PHY 481/581

Some classical/quantum physics for the nanometer length scale

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What is nano-science? … the science of materials whose properties scale with size … (J. W. Steed, J. L. Atwood)

One may extrapolate from know behavior at the length scale of human beings to some micro- or nano-scopic length scale, this is known as scaling

Linear or near linear relationships are easiest for human beings to comprehend

The scaling may not be linear, the phenomenon may be complex,

Rigid body behavior at the tens and hundreds of nanometer scale may be explained by classical physics concepts, human beings have particular problems with exponential scaling

How about Galileo’s statement: all bodies fall at the same rate …?
A macroscopic body freely falling in air, does theory I or II describe the phenomenon reasonably well?

Physical reality is often the superposition of more than one phenomenon, some of these phenomena may scale differently, classical physics typically looks for an isolated system, tries to neglect other influences if they are not too strong.

At the small scale, forces such as friction and surface tension often dominate over forces such as gravity, electrostatic forces may become strong, …

\[ v = v_0 - at \]
\[ s = \frac{-at^2}{2} + v_0 t \]

\( v_0 = 100 \text{ km/h}, \text{ deceleration: } 5 \text{ m/s}^2 \):
27,778 m traveled in reaction time 1 sec, 6.55 sec total and 105 m travelled before stop, what if 200 km/h? 12.11 sec but 364.2 m
The force $F$ needed to move a sphere of mass $m$, density $\rho$, radius $R$ at a velocity $v$ through a viscous medium of viscosity $\eta$ (Stoke’s Law) is given by:

$$F = 6\pi\eta R v$$  \hspace{1cm} (2.12)

**Kind of “friction”**

Terminal velocity when “friction force” is balanced by force of gravity

$$v_t = \frac{mg}{6\pi\eta R} = \frac{\frac{4}{3}\pi R^3 \rho g}{6\pi\eta R} = \frac{2\rho g R^2}{9\eta} \propto R^2$$

This applies for reasonably laminar flow, Reynolds number < 2000

1 mm radius sphere $7 \text{ g/cm}^3$ (small steel ball baring) falling through water ($\eta = 0.01 \text{ Pa s}$) reaches terminal velocity of about 1 m/s, if sphere has radius 1 µm, $v_t \approx 1$ µm/s, if sphere has radius 1 nm, $v_t \approx 1$ pm/s, so it takes 1,000 sec to fall by only one diameter. In addition, there will be forces “jostling” the nano-particle around due to movement of water molecules, (Einstein’s proof of the existence of molecules)

Sphere with 10 µm radius, density $7 \text{ g/cm}^3$ falls in air (with $v_t$) about 8 cm/s, but a 15 nm radius sphere is predicted to fall in air only about 175 nm/s !!

1 nm diameter sphere $7 \text{ g/cm}^3$

Brownian diffusion length about 9 µm, characteristic distance displaced every second !!!, so force of gravity is negligible for this particle

Brownian diffusion force

For $\text{Re} \ll 1$, viscous forced dominate

might be $m g$

Mass of sphere times acceleration

Langevin equation
Artist’s impression of a (very large) nanobot pinching a red blood cell (diameter 6 – 8 µm), note that micro-machines do scarcely exist, nano-machines and bots are truly science fiction, most importantly they may never look like some miniature version of the machines we are used to, ...

A bacterium swimming through water, viscosity $8.9 \times 10^{-4}$ Pa s velocity $50 \mu$m/s for $1 \mu$m size bacterium gives Reynolds number $5.6 \times 10^{-5}$ (a human would have Re number 10 orders of magnitude larger, so completely different regimes !!!, bacterium faces enormous “resistance by water”, viscosity of blood $2.7 \times 10^{-3}$ Pa s, so conditions are similar

There are also no airplanes with rigid wings smaller than about $1$ mm, insects just fly differently, beating their wings
“Friction, resistance to movement” is generally very high at the nanoscale, (almost everything is pretty sticky), but can be small for certain high symmetry arrangements, e.g. the inner tubes in a multi-walled carbon nanotube.

Sure nanobots will need moving parts, rotations about some axes, resistance to rotation is usually very high.
Cavenagh bridge, Singapore’s oldest suspension (cantilever) bridge

SEM image of an AFM cantilever, resonance frequencies 10 – 200 kHz, force constants 0.01 – 100 N/m, forces in the nN range can be detected

Resonance frequency of a cantilever scales with $L^{-2}$, $L = \text{Length}$

There are two basis modes for AFM, “contact” mode and non-contact mode. In contact mode (a few nm) the tip is very close to the sample and the interaction is repulsive (due to Pauli repulsion). In the not-contact mode, the tip is further away, some 100 nm, the interaction is attractive, the cantilever vibrates at a certain frequency, the change of this frequency due to the interaction is measured (instead of the bending of the cantilever).
$$\omega = \frac{2\pi 0.56}{L^2} \left( \frac{Ywt^3}{12\rho A} \right)^{1/2}$$

For a miniature diving board, i.e. some plank fixed on one side, vibrating perpendicular to the characteristic Length $L$

$L$: length of cantilever, $Y$: Young’s modulus, kind of a rigidity measure of a material, Force per unit area (pressure stress) per fractional deformation (GPa), $\rho$ mass density, $A$ cross section area, $w$ and $t$ widths and thickness of the cantilever,

In the absence of damping, a mechanical system will absorb energy that is delivered with a frequency that matches is resonance frequency very readily
Envelope: \(1/\left|1-\left(\frac{\omega_A}{\omega_0}\right)^2\right|\) when \(\delta = 0\)

Maximum Curve:
\(1/\sqrt{1-\left(\frac{\omega_A}{\omega_0}\right)^2}\)

\(\omega_0 = \text{Natural Frequency}\)
\(\omega_A = \text{Input Frequency}\)
Disastrous resonance when \(\delta = 0\) for \(\frac{\omega_A}{\omega_0} = 1\)

\(\delta = \text{Damping coefficient}\)
The original Tacoma Narrows Bridge roadway twisted and vibrated violently under 40-mile-per-hour (64 km/h) winds on the day of the collapse.

FT-IR spectrum of ethanol molecules in liquid state, molecular fingerprinting as these vibrations are characteristic for each molecule.

Covalent bond lengths 1 – 2 Å

(Wavenumber cm$^{-1}$ = Hz/c)

$10^{14}$ Hz, 100 THz, effective spring constants 2000 N/m
Another force that influences things at the nanometer length scale:

**Casimir force effect of Quantum Electrodynamics**

All magnetic and electric fields fluctuate due to Heisenberg’s uncertainty principle, this is even so in the absence of electric or magnetic fields, i.e. \( B = 0 \) or \( E = 0 \), in a perfect vacuum

\[ \Delta E \cdot \Delta t \approx h \]

so loosely speaking: below the threshold of the energy-time uncertainty relation, all kinds of “strange particles” (especially “virtual photons”) pop in and out of existence

we know of their existence indirectly because the measured gyro- magnetic-ratio of the electron, \( g = 2.00232 \) is slightly larger than 2 (as predicted by Dirac loosely speaking in the absence of such effects)

this experimental value agrees very well with the prediction of quantum electrodynamics, i.e. the theory of interaction between electromagnetic radiation and matter (charged particles in particular) that also merges quantum mechanics with special relativity), another consequence is the Casimir force

![Figure 6.14](image)

**Figure 6.14** Two parallel metal plates exhibit the Casimir effect even in empty space. Virtual photons of any wavelength can strike the plates from the outside, but photons trapped between the plates can have only certain wavelengths. The resulting imbalance produces inward forces on the plates.

The energy density is about 0.5 nJ/m\(^2\), for two metal plates at a distance of about 10 nm, this amounts to an extra pressure of about one atmosphere
A suitably designed cantilever of an atomic force microscope (AFM) can measure the Casimir force.

There is no energy extracted from the vacuum or the surrounding, the Casimir force is just an attractive interaction that is always there and can be used to move one specific mechanical element by another without contact, since there is no contact, there is also no friction.

**Fig. 8.4** AFM measurement of the Casimir force. The method for measuring the Casimir force with an AFM is shown schematically in the top figure. The AFM cantilever has a gold-coated sphere glued onto the end instead of having a sharp tip as normally used to scan a surface topography (see Chapter 4, Section 4.4.4). The deflection of the cantilever is determined in the usual way, that is, a laser beam is reflected from the back of a cantilever and the deflection of the reflected beam measured by a position sensitive detector. The bottom picture shows the 200 μm diameter gold-coated polystyrene sphere and cantilever used in the Mohideen experiment [7]. Reproduced with permission from the Institute of Physics pages physicsweb (http://physicsweb.org/articles/world/15/9/6).

**Fig. 8.7** Non-contact rack and pinion. The lateral Casimir force can be used as a method of transmitting force without contact as in this rack and pinion.

**Fig. 8.8** Casimir ratchet. Using asymmetric teeth on one side, an oscillatory motion in a normal direction can produce a unidirectional motion of the other side.