

RC Circuits and Bioelectrical Impedance Analysis

Objectives:

- Explain how resistors and capacitors behave in AC circuits.
- Relate resistance, reactance, impedance, and phase angle to each other.
- Explain why the body can be modeled as an RC circuit.
- Explain how the body's fat free mass (FFM) can be measured using BIA.
- Explain the multi-frequency electrical characteristics of the body.

Equipment:

- Assorted resistors and capacitors
- Banana cables and alligator clips
- LabQuest device connected to a computer running LoggerPro
- Force Plate
- Two differential voltage probes
- BIA device
- Hand held electrodes
- Frequency Counter

Part One: Resistors and Capacitors in AC Circuits

Ohm's law for resistors in DC circuits relates the potential across a resistor to the current through it as

$$R = \frac{V}{I}. \quad (1)$$

In AC circuits with resistors and capacitors the current is not in phase with the voltage across the capacitor, nor is it in phase with the input voltage. The concept of resistance used in DC circuits needs to be generalized to be applicable for AC circuits. In an AC circuit with a resistor and a capacitor, the resistance R needs to be replaced by the impedance Z

$$Z = \frac{V}{I}, \quad (2)$$

where V and I are either the root mean squares or the peak values of the oscillating voltage and current. The impedance for a circuit with a resistor and a capacitor is given by

$$Z = \sqrt{R^2 + X_c^2}, \quad (3)$$

where R is resistance and X_c is the reactance of a capacitor C . While R is independent of the frequency, X_c varies with the frequency (f) according to

$$X_c = \frac{1}{2\pi f C}. \quad (4)$$

- **Q1: Based on Eqs. (3-4), what will happen to the reactance of a capacitor as the frequency increases? What will happen to the total impedance of a circuit with a resistor and capacitor in series (RC circuit) as the frequency increases? Sketch the graph of reactance and resistance versus frequency.**

Phase Shift and Calculating Resistance and Reactance

The relationship between resistance, reactance, and total impedance can be graphically represented with a phasor diagram. In one type of phasor diagram, the resistances of the circuit are plotted on the x-axis and the reactances on the y-axis (Fig. 1). The angle that the total impedance makes with the resistance is called the phase shift or phase angle ϕ . The phase angle is given by

$$\tan \phi = \frac{X_C}{R} \quad (5)$$

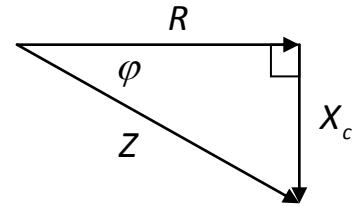


Figure 1 – Graphical representation of impedance, resistance and reactance.

Phasors can also be plotted on an x-y coordinate system. Using this method, each vector has its tail on the origin. These vectors rotate counter-clockwise around the axis so that the y-component of a given vector represents the instantaneous value. For an RC series circuit, the voltage across the capacitor is 90° behind or rotated 90° clockwise from, the current vector (current leads voltage).

Equations (3) and (5) can be solved for X_C and R , resulting in (try to verify this)

$$R = \sqrt{\frac{Z^2}{1+\tan^2\phi}} \quad X_C = R \tan \phi. \quad (6)$$

Therefore, if we measure Z and ϕ , we can calculate X_C and R .

- **Q2: Based on Eq. (5) and your answer to Q1, what will happen to the phase angle as the frequency increases? When would the phase angle be 0° ? When would it be 90° ?**

Measuring Total Impedance

When the components of an RC circuit are all in series, the current passing through each component is the same for both circuit elements. The current can be calculated as:

$$I = \frac{V_C}{X_C} = \frac{V_R}{R} = \frac{V_Z}{Z}$$

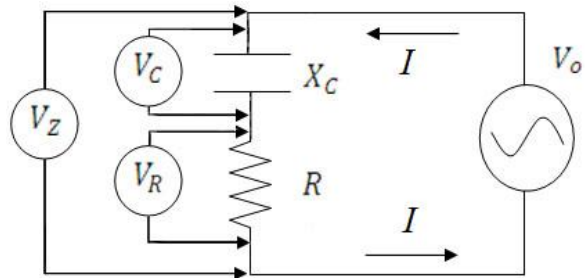


Figure 2 – Circuit to determine resistance and reactance of an unknown RC circuit.

