SUPERCONDUCTIVITY

Basic Phenomenon

If a material is described as a superconductor, below a certain temperature – the critical temperarure - it loses its electrical resistivity to become a perfect conductor.

Background History

Kammerlingh Onnes – liquefying of He in 1908.

$T_{\text{boiling point}}$ for He = 4.2K

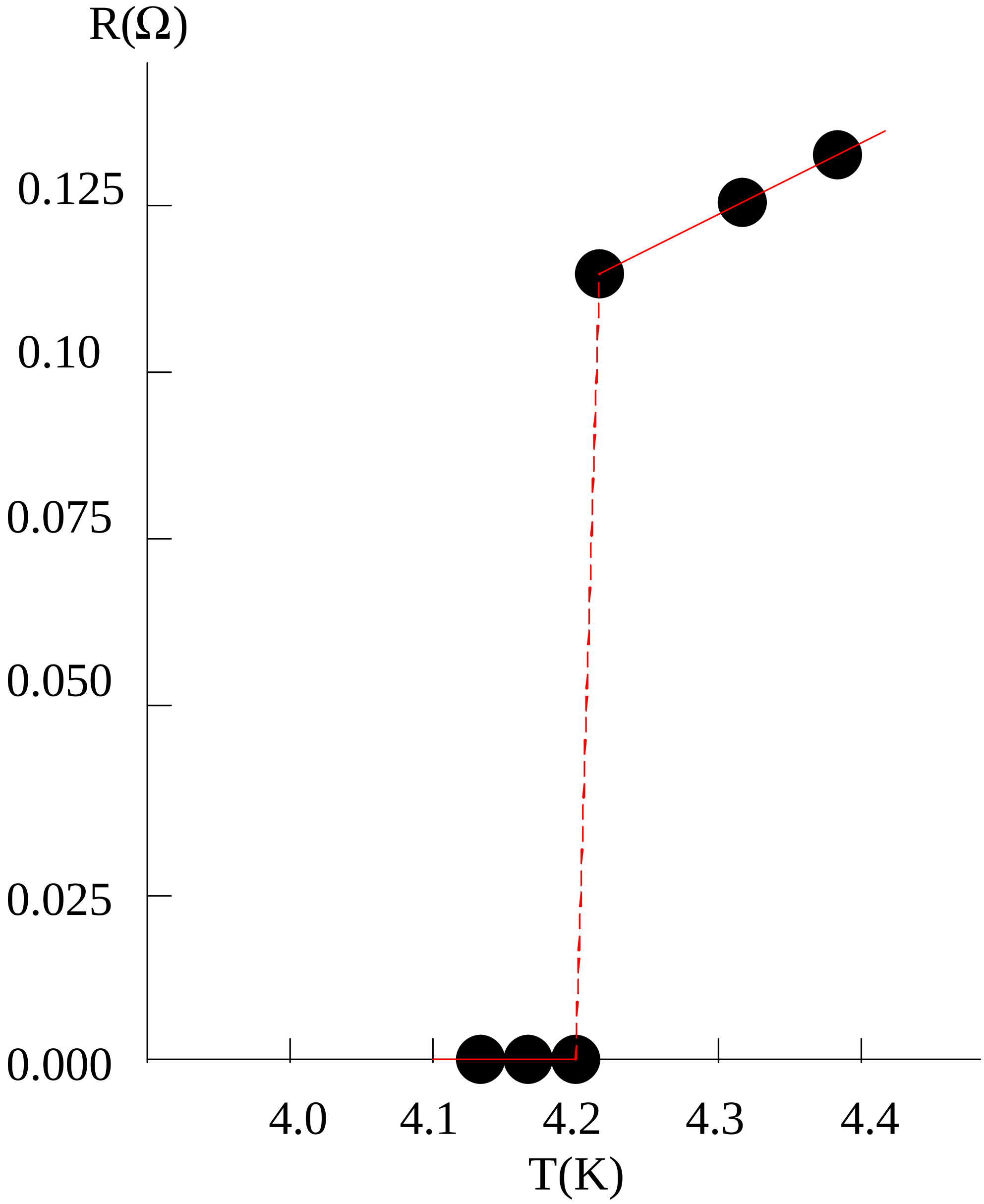
Study of properties of metals at low T.

→ including electrical properties e.g. resistivity

First indication of superconducting behaviour came from a mercury (Hg) sample.

Resistance of Hg sample versus T

Onnes 1911



Resistance falls sharply to zero at critical temp’ Tc (≈ 4.2K)

Superconducting state little affected by impurities.

Elemental Superconductors

$T_c < 0.1$ K for Hafnium (Hf) and Iridium (Ir)

$T_c = 9.2$ K for Nb (element with highest Tc)

Superconducting Alloys

Many metallic alloys were also found to be superconducting

e.g. MoC ($T_c = 14.3$K), V_3Ga ($T_c = 16.8$K),
Nb_3Sn ($T_c = 18.05$K), Nb_3Ga ($T_c = 21.0$K)

In 1972 Nb_3Ge → $T_c = 23.2$K

No improvement in T_c for 14 years.

“High T_c” Oxides

Large break through in 1986 - Bednorz and Müller

$T_c \approx 35K$ for $La_{2-x}Ba_xCuO_4$

Many similar materials since discovered with higher Tc

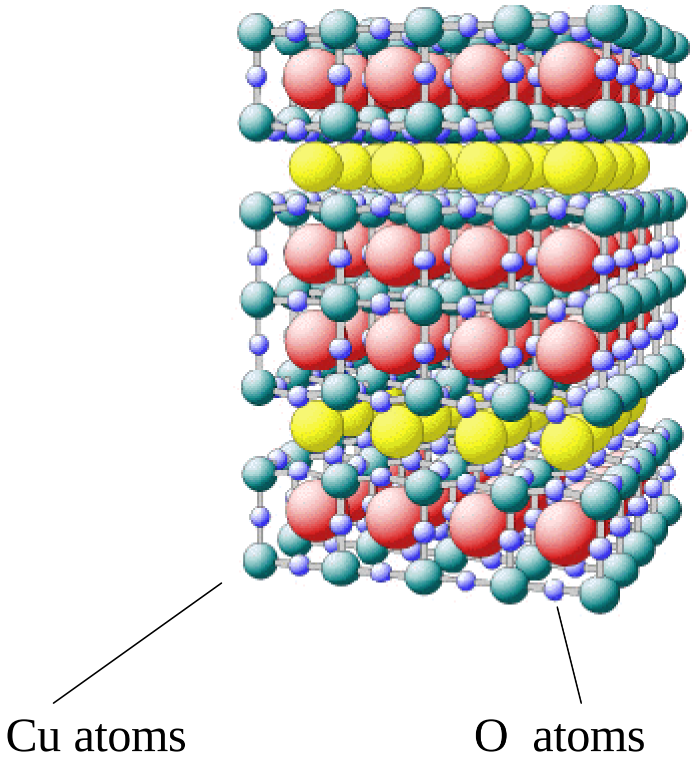
$YBa_2Cu_3O_{7-\delta}$ → $T_c = 92K$ (1987)
[“YBCO”]

$Tl_2Ba_2Ca_2Cu_3O_{10}$ → $T_c = 122K$ (1988)

$HgBa_2Ca_2Cu_3O_{8+\delta}$ → $T_c = 133.5K$

Referred to as “high-temperature superconductors” or “high-T_c superconductors”.

Structure of YBaCuO



Common feature in most of these materials:

crystal structures contain planes of CuO_2

Believed to play crucial role in the conductivity and superconductivity of high-T_c materials

Oxygen content is critical

e.g. $YBa_2Cu_3O_{7-\delta}$

$\delta = 1$ → $YBa_2Cu_3O_6$ - insulator

$\delta = \sim 0.6$ → $YBa_2Cu_3O_{6.4}$ - metallic
(metal-insulator transition)

δ just less than 0.6 - superconducting ($T_c \approx 40K$)

As δ decreased further, T_c increases.

$\delta \approx 0.1$ → $YBa_2Cu_3O_{6.9}$ - $T_c = 92K$

[Not possible to prepare $YBa_2Cu_3O_{7-\delta}$ for δ less than ~ 0.1 without changes in basic crystal structure].

Advantages/Potential Problems of High T_c Materials

For high T_c oxide materials, T_c > boiling point of N_2
"YBCO" $T_c = 92K$

Boiling point of liquid N_2 - 77K

Liquid N_2 much cheaper as a coolant than liquid He.

Problems - oxide materials most easily prepared as a ceramic (i.e. many small crystallites bonded together).

Performance degraded by poor contact between crystallites.

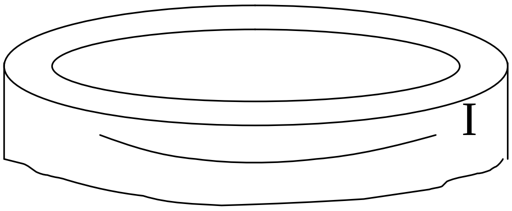
Brittleness and toxicity of the materials also lead to problems.

How Superconducting?

How superconducting are these materials?

Can we measure a (small) finite resistance in the superconducting state?

Sensitive method for detecting small resistance– look for decay in current around a closed loop of superconductor.



Set up current I in superconducting loop using e.g. B-field

If loop has resistance R and self-inductance L, current should decay with time constant τ

where $\tau = L/R$

Failure to observe decay

→ upper limit of 10^{-26} Ωm for resistivity ρ in superconducting state

c.f. $\rho = 10^{-8}$ Ωm for Cu at room temp'

Magnetic Properties

Superconductors also show novel magnetic behaviour.

They behave in 1 of 2 ways.

Classified into:

Type 1 superconductors (all elementals s/c's except Nb)

Type 2 superconductors (high-T_c oxides)

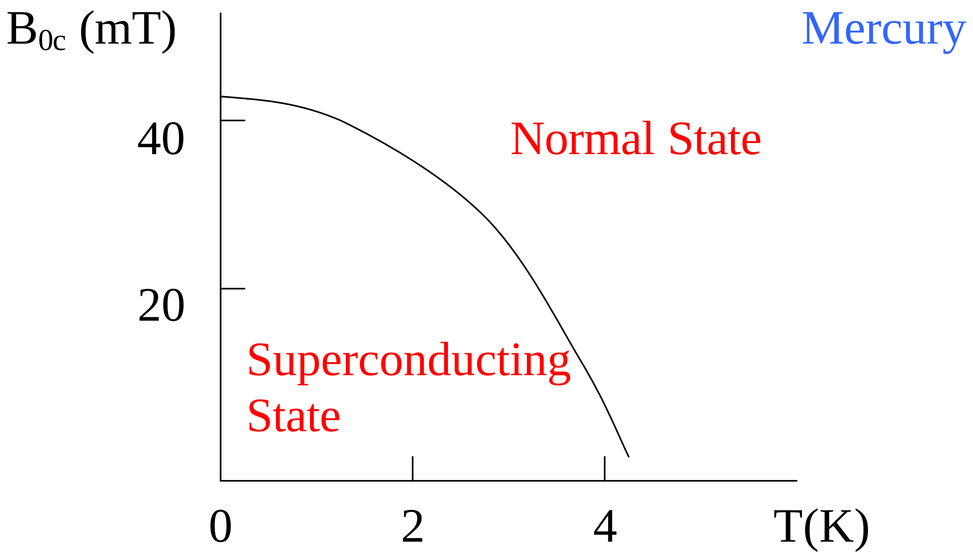
Type 1 Superconductors

Super conductivity destroyed by modest magnetic field – critical field B_{0c}.

B_{0c} depends on temperature T according to:

$$B_{0c}(T) = B_{0c}(0)[1-(T/T_c)^2]$$

e.g. for mercury

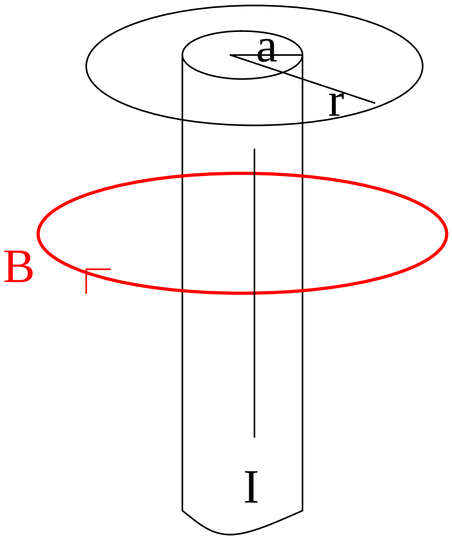


Critical Current in Superconducting Wire

Existance of critical field B_{0c} implies that for a superconducting wire, there will be a critical current I_c [since current carrying wire generates a B-field].

For currents $I > I_c$, superconductivity is destroyed.

Wire radius - a
Current I wire - I



B-field lines – concentric circles centred around wire axis

Can calculate field magnitude using Ampere’s law:

$$\oint B.dl = \mu_0 I \qquad [\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Hm}^{-1}]$$

At wire surface: $$B = \frac{\mu_0 I}{2\pi a}$$

Typical values: wire diameter = 2a = 1mm
critical field B_{0c} = 20 mT

This gives: I_c = 50 A

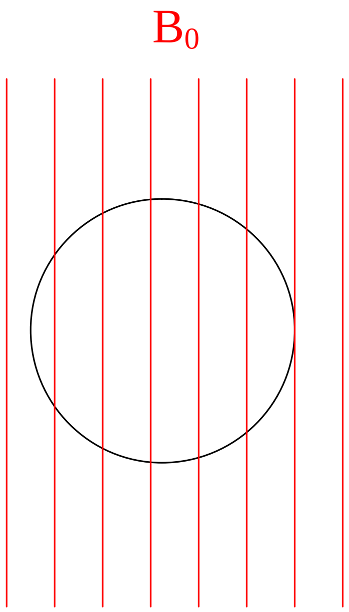
Meissner Effect

What happens to magnetic field inside superconductor?

Consider effect of applying a magnetic field (flux density) B_0 to the material.

In normal (non-superconducting) state

$T > T_c$



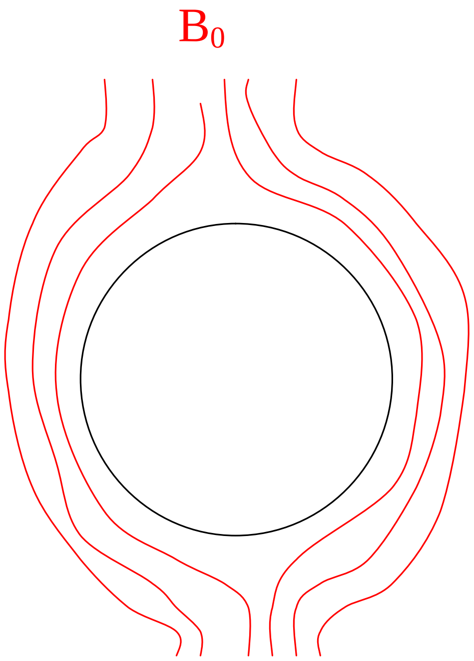
Field passes through material with essentially no change (or only very small change).

Field B inside material relates to B_0 and magnetisation M of the material by

$$B = B_0 + \mu_0 M$$

So in normal state M is essentially zero.

In superconducting state



$T < T_c$

Field is excluded from superconductor.

Meissner and Ochsenfeld 1933.

So field B inside superconductor is zero.

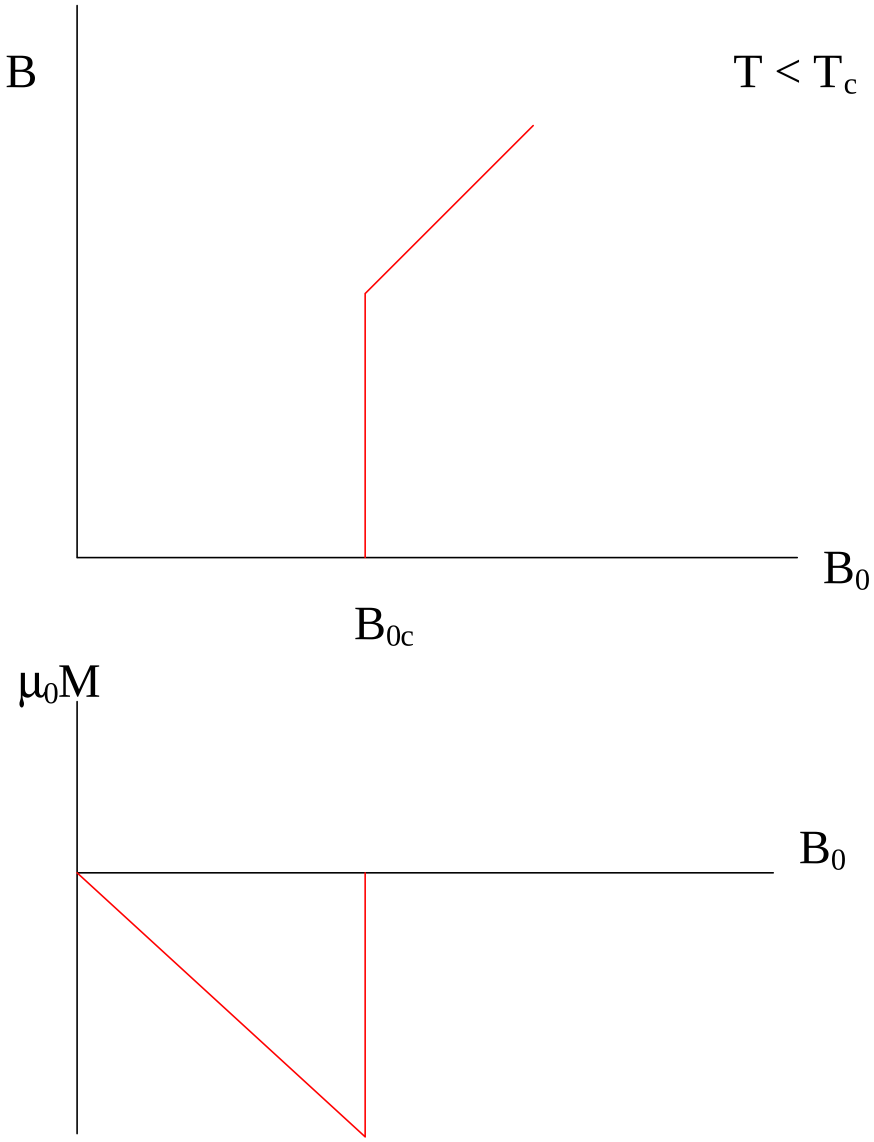
$$\text{i.e. } B = B_0 + \mu_0 M = 0$$

$$\rightarrow \quad M = -B_0/\mu_0$$

So magnetic susceptibility $\chi = \mu_0 M/B_0 = -1$
i.e. perfect diamagnetic

Referred to as Meissner effect.

Graphically

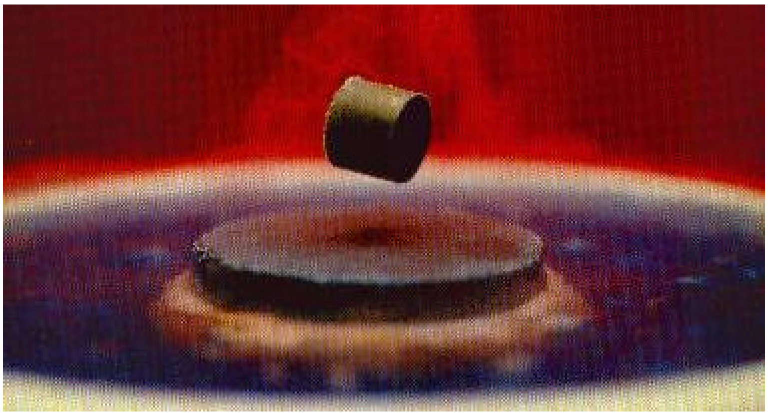


What’s actually happening?

In the superconducting state:

> screening currents flow on the surface of the superconductor in such a way as to generate a field inside the superconductor equal and opposite to the applied field.

Helps to explain levitation of superconductor that can can occur in a magnetic field. Results from repulsion between permanent magnet producing the external field and the magnet fields produced by the screening currents.



Type 2 Superconductors

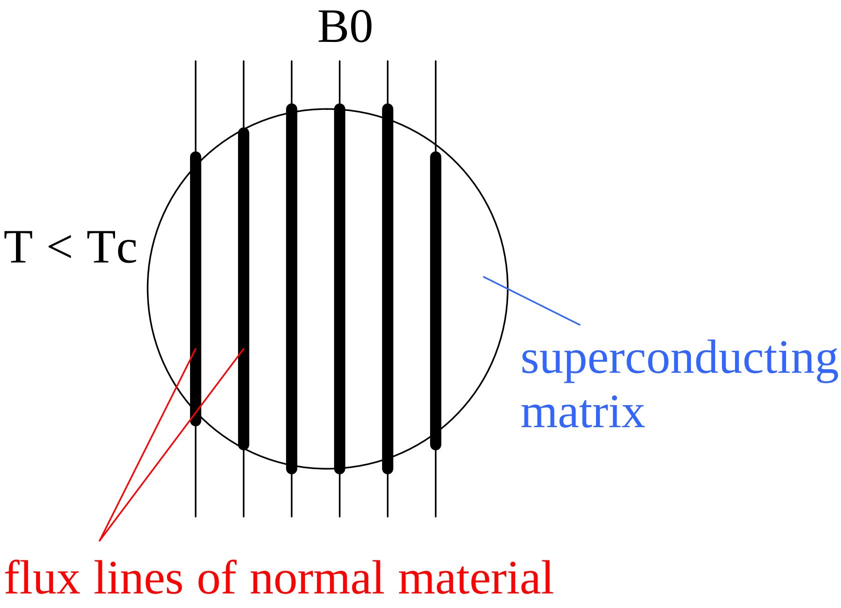
Critical fields B_{0c} found to be small for Type 1 superconductors → potential current densities in material (before reverting to normal state) are small. (Most elemental s/c's)

Certain superconducting compounds → capable of carrying much higher current densities in superconducting state.

These also display different magnetic properties.

At low fields, Meissner effect is observed (as described above).

At critical field $B_{0c}(1)$, magnetic field starts to enter the specimen. However, field does not enter uiniformly- but does so along flux lines of normal material contained in superconducting matrix.



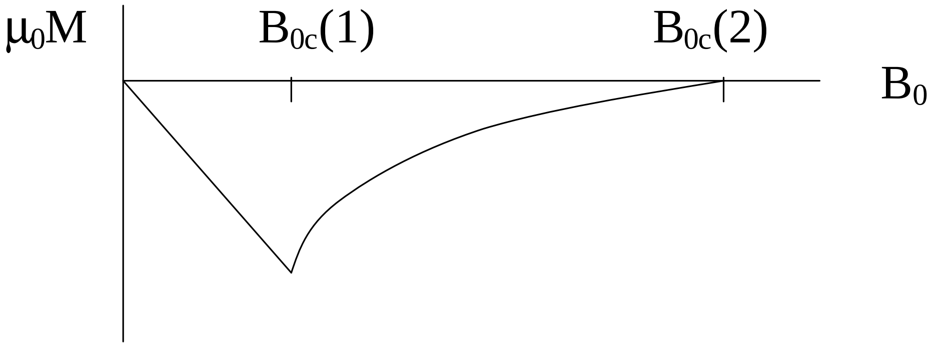
Mixed state described as vortex state.

Can persist over a large field range.

As external field B_0 is increased above $B_{0c}(1)$, density of flux lines increases.

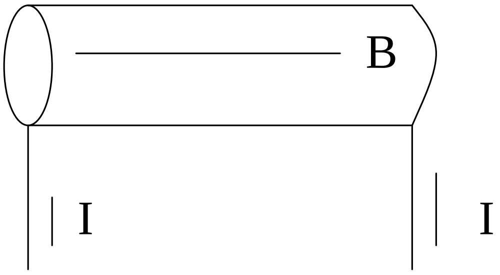
Eventually, at second critical field $B_{0c}(2)$, flux fully penetrates the sample – reverts to normal state.

Graphically



Possible Applications of Superconductors

Superconducting Magnets



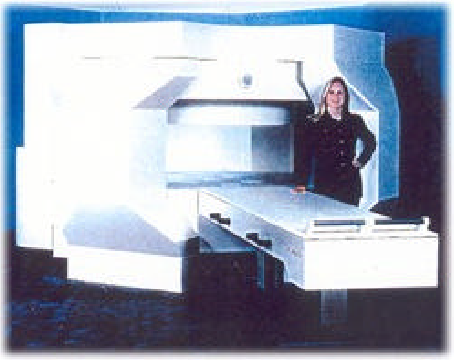
Solenoid

N turns
Current I

$$B = \mu_0(N/L)I$$

Superconducting Material → Large I

Hence → can get large B!



Magnetic Resonance Imaging Unit

Uses large superconducting magnet – can provide detailed images inside human body.

MagLev Transport

Use Meissner effect to get vehicles to “float” on strong superconducting magnets.

e.g. Yamanasi Maglev Test Line

Virtually eliminates friction between train and track.

Thermal Properties

Can describe thermal properties of superconductors using classical thermodynamics.

For Example:

Can show that there is a latent heat L associated with the normal - superconducting transition, given by

$$L = -VTB_{0c}(dB_{0c}/dT)/\mu_0$$

[V = volume of superconductor, T = temperature]

Can also show that there is a discontinuity in the specific heat capacity C at the phase transition (in zero field). [Good agreement with experimental data for metallic superconductors].

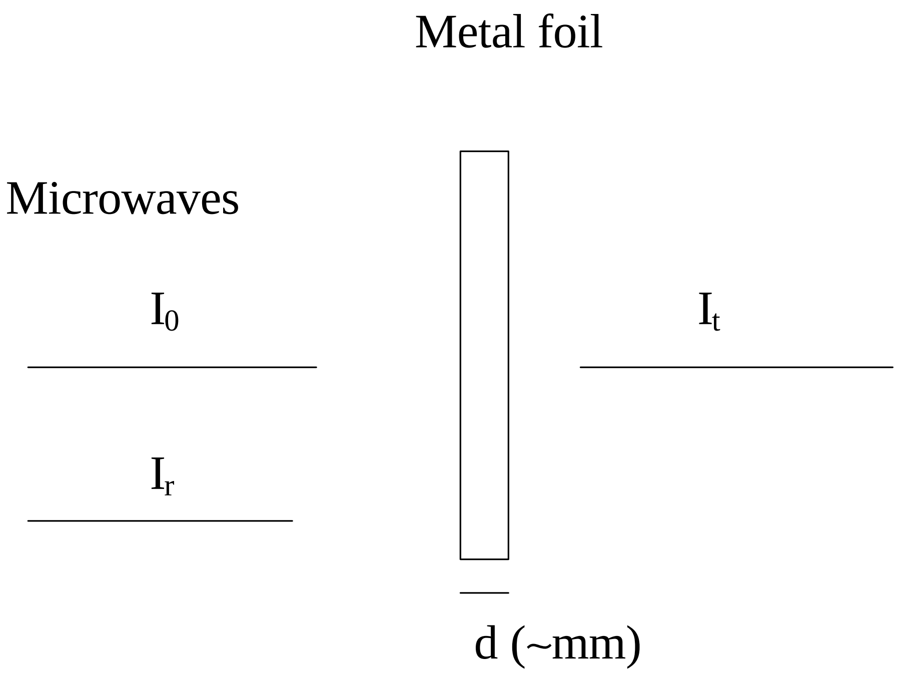
For metal, C has contribution from lattice vibrations and an electronic contribution. Measurements of electronic part of C in superconductors reveals that it varies as

$$\underline{\exp\{-E_{gs}/kT\}}$$

Suggests presence of energy gap E_{gs}. [2Δ in Tanner]

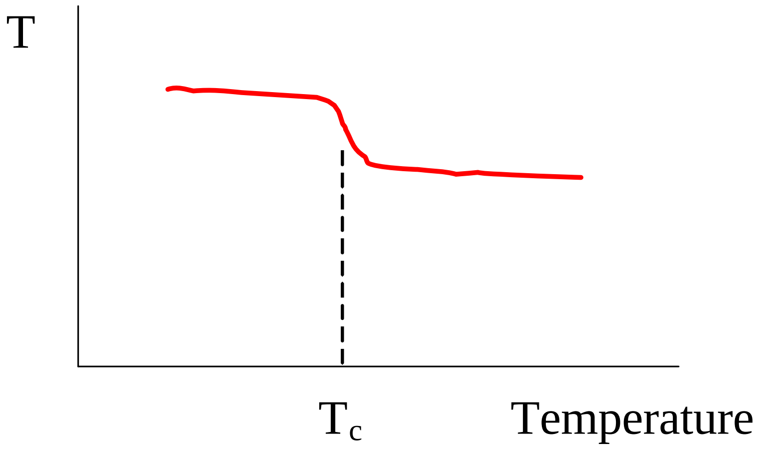
Microwave and Infra-Red Absorption

Reinforces idea that an energy gap may be present in superconductors.



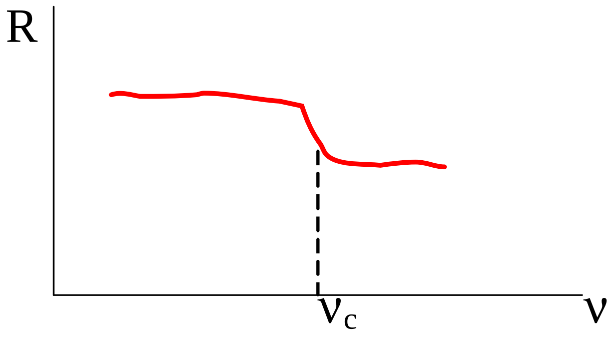
Transmission $T = I/I_0$ Reflectivity $R = I_r/I_0$

Transmission T



T increases as foil is cooled through its transition temperature T_c. Suggests incident photons don't now have enough energy excite electrons across some energy gap.

Reflectivity R



Similarly, if infra-red reflectivity R is measured as function of frequency ν (for superconducting matertial), get sharp increase in R at specific ν-value ν_c.

Again, suggests energy gap given by

$$E_{gs} = h\nu_c$$

Theoretical Models for Superconductivity

A bit hard ……

Microscopic Theory

Bardeen, Cooper, Schrieffer 1957

First successful microscopic theory – BCS theory.

Key points:

Electrons in a superconductor at low T are coupled in pairs

Coupling comes about due to interaction beween electrons and crystal lattice

In (a little) more detail:

one electron interacts with lattice and perturbs it (positive ions attracted slightly towards electron, thus deforming lattice). Under certain conditions, this deformation may be such that the net charge seen by another electron in the vicinity is positive.

Hence there can be a net attraction between electrons.

Electrons form a bound state, known as a Cooper pair.

Electrons in a Cooper pair have opposite spins – hence Cooper pair has zero spin i.e. acts as a boson

Bosons do not obey exclusion principle → can all occupy same quantum state of same energy

Consequences: Cooper pairs act in a correlated way. So, in this collective state, they can all move together. Binding energy of Cooper pair is largest when all are in same state. A cooperative phenomenon.

Energy required to break a Cooper pair is

$$E_{gs}$$

Referred to as superconducting energy gap.

Theory predicts that E_{gs} temperature-dependent, but at $T = 0$ K, $E_{gs} = 3.5kT_c$

Energy gap of E_{gs} opens up in density of states at Fermi level.