

Portland State University
General Physics Workshop
Problem Set 6

Alternating Current

Equations and Relations:

Force on a moving charged particle:

$$F = qvB \sin(\theta)$$

Ampère's law: $\sum B_{\parallel} \Delta l = \mu_0 I$

Coil rotating in a magnetic field:

$$emf = \omega N B A \sin(\omega t)$$

Faraday's law: $emf = -\frac{\Delta \Phi}{\Delta t}$

Energy stored in an Inductor: $U = \frac{1}{2} L I^2$

Time constant: $\tau = \frac{L}{R}$

Current after closing LR circuit:

$$I(t) = I_0 (1 - e^{-t/\tau})$$

Current after opening LR circuit:

$$I(t) = I_0 e^{-t/\tau}$$

Capacitive reactance: $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Inductive reactance: $X_L = \omega L = 2\pi f L$

Impedance: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase shift: $\tan(\phi) = \frac{X_L - X_C}{R}$

Average power: $P_{av} = I_{rms} V_{rms} \cos(\phi)$

Power factor: $\cos(\phi) = \frac{R}{Z}$

Resonance: $\omega_0 = \frac{1}{\sqrt{LC}}$

1. The power factor of an RL circuit with a 100-Ω resistor and a certain inductor is 0.60. What is the impedance of the circuit?
2. A 120 V-rms voltage is applied across a 6.0-μF capacitor and a 100-Ω resistor. If the frequency of the generator is 60 Hz, what is the rms value of the current in the circuit?
3. A certain ac signal at 1000 Hz is applied across an inductor and a 100-Ω resistor. If the power factor of the circuit is 0.400, what is the inductance of this circuit?
4. A series RLC circuit has a 100-Ω resistor, a 0.100-μF capacitor and a 2.00-mH inductor connected across a 120-V rms ac voltage source operating at (1000/π) Hz. What is the resonant frequency of this circuit?
5. A series LR circuit has an inductance of 18 mH and a resistance of 12 Ω. The current is connected to a 120-V 60-Hz wall outlet.
 - a.) Calculate the impedance of the circuit.
 - b.) Determine the rms voltages across the inductor and the resistor.
 - c.) Draw a phasor diagram for the circuit.
 - d.) Calculate the phase angle between the current and voltage.

6. The capacitance in a series RCL circuit is C_1 . The generator in the circuit has a fixed frequency that is less than the resonant frequency of the circuit. You want to lower the resonant frequency of the circuit to match that of the generator, and you do so by adding a second capacitance C_2 .
- Should the circuit capacitance be increased or decreased?
 - Should C_2 be added in parallel or in series with C_1 ? Why?
 - The initial circuit capacitance is $C_1 = 2.60 \mu\text{F}$, and the corresponding resonant frequency is $f_{01} = 7.30 \text{ kHz}$. The generator frequency is 5.60 kHz . What is the value of the capacitance C_2 that should be added so that the circuit will have a frequency matching the generator?
7. A light dimmer used to adjust the light level of a light bulb consist of a variable inductance L . In this example the inductance can change from zero to L_{max} and the light bulb is connected in series with the inductor. The light bulb is rated as 60W at 120V and connected to a standard outlet (120V , 60 Hz). Assume that the resistance of the bulb is independent of the temperature.
- Calculate the current in the circuit for $L=0$.
 - How much energy does the bulb dissipate in one hour?
 - Calculate L_{max} such that the energy dissipation of the bulb can be varied by a factor of 10.
 - How much energy is dissipated by the inductor in one hour? How much energy is dissipated by the whole circuit in one hour?
- *Could you use a variable resistor instead of the variable inductor to change the light intensity? If yes, which method of dimming would you recommend?*
- *A series RCL circuit contains three components – a resistor, a capacitor, and an inductor. Which of these components, on average, dissipates power?*
8. In a hand-cranked generator, a coil of 50 turns of wire, each enclosing an area of 10 cm^2 , is rotated in a 0.6 T magnetic field. With effort, you are able to turn the handle five full revolutions each second.
- What is the maximum *emf* you can generate?
 - What is the instantaneous value of the *emf* at $t = 0 \text{ s}$, $t = \frac{1}{20} \text{ s}$, $t = \frac{2}{20} \text{ s}$, $t = \frac{3}{20} \text{ s}$, $t = \frac{4}{20} \text{ s}$? (Assume $t = 0$ at an instant when the *emf* is going from - to +.)
- *How often does the current reverse for one full revolution of the coil?*

Additional Questions

1. How is the current in the circuit related to the generator voltage and the impedance of the circuit?
2. What is the impedance of the circuit at resonance?
3. At resonance, how is the average dissipated power related to the generator voltage and the resistance?
4. How do you change the station when you turn the tuning knob on your radio?