

Homework 5

For the following exercises, (a) identify the distribution, including the values of all parameters, and (b) find the probability.

Reminder (this should be in your sheet of notes for the quiz): the identification and parameters for part (a) should look like one of these:

Binomial($n = \underline{\hspace{1cm}}$, $p = \underline{\hspace{1cm}}$)

Poisson($\mu = \underline{\hspace{1cm}}$)

Hypergeometric($N = \underline{\hspace{1cm}}$, $r = \underline{\hspace{1cm}}$, $n = \underline{\hspace{1cm}}$)

1. A 2011 poll found that 35% of U.S. adults do not work at all while on summer vacation. In a random sample of 10 U.S. adults, find the probability that 2 or fewer do not work during summer vacation, assuming independence.

(a) This is not Poisson - I am not told about an average number of occurrences over a unit of time or space. This is a Binomial($n = 10$, $p = 0.35$). I would need the population size in order to treat this as a Hypergeometric, which was not given, and since the trials are independent it can't be Hypergeometric.

b) $P(X \leq 2) = \text{binomcdf}(10, 0.35, 2) = 0.2616$

2. A website has an average of 6 hits per minute. In a 2 minute period, what is the probability that it gets exactly 11 hits?

(a) I am told an average number of occurrences over a unit of time, so this is Poisson. My given average is 6 hits per minute, but I want to find probabilities in a 2 minute period, so my new mean is 12 hits per 2 minutes. This is Poisson($\mu = 12$).

(b) $P(X = 11) = \text{poissonpdf}(12, 11) = 0.1144$

3. The Federal Deposit Insurance Corporation (FDIC) normally insures deposits of up to \$100,000 in banks that are members of the Federal Reserve System against losses due to bank failure or theft. Over the ten years from 2000 - 2010, the average number of bank failures per year was 45. Assuming that this yearly trend of an average of 45 failures per year continues, what is the probability that less than 60 banks will fail in the next year?

(a) I am told an average number of occurrences per unit of time, so this is a Poisson. I am given the average per year, and I want to find the probability over a year, so I do not have to adjust the mean. This is Poisson($\mu = 45$).

(b) $P(X < 60) = P(X \leq 59) = \text{poissoncdf}(45, 59) = 0.9813$

4. The probability that you will win a certain game is 0.3, independent of past wins and losses. If you play the game 20 times, what is the probability that you will win 3 or fewer times?

(a) The probability of winning remains the same in every game, and the trials are independent. This is

a Binomial($n = 20, p = 0.3$).

(b) $P(X \leq 3) = \text{binomcdf}(20, 0.3, 3) = 0.1071$

5. A 2010 American Community Survey estimates that 47.1% of women ages 15 and over are married. We randomly select five women between these ages. What is the probability that the fifth woman selected is the only one that is married?

(a) There is no average over space or time, so this isn't a Poisson. I am not finding the probability that one of the five is not married – I am finding the probability that the first four are not married and the fifth one is married. In the binomial, the order in which the “successes” happen doesn't matter. This is a geometric experiment with $p = 0.471$. Define success to be selecting a married woman, and find the probability of getting the first success on the 5th trial.

(b) $P(X = 5) = \text{geompdf}(0.471, 5) = 0.03688$

6. Many primary care doctors feel overworked and burdened by potential lawsuits. In fact, the Physicians' Foundation reported that 60% of general practice physicians in the United States do not recommend medicine as a career (Reuters, Nov 18, 2008). In a random sample of 5 general practice physicians, find the probability that at least one does not recommend medicine as a career, assuming independence.

(a) Here we have a sample of 5 from a much larger population, and we are assuming independence, so this is a Binomial($n = 5, p = 0.60$). We are counting the number of doctors that don't recommend medicine as a career, so our p needs to be the probability that a doctor doesn't recommend medicine as a career, which is $60\% = 0.6$.

(b) $P(X \text{ is at least one}) = 1 - P(X = 0) = 1 - \text{binompdf}(5, 0.6, 0) = 1 - 0.0124 = 0.9898$

7. A husband and wife both have brown eyes but carry genes that make it possible for their children to have brown eyes (probability 0.75), blue eyes (probability 0.125), or green eyes (probability 0.125). What is the probability that their third child is the first one with green eyes?

(a) There is no average over space or time, so this isn't a Poisson. I am finding the probability for the first occurrence of an event, so this is geometric. Note that the probability of getting a green eyed child has the same probability for each child, so the parameter is $p = 0.125$

(b) $P(X = 3) = \text{geomet}(0.125, 3) = 0.0957$

8. An industrial fabric production machine makes an average of 0.7 defects per square yard. What is the probability of finding exactly 6 defects in the next 5 square yards of fabric?

(a) Here we are told an average over a unit of space (here it is area), so this is Poisson. We are told the average per square yard, but we want the probability per 5 square yards. If we get 0.7 defects per square yard, then we get $0.7 * 5 = 3.5$ defects per 5 square yards. This is Poisson($\mu = 3.5$).

(b) $P(X = 6) = \text{poissonpdf}(3.5, 6) = 0.0771$

9. California highway patrol reports show that an estimated 7.8% of drivers exceed the speed limit of

70 mph on I-5. Suppose a highway patrol officer is hidden on the side of the freeway. What is the probability that the first speeder she observes is within the first 5 cars that pass her?

(a) Here we are finding probabilities for the first occurrence of an event, where individual trials are assumed to be constant and independent. This is a geometric experiment with $p = 0.078$.

(b) $P(\text{first speeder is within the first 5}) = P(X \leq 5) = \text{geometcdf}(0.078, 5) = 0.3337$