision makers, what other information would you like to have before making a decision or recommendation? Would it take a long time to get this information? What effect might waiting have on decision making?

4-17 Refer to problem 4-15. Suppose that a consequence of the influenza and the vaccination reaction is not death, but rather critical illness. Since it is not completely known how the influenza affects the heart muscles, the severity of the illness is uncertain. The uncertainty includes whether the effect is temporary or permanent, whether the effect is on the heart muscles in general or on a particular muscle, and how the influenza in combination with some other preexisting condition will affect the individual. Presumably, some persons with particular respiratory or circulatory difficulties would succumb. Under these new conditions, which type of program would you recommend? (Assume no numeric changes from problem 4-15.)

4-18 Think of a decision situation in which you have been involved.

(*a*) Was there an identifiable objective?

(*b*) Was there a workable criterion?

(*c*) Were alternatives explicitly sought and found? List them.

(*d*) Were uncontrollable but related events (states of nature) uncovered? What were they?

(*e*) Were the consequences of the alternatives investigated? Summarize them.

(*f*) Can you summarize the decision situation in a decision table?

(*g*) Is there an appropriate probability distribution for the states of nature (the uncontrollables)?

(*h*) Which alternative would you pick based on the summary?

(*i*) Which alternative was chosen?

(*j*) If there is a discrepancy between the answers to parts *h* and *i*, is it due to a problem feature that cannot be adequately represented in the decision table?

(*k*) Discuss the role of such a summary in decision making.

CHAPTER

FIVE

DECISIONS AND REVISED PROBABILITIES

The search for and evaluation of decision alternatives often reveals new information. When new information concerns the identification of alternatives, there are revisions to and expansions of the list of choices. When it concerns the effects, consequences are restated. When the uncontrollable factors are involved, either the states of nature themselves are reconsidered or their likelihoods are revised. Here we will explore the fundamentals of how to revise probabilities on the basis of new information. The value of the new information is seen in its impact on the expected payoff. The expected value and the cost of the new information are compared to determine whether it is worth acquiring.

The ideas and techniques of probability revision are presented through a previously encountered (problem 4-15) government vaccination decision situation. The decision to fund and implement a widespread vaccination program has severe and long-lasting consequences. The sheer magnitude of the consequences is sufficient to suggest that efforts be made to acquire new information on the situation and use that new information, not to replace prior information and probabilities but to update them. By incorporating new information, probability estimates can be revised. This chapter focuses on the role of new information in revising rather than replacing previously estimated probabilities.

NEW INFORMATION AND REVISED PROBABILITIES

Problem 4-15 is used for illustrative purposes. The essence of the problem follows. There is a possibility of an imminent influenza outbreak. As a preventative step federal officials can choose one of three alternatives: