PHY 481/581

Some classical/quantum physics for the nanometer length scale

What is nano-science? … the science of materials whose properties scale with size … (J. W. Steed, J. L. Atwood)

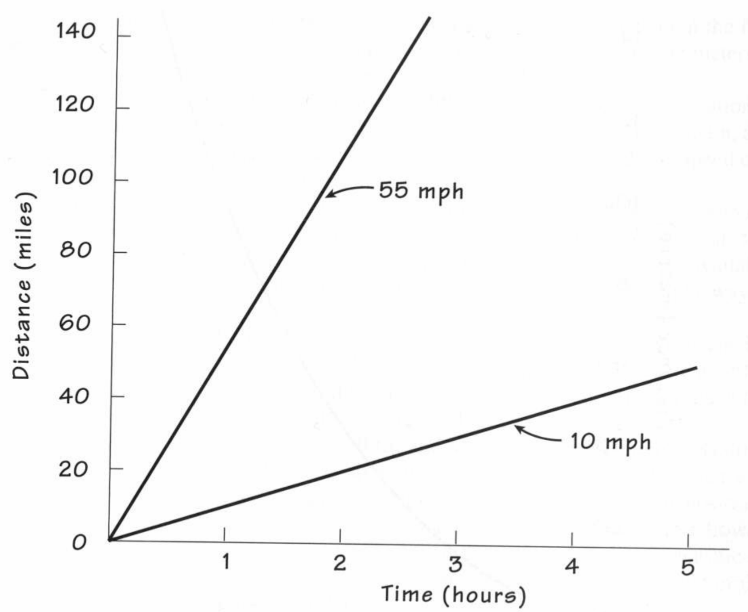


FIGURE 1.2. Graphs of motion at constant speed.

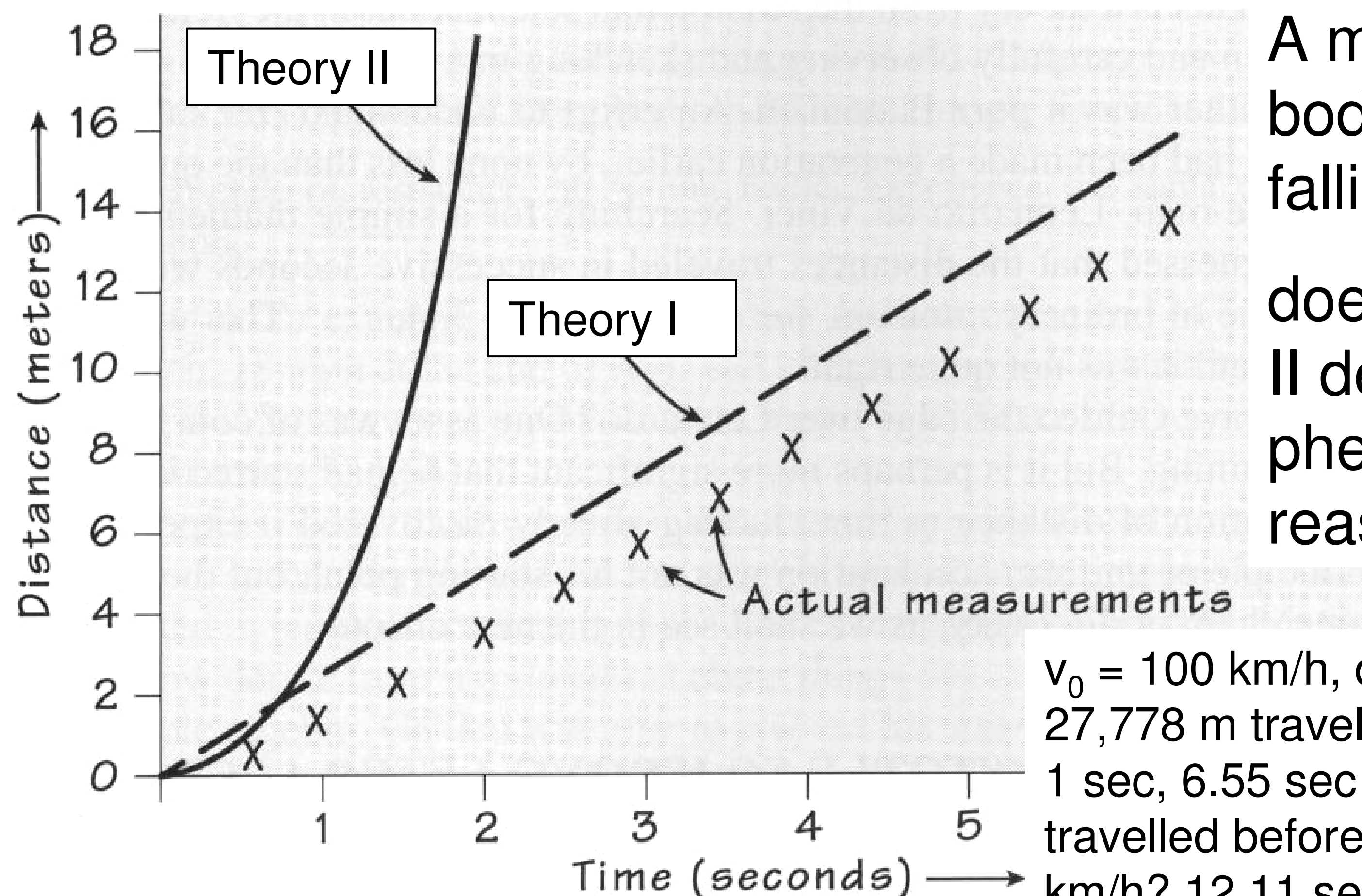
One may extrapolate from know behavior at the length scale of human beings to some micro- or nano-scopic length scale, this is known as scaling

Linear or near linear relationships are easiest for human beings to comprehend

The scaling may not be linear, the phenomenon may be complex,

Rigid body behavior at the tens and hundreds of nanometer scale may be explained by classical physics concepts, human beings have particular problems with exponential scaling

How about Galileo's statement: all bodies fall at the same rate …?



A macroscopic body freely falling in air,

does theory I or II describe the phenomenon reasonably well?

v_0 = 100 km/h, deceleration: 5 m/s^2: 27,778 m traveled in reaction time 1 sec, 6.55 sec total and **105 m** travelled before stop, what if 200 km/h? 12.11 sec but **364.2 m**

$$v = v_0 - at \qquad s = \frac{-at^2}{2} + v_0 t$$

Physical reality is often the superposition of more than one phenomenon, some of these phenomena may scale differently, classical physics typically looks for an isolated system, tries to neglect other influences if they are not too strong

At the small scale, forces such as friction and surface tension often dominate over forces such as gravity, electrostatic forces may become strong, …

The force F needed to move a sphere of mass m, density ρ, radius R at a velocity v through a viscous medium of viscosity η (Stoke's Law) is given by:

Kind of "friction" $$F = 6\pi\eta R v \quad (2.12)$$

Terminal velocity when "friction force" is balanced by force of gravity

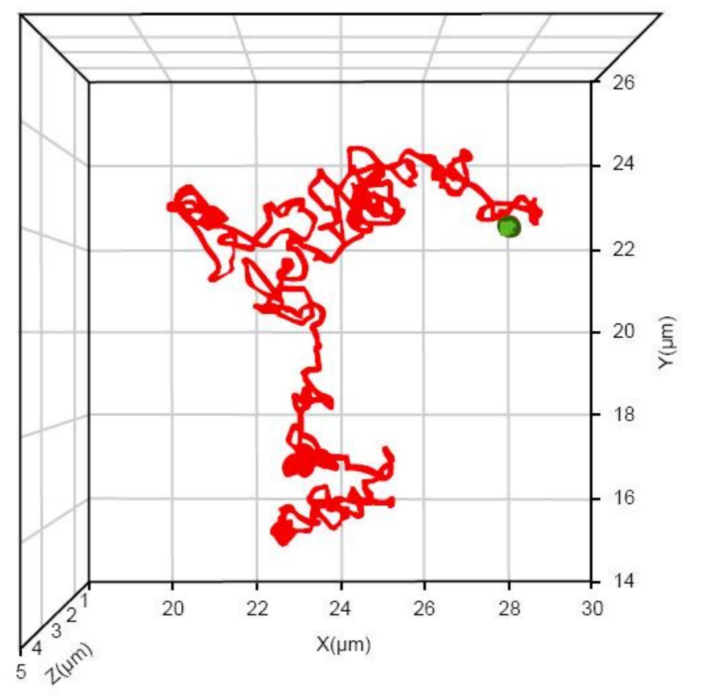
$$v_t = \frac{mg}{6\pi\eta R} = \frac{\frac{4}{3}\pi R^3 \rho g}{6\pi\eta R} = \frac{2\rho g R^2}{9\eta} \propto R^2$$

Small particles will fall much more slowly !! There will be additional effects !

This applies for reasonably laminar flow, Reynolds number < 2000 $Re = \frac{2R\rho v}{\eta} = \frac{Inertial \cdot forces\ (\rho v)}{Viscous \cdot forces\ (\eta/2R)}$

1 mm radius sphere 7 g/cm³ (small steel ball baring) falling through water (η = 0.01 Pa s) reaches terminal velocity of about 1 m/s, if sphere has radius 1 μm, $v_t \approx$ 1 μm/s, if sphere has radius **1 nm, $v_t \approx$ 1 pm/s, so it takes 1,000 sec to fall by only one diameter**. In addition, there will be forces "jostling" the nano-particle around due to movement of water molecules, (Einstein's proof of the existence of molecules)

Sphere with 10 μm radius, density 7 g/cm³ falls in air (with v_t) about 8 cm/s, but a 15 nm radius sphere is predicted to fall in air only about 175 nm/s !!



1 nm diameter sphere 7 g/cm³ Brownian diffusion length about **9 μm, characteristic distance displaced every second** !!!, so force of gravity is negligible for this particle

Brownian diffusion force

For Re <<1, viscous forced dominate

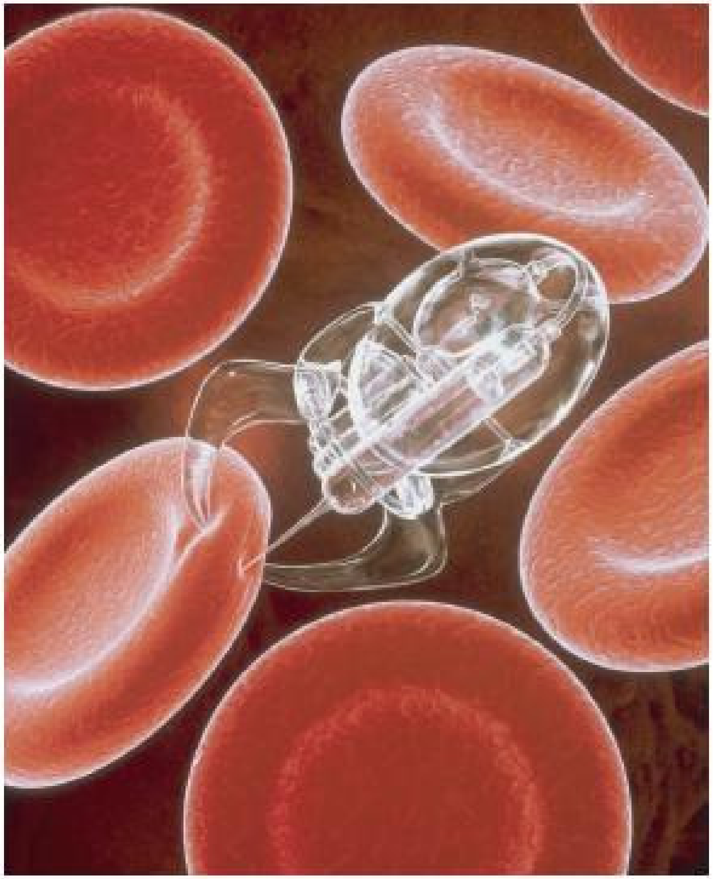
$$F_{ext} + F(t) - (6\pi\eta R)\frac{dx}{dt} = \left(\frac{4\pi R^3 \rho}{3}\right)\frac{d^2x}{dt^2} \quad (2.12)$$

might be m g

Mass of sphere times acceleration

Langevin equation

Nanobot ??? – Artist's Impression



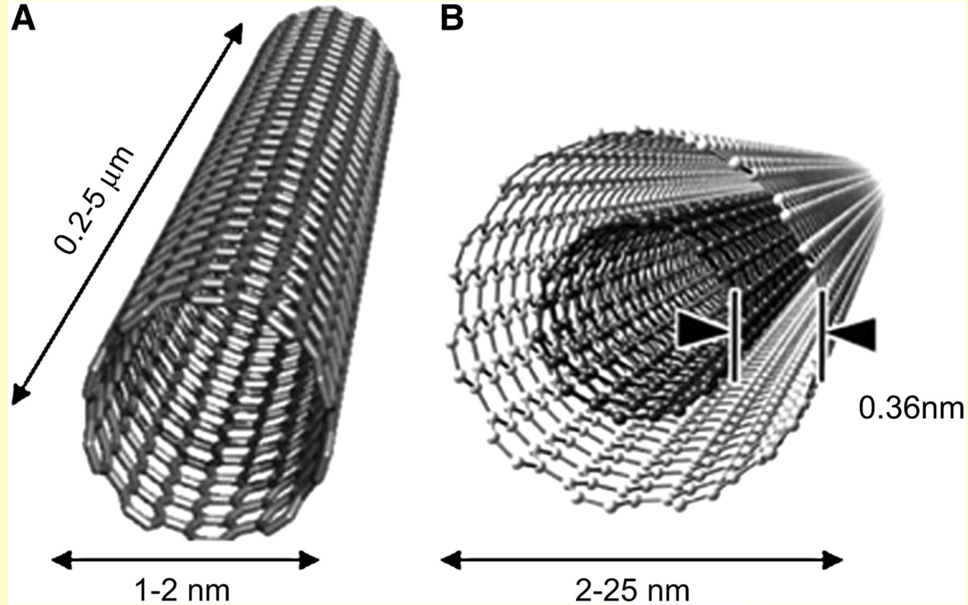
Artist's impression of a (very large) nanobot pinching a red blood cell (diameter 6 – 8 μm), note that micro-machines do scarcely exist,

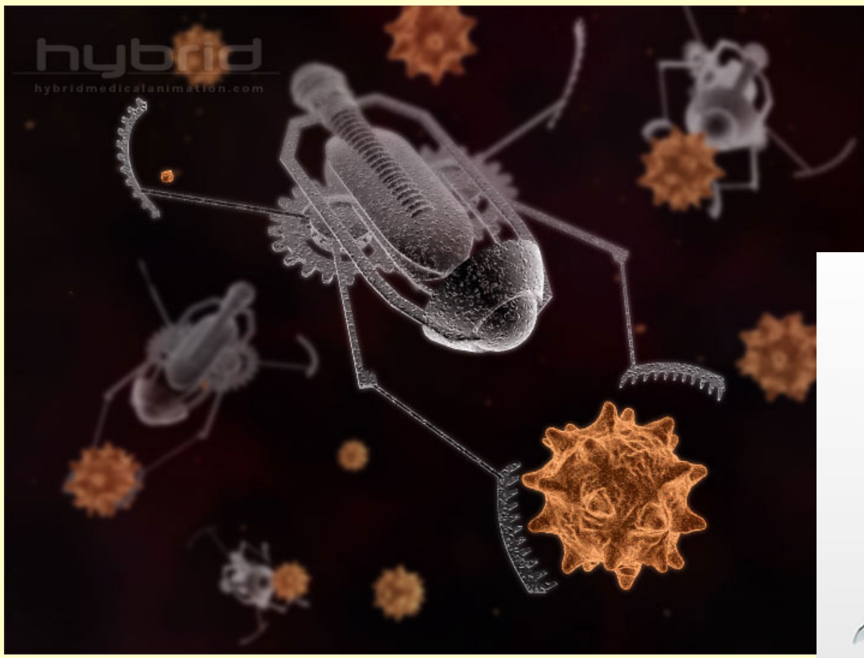
nano-machines and bots are truly science fiction, most importantly they may never look like some miniature version of the machines we are used to, …

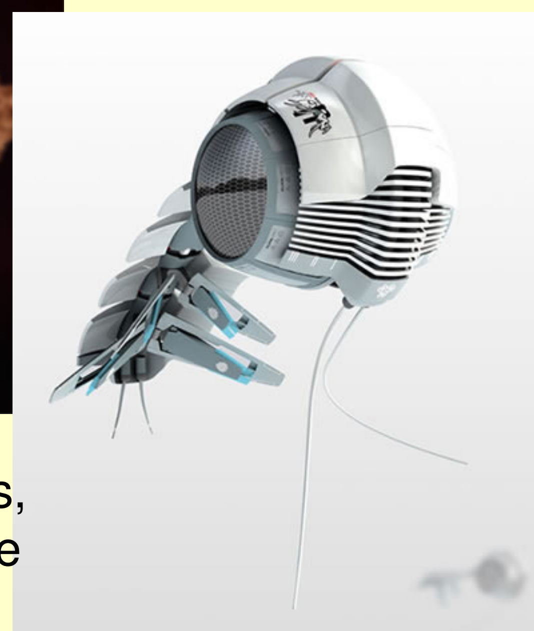
A bacterium swimming through water, viscosity 8.9 10^{-4} Pa s velocity 50 μm/s for 1 μm size bacterium gives Reynolds number 5.6 10^{-5} (a human would have Re number 10 orders of magnitude larger, so completely different regimes !!!, bacterium faces enormous "resistance by water", viscosity of blood 2.7 10^{-3} Pa s, so conditions are similar

There are also no airplanes with rigid wings smaller than about 1 mm, insects just fly differently, beating their wings

“Friction, resistance to movement” is generally very high at the nanoscale, (almost everything is pretty sticky), but can be small for certain high symmetry arrangements, e.g. the inner tubes in a multi-walled carbon nanotube



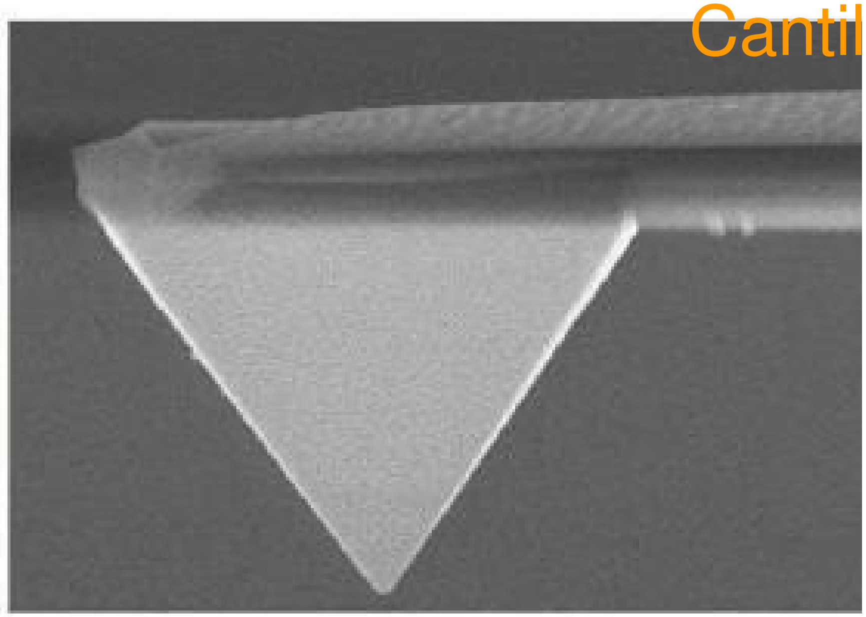




Sure nanobots will need moving parts, rotations about some axes, resistance to rotation is usually very high

Cantilevers

Cavenagh bridge, Singapore's oldest suspension (cantilever) bridge



SEM image of an AFM cantilever, resonance frequencies 10 – 200 kHz, force constants 0.01 – 100 N/m, forces in the nN range can be detected

Resonance frequency of a cantilever scales with L^{-2}, L = Length

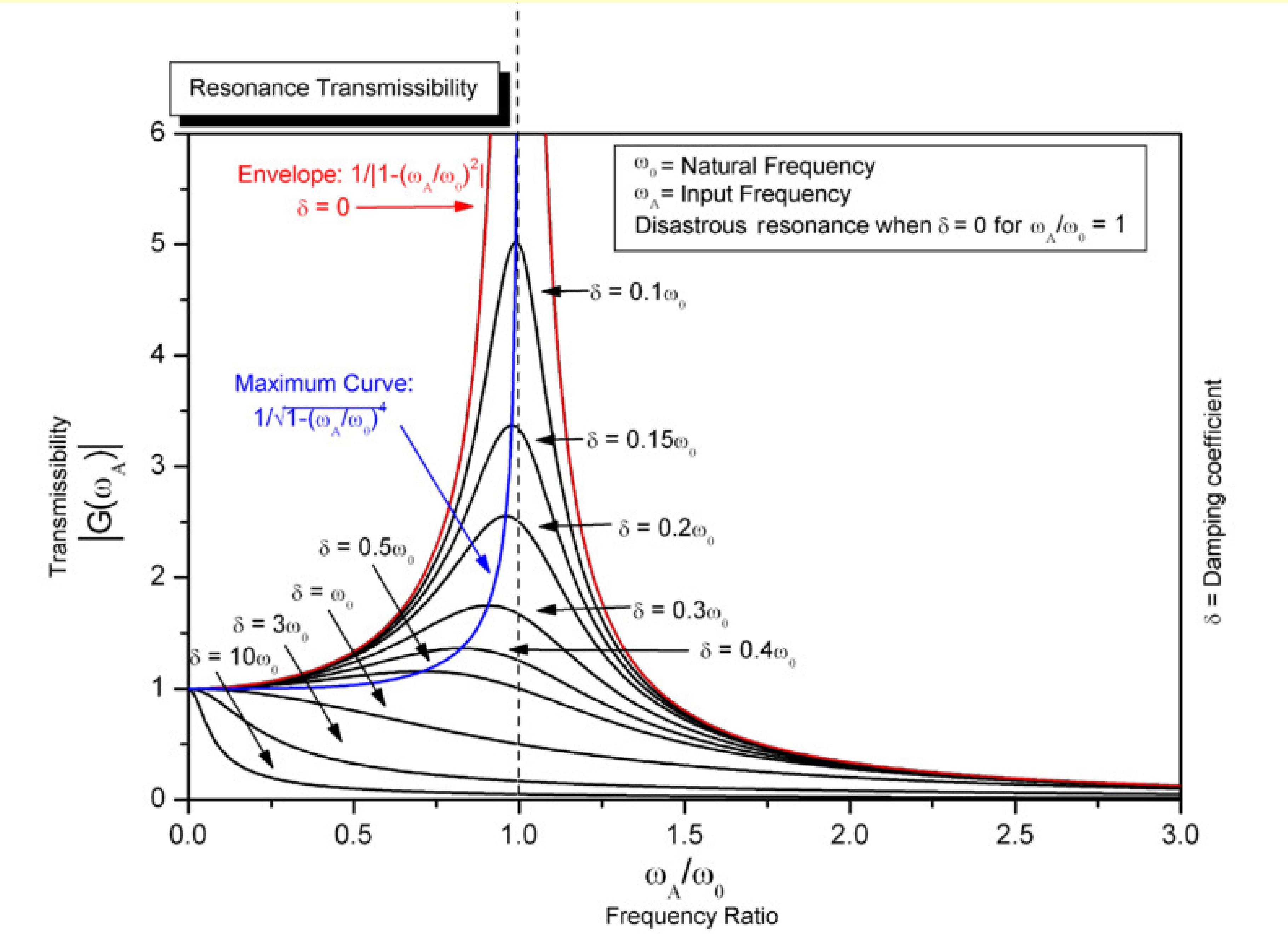
There are two basis modes for AFM, "contact" mode and non-contact mode. In contact mode (a few nm) the tip is very close to the sample and the interaction is repulsive (due to Pauli repulsion). In the not-contact mode, the tip is further away, some 100 nm, the interaction is attractive, the cantilever vibrates at a certain frequency, the change of this frequency due to the interaction is measured (instead of the bending of the cantilever.

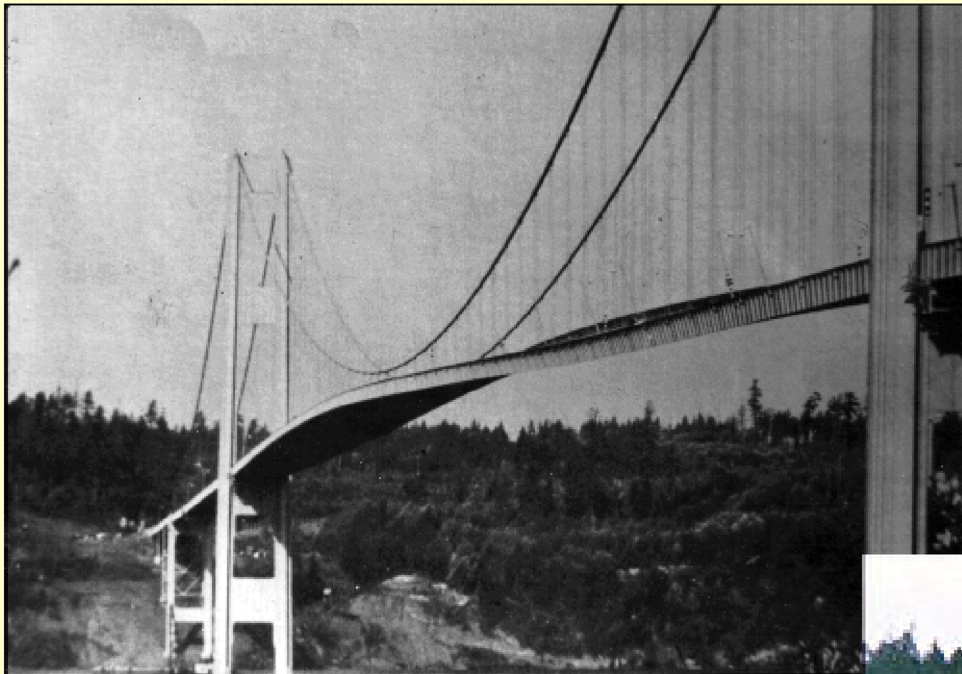
$$\omega = 2\pi \frac{0.56}{L^2} \left(\frac{Ywt^3}{12\rho A} \right)^{1/2}$$

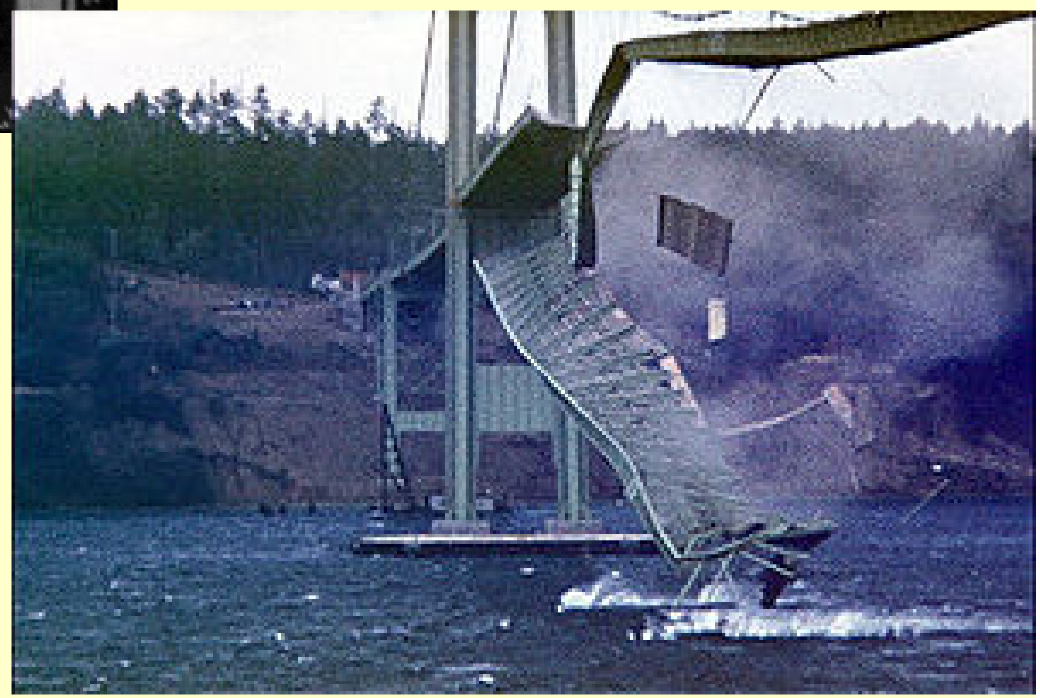
For a miniature diving board, i.e. some plank fixed on one side, vibrating perpendicular to the characteristic Length L

L: length of cantilever, Y: Young's modulus, kind of a rigidity measure of a material, Force per unit area (pressure stress) per fractional deformation (GPa), ρ mass density, A cross section area, *w* and *t* widths and thickness of the cantilever,

In the absence of damping, a mechanical system will absorb energy that is delivered with a frequency that matches is resonance frequency very readily


Resonance Transmissibility
Envelope: 1/|1-(ω_A/ω_0)^2|
δ = 0
ω_0 = Natural Frequency
ω_A = Input Frequency
Disastrous resonance when δ = 0 for ω_A/ω_0 = 1
δ = 0.1ω_0
δ = 0.15ω_0
δ = 0.2ω_0
δ = 0.3ω_0
δ = 0.4ω_0
δ = 0.5ω_0
δ = ω_0
δ = 3ω_0
δ = 10ω_0
Maximum Curve:
1/√1-(ω_A/ω_0)^4
Transmissibility
|G(ω_A)|
δ = Damping coefficient
ω_A/ω_0
Frequency Ratio
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
1
2
3
4
5
6



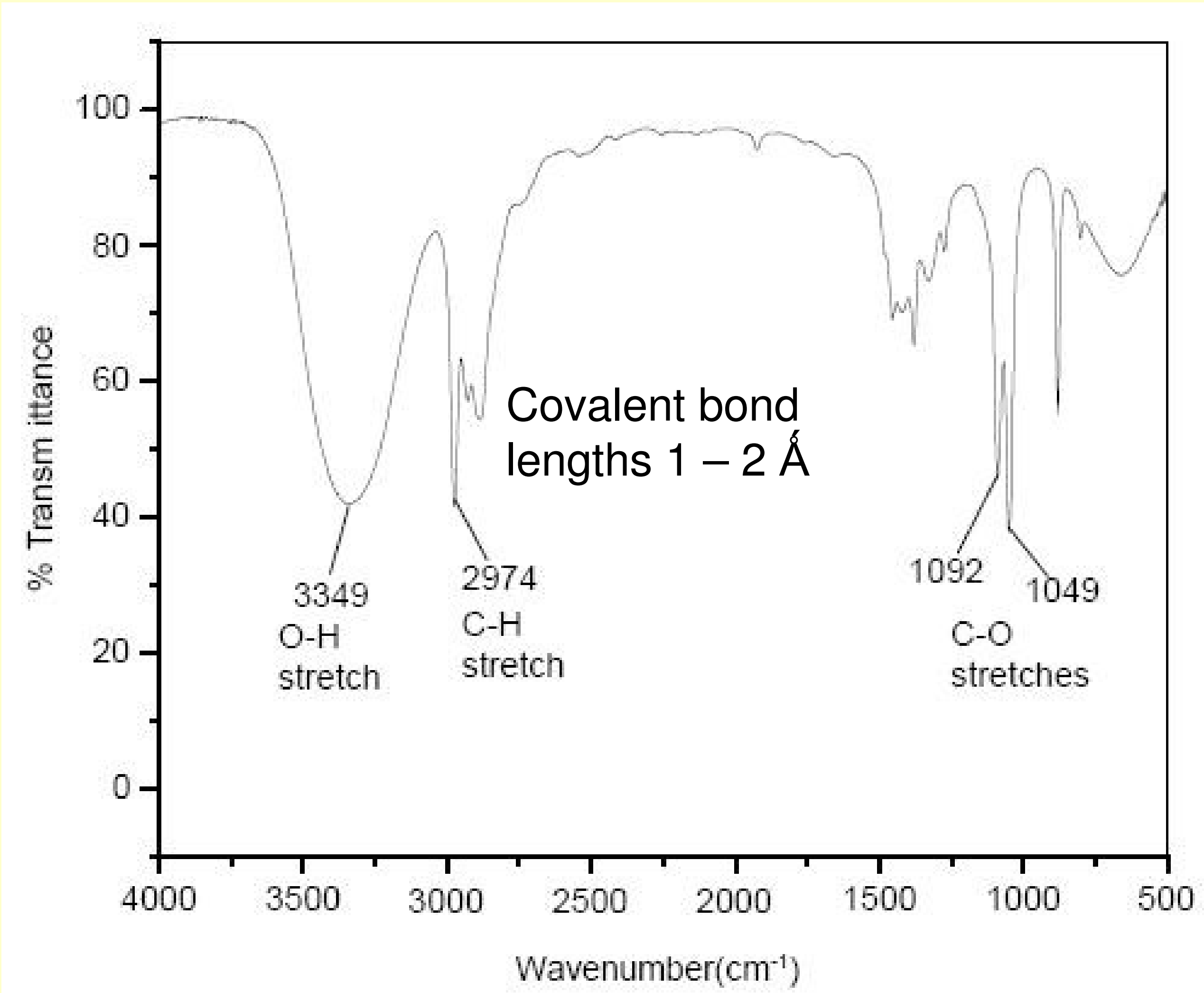


The original Tacoma Narrows Bridge roadway twisted and vibrated violently under 40-mile-per-hour (64 km/h) winds on the day of the collapse

http://en.wikipedia.org/wiki/File:Tacoma_Narrows_Bridge_destruction.ogg

Molecular Vibrations

10^{14} Hz, 100 THz, effective spring constants 2000 N/m



Covalent bond lengths 1 – 2 Å

(Wavenumber cm^{-1} = Hz/c)

FT-IR spectrum of ethanol molecules in liquid state, molecular fingerprinting as these vibrations are characteristic for each molecule

Another force that influences things at the nanometer length scale:

Casimir force effect of Quantum Electrodynamics

All magnetic and electric fields fluctuate due to Heisenberg's uncertainty principle, this is even so in the absence of electric or magnetic fields, i.e. B = 0 or E = 0, in a perfect vacuum

$$\Delta E \cdot \Delta t \approx h$$

so loosely speaking: below the threshold of the energy-time uncertainty relation, all kinds of "strange particles" (especially "virtual photons") pop in and out of existence

we know of their existence indirectly because the measured gyro- magnetic-ratio of the electron, g = 2.00232 is slightly larger than 2 (as predicted by Dirac loosely speaking in the absence of such effects)

this experimental value agrees very well with the prediction of quantum electrodynamics, i.e. the theory of interaction between electromagnetic radiation and matter (charged particles in particular) that also merges quantum mechanics with special relativity), another consequence is the Casimir force

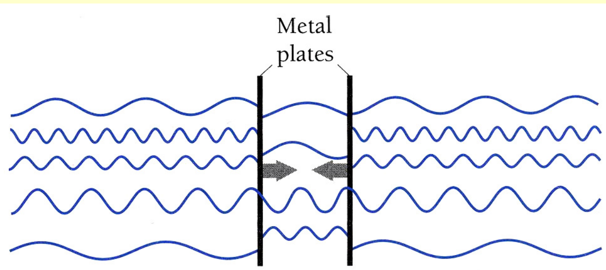


Figure 6.14 Two parallel metal plates exhibit the Casimir effect even in empty space. Virtual photons of any wavelength can strike the plates from the outside, but photons trapped between the plates can have only certain wavelengths. The resulting imbalance produces inward forces on the plates.

The energy density is about 0.5 nJ/m^2, for two metal plates at a distance of about 10 nm, this amounts to an extra pressure of about one atmosphere

A suitably designed cantilever of an atomic force microscope (AFM) can measure the Casimir force

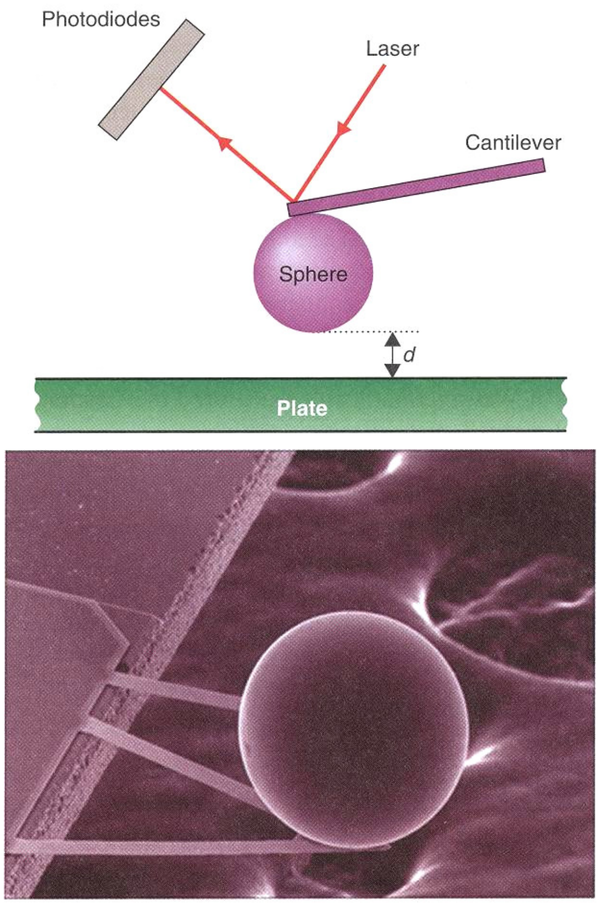


Fig. 8.4 AFM measurement of the Casimir force. The method for measuring the Casimir force with an AFM is shown schematically in the top figure. The AFM cantilever has a gold-coated sphere glued onto the end instead of having a sharp tip as normally used to scan a surface topography (see Chapter 4, Section 4.4.4). The deflection of the cantilever is determined in the usual way, that is, a laser beam is reflected from the back of a cantilever and the deflection of the reflected beam measured by a position sensitive detector. The bottom picture shows the 200 µm diameter gold-coated polystyrene sphere and cantilever used in the Mohideen experiment [7]. Reproduced with permission from the Institute of Physics pages *physicsweb* (http://physicsweb.org/articles/world/15/9/6).

There is no energy extracted from the vacuum or the surrounding, the Casimir force is just an attractive interaction that is always there and can be used to move one specific mechanical element by another without contact, since there is no contract, there is also no friction

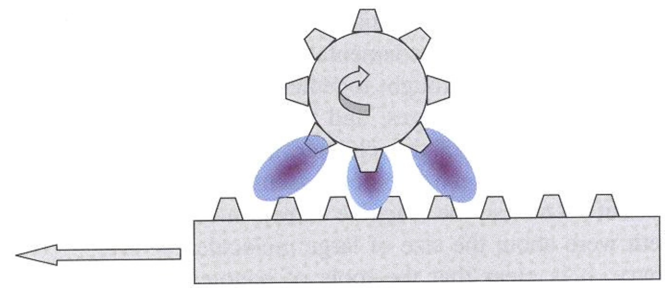


Fig. 8.7 Non-contact rack and pinion. The lateral Casimir force can be used as a method of transmitting force without contact as in this rack and pinion.

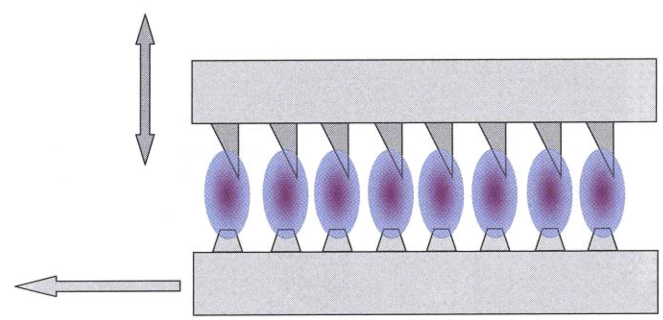


Fig. 8.8 Casimir ratchet. Using asymmetric teeth on one side, an oscillatory motion in a normal direction can produce a unidirectional motion of the other side.