- **Q3: Apply the following values to the circuit in Fig. 2: $X_C = 300 \Omega$, $R = 150 \Omega$, and $V_0 = 1 \text{ V}$. Use these values and the equations above to find the total impedance Z , the current I , and the voltages across each component. Draw a phasor diagram showing the voltages in the circuit V_C , V_R and V_Z , and the current I so that the tail of each vector is at the origin of an x-y axis (vector magnitudes should be correct relative to each other but an exact scale is not necessary). Identify the phase angle ϕ .**
- **Q4: Using your phasor diagram, what is the equation for V_Z in terms of V_R and V_C ?**

Part Two: Impedance and Phase Shift of an RC Circuit

In this section you will use two methods to experimentally determine the impedance and phase of an RC circuit. Both methods involve comparing the potential across the RC circuit to the potential across a known reference resistor.

Experimental Procedure

LabQuest and LoggerPro Setup

- 1) Connect a differential voltage probe to CH1 and CH2 on the LabQuest, and then open LoggerPro.
- 2) To zero the voltage probes connect the positive and negative terminals to each other. In LoggerPro under the Experiment menu select *Zero*. Make sure that both CH1 and CH2 are selected and click *OK*.
- 3) To set the collection rate and duration click on *Data Collection* under the Experiment menu. Change *length* to 0.04 seconds and *samples/second* to 10,000. The box *Repeat* should be unchecked; *Continuous Data Collection* should be unchecked; triggering should be disabled. Click *Done*.

Frequency Counter

- 1) Turn on the frequency counter and press the *Function* button until the display reads three, then press *Confirm*. Connect the red terminal of the frequency counter to V+ on the BIA device and the black terminal to GND. Do not make any other connections.
- 2) Switch the frequency range on the BIA device to low and the frequency selector knob to the lowest setting. Turn on the BIA device.
- 3) Record the value of the frequency at this setting in the Excel sheet (e.g., “Low 1” for “Knob #”). The value of the frequency should be around 20-30 Hz. Repeat this procedure with the frequency range switched to high and the selector knob at the lowest setting; the frequency should be around 200-300 Hz.
- 4) Continue this procedure to fill in the columns for “Knob #” and “f(Hz)” in the Excel sheet. The remaining values for frequency should begin around 10,000 Hz and may go up to 400,000 Hz, but need not be at sequential settings on the BIA device. However, in Part IV you will need to take a measurement as close to 50,000 Hz as possible, so include this setting as one of the frequencies in the Excel sheet for Part II.
- 5) Disconnect and turn off the frequency counter when finished.

An RC Series Circuit and Known Resistor – External Voltage Probes

- 1) Set up an RC series circuit and connect it to the BIA device as shown in Fig. 3(a). Use components with the following values: $R = 470 \Omega$, $C = 1 \mu\text{F}$. Use extra banana cables to connect I+ to V+ and I- to V-. Note that R_{ref} is a reference resistor internal to the BIA device.
- 2) Now connect the CH1 probe to V+ and V- and the CH2 probe to I- and GND. Make sure the polarity of the voltage probes matches that of the BIA device.
- 3) Make sure the BIA device frequency range is set to low, the frequency selector knob is at the lowest setting, and turn the device on and.

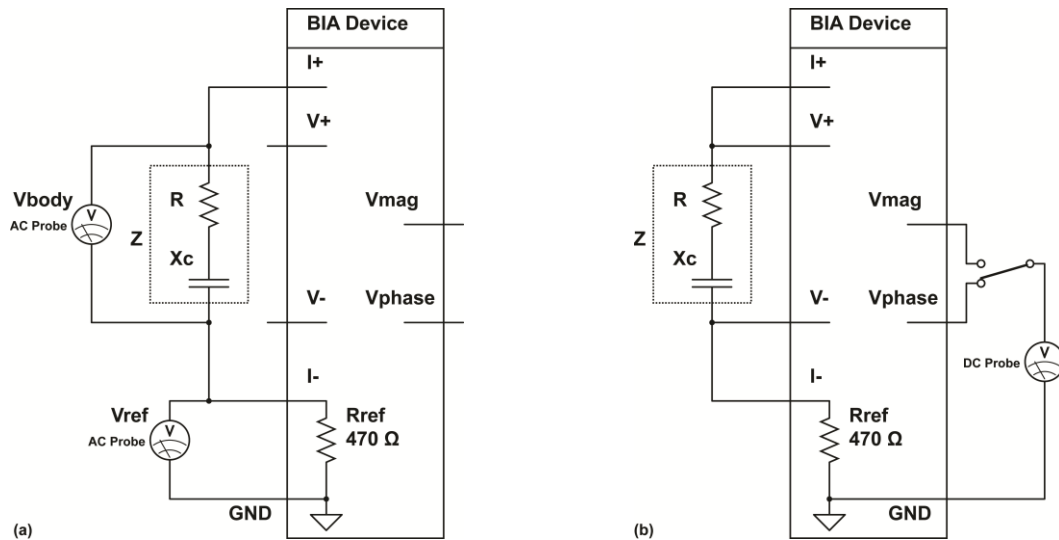


Figure 3 – Experimental setup to measure the impedance of an RC series circuit (boxed region) using (a) two external differential voltage probes and (b) the BIA device.

- 4) Click the green collect button on LoggerPro. You should now be able to see two sine waves, which represent the potentials across the external RC circuit and the internal reference resistor (try zooming in on the y-axis if necessary).
 - a. Under the Analyze menu click on *Curve Fit*.
 - b. Select a sine wave fit for the CH1 data, and then click *Try Fit* and *OK*.
 - c. Create a sine wave fit for the CH2 data as well.
- 5) The amplitude of the voltage signal is the coefficient A from the fit; the phase is the coefficient C.
 - a. Record the amplitudes and phases of the two waveforms in Table 2.1 on the Excel sheet provided.
- 6) The Excel sheet calculates impedance Z and phase shift of the RC circuit from the measured amplitudes and phases. The impedance is given by the equation

$$Z = \left(\frac{V_{pp1}}{V_{pp2}} \right) * 470 \Omega.$$

This equation is derived using the relation $Z/R = V_Z/V_R$, which was discussed in Part One. Is V_{pp1} equivalent to V_R , V_C or V_Z ? Since the phase information reported by LoggerPro is in radians, not degrees, it is automatically converted by the formula in Excel.

- 7) Keep the frequency knob on the lowest setting, flip the frequency range switch to the high range and repeat steps 4-6.

The theoretical values for Z and phase shift are calculated in Excel using Eqs. (3-5) and the actual values for frequency, resistance, and capacitance.

- **Q5: Describe how the sine waves changed as the frequency increased. Does this agree with your answers to Q1 and Q2?**

An RC Series Circuit and Known Resistor – BIA Device

The maximum sampling rate of the LabQuest device is 10 kHz. The device is unable to measure signals with high frequencies close to or above its sampling rate. Therefore, if we wish to experiment with higher frequencies we must use an alternative method of data collection. Inside the BIA device is an integrated circuit that takes two sinusoidal waveforms as inputs and outputs two DC potentials proportional to the phase between the waves and the logarithm of the ratio of the sine waves' amplitudes. The relationships between the DC potentials V_{mag} and V_{phs} and impedance and phase are given by the equations

$$\varphi = \frac{V_{phs} - 1.888 V}{-0.011 V} \quad Z = 470 \Omega \cdot 10^{\left(\frac{V_{mag} - 0.94 V}{0.61 V}\right)} \quad (7)$$

Therefore the BIA device can measure the phase and impedance of a potential across an external circuit by comparing it to the potential across a known reference resistor [Fig. 3(b)]. Although the BIA device can be used for high frequencies, it will be inaccurate for low frequencies. This is why LabQuest and LoggerPro are used when the frequency knob is on its lowest setting.

- 8) Replace the 1 μF capacitor in the RC circuit with a 10 nF capacitor.
- 9) Connect the CH1 probe to V_{phs} and GND and the CH2 probe to V_{mag} and GND . During the experiment, you will record the voltages displayed on the bottom left corner of the LoggerPro window in the Excel table for V_{phs} and V_{mag} .
- 10) Take data at the settings/frequencies you have listed in Table 2.2 on the Excel sheet. In the Excel table, Eq. (7) is used to calculate Z and the phase shift from the measured voltages you enter. The theoretical values for Z and phase shift are calculated in Excel using Eqs. (3-5) and the known values for frequency, resistance, and capacitance. Use the plots (total impedance vs. frequency and phase shift vs. frequency) to compare your experimental results to the expected values.

➤ **Q6: How does the measured data compare to the expected calculations? Does it agree with your answers to Q1 and Q2?**

Part Three: Single Frequency Bioelectrical Impedance Analysis

It is useful for researchers and medical personnel to know what percentage of a patient's body mass is due to fat cells. While there are many methods to measure body fat percentage, they either suffer from having low accuracies (manual measurement or caliper tests), being difficult to perform (water immersion or densitometry) or are potentially harmful (DEXA, dual energy x-ray absorptiometry). An alternative method called Bioelectrical Impedance Analysis (BIA) takes advantage of the conductive properties of the body.

The way in which the body responds to electric current is similar to an RC circuit, where body fluids (intra- and extracellular water, or ICW and ECW) are represented by resistors and cell walls by capacitors. Fat cells have very high impedance compared to muscle cells, organ cells, extracellular components (including connective tissue). Therefore, electric current will mostly propagate through the fat-free body mass. This means that BIA measures the volume of fat free mass V_{FFM} .

The quantity that relates an object's resistance to its dimensions is called resistivity ρ . The resistance of a uniform cylinder of length L and cross sectional area A is

$$R = \rho \frac{L}{A} \quad (8)$$

This can be rewritten in terms of volume V as (try to verify this)

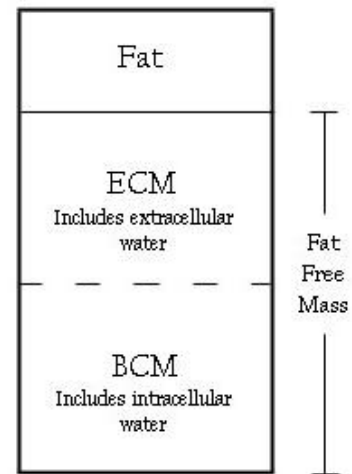


Figure 4 – Compartments of the body measured by BIA. ECM = extra-cellular mass, BCM = body cell mass.

$$R = \rho \frac{L^2}{V} \quad (9)$$

The volume of the human body can be approximated as a combination of cylinders containing a material with a certain resistivity ρ_{body} . A single cylinder can be used to represent the volume that includes a person's arms and chest for BIA measurements made hand-to-hand (hand-to-foot or foot-to-foot measurements can also be made). Noting that a person's height (H) is approximately equal to the distance from fingertip to fingertip, the length of the cylinder is equal to a person's height. Substituting H for L and solving for volume V , Eq. (9) can be rewritten as

$$V_{FFM} = \rho_{body} \frac{H^2}{R}$$

The fat free mass (FFM) can be calculated using the average density of the fluid in the body (d_{body}). Noting that $V_{FFM} = FFM / d_{body}$,

$$FFM = d_{body} \cdot \rho_{body} \frac{H^2}{R}$$

While this equation is a good first approximation, it does not account for variations in gender, age, weight, and other factors. In order to account for these, it is necessary to measure a large sample of people and correlate those measurements to a "gold standard" for determining FFM, such as densitometry. Various studies in BIA have used information such as reactance, age, weight, and gender to empirically fit data, adding correction terms to the equation above. Collecting data from a large sample of volunteers and comparing it to the values obtained with the commercially available OMRON device, an equation for FFM as measured by our BIA device was empirically found:

$$FFM = 0.36 \frac{Height^2}{R} + 0.162Height + .0289Weight - 0.134Age - 6.83 + 4.86 * Gender(0 \text{ female}, 1 \text{ male}) \quad (10)$$

Although BIA is a simple procedure and poses no danger to the patient because of the very small electrical current used ($< 800 \mu A$), it does have precision limitations. The density and resistivity of the body's fluid vary depending on hydration level, food intake, exercise, alcohol consumption, and even body positioning. Therefore accurate results require the patient to follow a strict regime prior to the test. Because this experiment does not follow the strict guidelines required for a precise measurement, there will most likely be some error in your calculated fat free mass.

BIA calculations are based on the impedance of deep body tissue. If only two electrodes are used, as with the circuit in Part One, the impedance of the skin-electrode contact affects the measurements. This is due to the fact that current is injected through the same electrodes that measure potential. To avoid this, four electrodes are used: two to inject current and two to measure the potential (Fig. 5). The voltage probe electrodes have very high input impedance and draw very little current, so skin-electrode impedance is avoided.

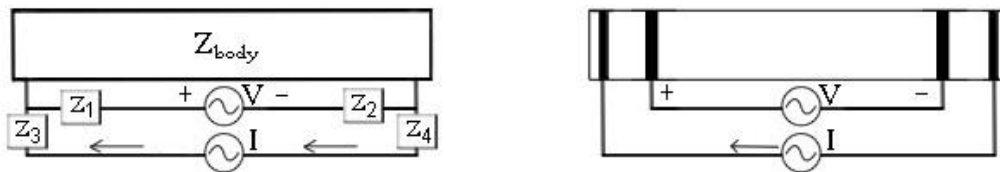


Figure 5 – (a) Two electrode method and (b) four electrode method to measure deep body tissue. Very little current is drawn by the voltage probe electrodes, so Z_1 and Z_2 are negligible.

Experimental Procedure

Single Frequency BIA

- 1) Change the frequency knob of the BIA device to the setting from Part Two that is closest to 50 kHz.
- 2) Connect the leads of one of the hand held electrodes to I+ and V+. Connect the other hand held electrode to I- and V-. While standing, hold the electrodes with outstretched arms as shown in Fig. 6. Make sure your palms are touching both electrodes.
- 3) Use LoggerPro to measure both V_{phs} and V_{mag} . Enter these into the Excel sheet provided. Z and the phase angle are calculated in the Excel columns using Eq. (7), as in Part Two. The body's resistance and reactance are calculated from Z and the phase angle using Eq. (6).
- 4) Enter the height, weight, age, and gender of the volunteer into the appropriate columns in the Excel sheet. The body's FFM is calculated by Excel using Eq. (10). Use a force plate to measure the volunteer's weight.
- 5) Measure the percentage body fat using the OMRON device and compare the results with those of the BIA device.

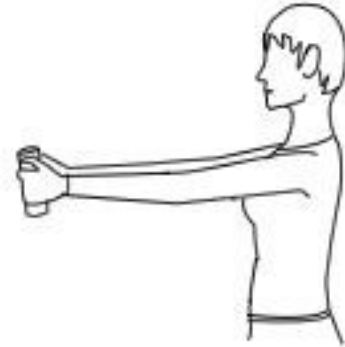


Figure 6 – How to hold the BIA electrodes when taking a reading.

Part Four: Multi-Frequency BIA

As we have seen, the electrical characteristics of the body can be approximated with an RC circuit. The parallel circuit shown in Fig. 7 is a more accurate representation of the body's circuit, where R_e and R_i represent the resistances of the body's extra- and intracellular water respectively, and capacitance C is due to cell membranes.

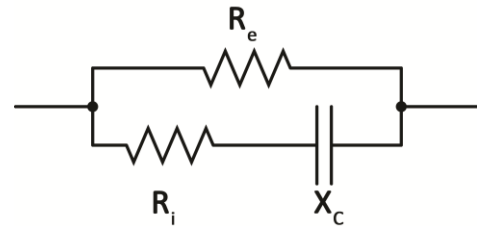


Figure 7 - Circuit model of the body used in multi-frequency analysis

Two resistors connected in parallel can be represented by a single equivalent resistor with the equation $R_{eq} = (R_1^{-1} + R_2^{-1})^{-1}$. In a similar way, it is possible to mathematically convert a parallel circuit such as this one into an equivalent series circuit with one value for effective resistance R_{eff} and one for effective reactance $X_{C,eff}$ Eq. (11). Due to the reactance of the capacitor, this mathematical conversion is dependent on frequency. Therefore the effective resistance and reactance change as the frequency changes, even though the physical components in the parallel circuit are constant. The series circuit model used in Part Three represents the series equivalent of the parallel circuit in Fig. (7) for a single frequency.

The methods of complex analysis are needed to derive the series equivalent equations; the results are shown below for the circuit in Fig. (7) for your reference (e is for extracellular and i is for intracellular):

$$R_{eff} = \frac{R_e R_i (R_e + R_i) + R_e X_i^2}{(R_e + R_i)^2 + X_i^2} \quad X_{C,eff} = \frac{R_e^2 X_i}{(R_e + R_i)^2 + X_i^2} \quad (11)$$

- **Q7: Based on Eq. (4), what do you think is the effective resistance and reactance for the circuit in Fig. 7 when a DC potential ($f=0$) is applied? What would they be for very high frequencies ($f=\infty$)?**

Experimental Procedure

- 1) Connect a circuit similar to Fig. 7 to the BIA device in place of the series RC circuit from Fig. 3. Use the following components: $R_{ecw}=470 \Omega$, $R_{icw}=100 \Omega$ and $C=10 \text{ nF}$.

- 2) Using LoggerPro, measure V_1 , V_2 , phase shift 1 and phase shift 2 as you did in Part Two for the two lowest frequencies (~ 50 and ~ 500 Hz), entering the data in Table 5.1 of the Excel sheet. For the remaining frequencies (choose from the same settings/frequencies you used in Part II), record V_{phs} and V_{mag} .

Z and the phase shift are calculated in Excel just as they were in Part Two. Measured R_{eff} and $X_{c,eff}$ are calculated according to Eq. (6). Theoretical values for R_{eff} and $X_{c,eff}$ are calculated using Eq. (11). Use the Cole-Cole plot in the Excel sheet to answer Q8.

- **Q8: The equivalent resistances for very high and very low frequencies are called R_∞ and R_0 respectively (Fig. 7). Based on your Cole-Cole plot, what are R_∞ and R_0 for your circuit? How does this compare to your answer for Q7?**

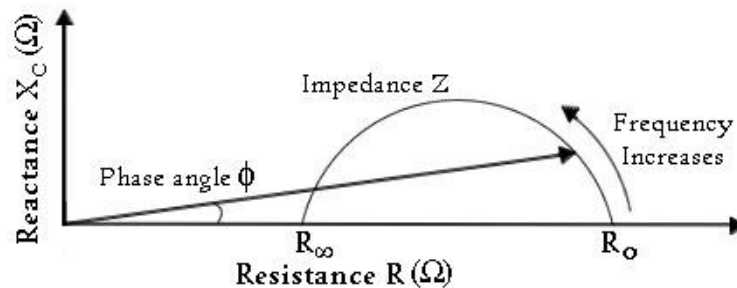


Figure 8 - Cole-Cole plot used in multi-frequency BIA analysis

- 3) Repeat the multi-frequency analysis using the hand held electrodes in place of the parallel circuit and enter your data in Table 5.2. Use the Cole-Cole plot from Table 5.2 to answer Q9.
- **Q9: Based on your Cole-Cole plot, what are R_∞ and R_0 for the body?**
- **Q10: Based on the model circuit in Fig. 7, explain why the equivalent resistance and reactance of the body changes as the frequency increases. Would you use high or low frequencies if you wanted to measure extra-cellular water only?**
- **Q11: Using your data, why do you think single frequency BIA is done at 50 kHz (Hint – find where 50 kHz occurs on the Cole-Cole plot for the body)?**